

# Submarine Cable 4 — Inductive Effects

## Introduction

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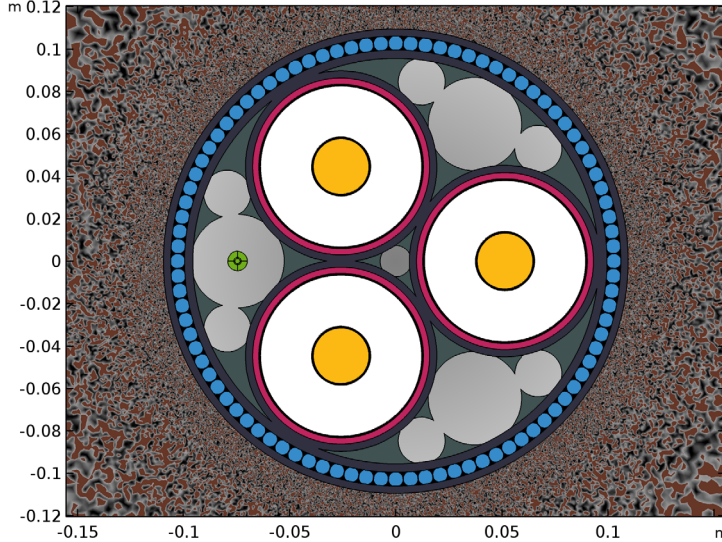
Results from the *Capacitive Effects* and *Bonding Capacitive* tutorials (the previous tutorials in this series), show there is only a weak coupling between the inductive and capacitive parts. Additionally, 3D twist models show that losses from in-plane eddy currents are comparatively small.

This justifies an inductive model that only includes out-of-plane currents (and consequently, in-plane magnetic fields). The model focuses on the importance of wire twist with respect to both phase conductors and armor, and the corresponding losses. Verification is included, results are compared to the cable's official specifications.

## Model Definition

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The model uses the same geometry as the *Capacitive Effects* model (see [Figure 1](#)). A large number of detailed material properties is included for metals, polymers, and the sea bed.



*Figure 1: The cable's cross section, including the three phases (yellow), screens (red), the XLPE (white), the armor (blue), and the fiber (green).*

### THEORETICAL BASIS

The model solves Maxwell–Ampère's law in 2D, using the out-of-plane magnetic vector potential  $\mathbf{A}$  as a dependent variable. When solving, all Maxwell's equations are either directly or indirectly involved, together with three *constitutive relations* (the ones containing the material properties  $\mu$ ,  $\epsilon$ , and  $\sigma$ ). In the following derivation, we use the *differential form*, together with the *SI unit system*.

Let's start with the notion, that when current is not conserved, we get a build-up of charge. This is given by  $\nabla \cdot \mathbf{J} = -\partial\rho/\partial t$  in the time domain, and  $\nabla \cdot \mathbf{J} = -j\omega\rho$  in the frequency domain. You can combine this with Gauss's law;  $\nabla \cdot \mathbf{D} = \rho$ , to get an adjusted current conservation law:

$$\nabla \cdot (\mathbf{J} + j\omega\mathbf{D}) = 0. \quad (1)$$

For the time domain, this would be  $\nabla \cdot (\mathbf{J} + \partial\mathbf{D}/\partial t) = 0$ , or, when using the electric field:

$$\nabla \cdot (\mathbf{J} + \epsilon\partial\mathbf{E}/\partial t) = 0, \quad (2)$$

where we have used our first constitutive relation;  $\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E} = \epsilon\mathbf{E}$ .

What this result tells us is that a time varying electric (displacement) field, is just another kind of current; the *displacement current*. This seems reasonable. After all, if you picture a charging capacitor, you will see currents flowing from the terminals to the capacitor plates, but not from one plate to the other. If you stick to the convection current is conserved at all times, the increasing electric field between the plates must be another kind of current.

You might have noticed this in the *Capacitive Effects* tutorial, where we have plotted both the displacement and the conduction current density: In the insulators the displacement current density is prominent, in the conductors the conduction current density is prominent, and the sum of both is conserved at all times. This is why some textbooks include the displacement current in the definition of the current density:

$$\mathbf{J}' = \mathbf{J} + j\omega\mathbf{D} = \sigma\mathbf{E} + j\omega\epsilon\mathbf{E} = (\sigma + j\omega\epsilon)\mathbf{E}, \quad (3)$$

where we have used our second constitutive relation;  $\mathbf{J} = \sigma\mathbf{E}$ . This relation suggests  $\omega\epsilon$  is some sort of imaginary "conductivity"; one that doesn't involve losses (in the time domain, this is less obvious). It also explains why some people tend to use a complex permittivity in order to model resistive effects.

The second part of our derivation starts with Maxwell–Ampère's law;  $\nabla \times \mathbf{H} = \mathbf{J}'$ , which basically states there is a direct relation between the magnetic field  $\mathbf{H}$  that encircles a

conductor and the current that runs through it<sup>1</sup> (in fact, the definition of the Ampère is based on this). If we take the *divergence* of that, we get something very convenient:

$$\nabla \cdot (\nabla \times \mathbf{H}) = \nabla \cdot \mathbf{J}' = \nabla \cdot (\mathbf{J} + j\omega\mathbf{D}) = 0. \quad (4)$$

This is true by definition; from basic vector calculus follows that the divergence of the curl of *any* vector field must always be zero. In other words, if we define the currents to be equal to the curl of  $\mathbf{H}$ , we get a current conservation law for free (that is, there is no additional equation required to enforce this). We have now applied Gauss's law and Maxwell–Ampère's law. If you add the third constitutive relation<sup>2</sup>;  $\mathbf{B} = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H}$ , you'll end up with the following:

$$\nabla \times \mathbf{H} = \nabla \times (\mu^{-1}\mathbf{B}) = \mathbf{J}' = (\sigma + j\omega\epsilon)\mathbf{E}. \quad (5)$$

Next, we introduce the concept of a vector field  $\mathbf{A}$ , the *magnetic vector potential* (with SI unit Vs/m), who's curl happens to be equal to the magnetic flux density  $\mathbf{B}$ . That is;  $\nabla \times \mathbf{A} = \mathbf{B}$ . The magnetic vector potential is not a directly measurable field or anything, nor is it unique; without any additional constraints there are many fields  $\mathbf{A}$  that fulfill the requirement<sup>3</sup>  $\nabla \times \mathbf{A} = \mathbf{B}$ .

This is similar to the electric scalar potential  $V$ . Even in electrostatic conditions, there is no such thing as an absolute (unique) electric potential: The potential is always measured with respect to some arbitrary reference (ground). What matters is the potential *difference*, or the potential *gradient*. The same goes for the magnetic vector potential. Using the relation between  $\mathbf{A}$  and  $\mathbf{B}$ , and taking the divergence once again, we get:

$$\nabla \cdot (\nabla \times \mathbf{A}) = \nabla \cdot \mathbf{B} = 0. \quad (6)$$

That is, if we define  $\mathbf{B}$  in terms of  $\mathbf{A}$ ;  $\mathbf{B} = \nabla \times \mathbf{A}$ , we get magnetic flux conservation (Gauss's law for magnetism) for free. This is for the exact same reason we got current conservation for free. If you now substitute this new definition of  $\mathbf{B}$  in Maxwell–Faraday's equation of electromagnetic induction;  $\nabla \times \mathbf{E} = -j\omega\mathbf{B}$ , you'll get:

$$\nabla \times \mathbf{E} = -j\omega(\nabla \times \mathbf{A}) = \nabla \times (-j\omega\mathbf{A}). \quad (7)$$

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1. Notice that initially, James Clerk Maxwell did not include the displacement currents, leading to a law that is valid under stationary conditions only (known as Ampère's circuital law). He added the displacement currents a couple of years later — at equation (112) in his 1861 paper On Physical Lines of Force — resulting in what we now refer to as "Maxwell–Ampère's law".

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2. Since this model runs in the frequency domain, we are restricted to linear material properties. It's possible to approximate the effect of a nonlinear material though; using effective nonlinear magnetic curves. For more on this, check the AC/DC Module users guide.

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3. This phenomenon leads to *gauge freedom*. For more info, see the AC/DC Module users guide.

This gives you the last piece of the puzzle<sup>4</sup>:  $\mathbf{E} = -j\omega\mathbf{A}$ . Now, both  $\mathbf{B}$  and  $\mathbf{E}$  can be expressed in terms of  $\mathbf{A}$ . If you substitute this result in Equation 5, you will find:

$$\nabla \times (\mu^{-1}\nabla \times \mathbf{A}) = (\sigma + j\omega\epsilon)(-j\omega\mathbf{A}). \quad (8)$$

Finally, if you swap about some terms and put everything on the left hand side, you'll get the following *2D partial differential equation* for the dependent variable  $\mathbf{A}$ :

$$-\omega^2 \epsilon\mathbf{A} + j\omega\sigma\mathbf{A} + \nabla \times (\mu^{-1}\nabla \times \mathbf{A}) = 0. \quad (9)$$

Due to the double curl, this is known as a curl-curl-type equation. The **Magnetic Fields** interface uses this equation in the domains to determine the value of  $\mathbf{A}$  (and consequently, the value of all the fields derived from it;  $\mathbf{E}$ ,  $\mathbf{D}$ ,  $\mathbf{J}$ ,  $\mathbf{H}$ ,  $\mathbf{B}$ ).

Notice that the direction of  $\mathbf{A}$  is the same as the direction of  $\mathbf{E}$  (and  $\mathbf{J}$ , assuming isotropic material properties). If we choose to include the out-of-plane component of the magnetic vector potential only (setting the other components equal to zero), we will be neglecting in-plane currents and electric fields (from what we've seen so far, this seems justified). Consequently, the 2D model treated here will only produce in-plane magnetic fields. Omitting the in-plane components of  $\mathbf{A}$  significantly simplifies and stabilizes the problem.

For the outer boundaries, the default condition  $\mathbf{n} \times \mathbf{A} = 0$  is used, which sets  $\mathbf{A}$  in the direction of the surface normal. Therefore,  $\mathbf{B}$  will be perpendicular to the surface normal; the magnetic flux lines will flow along the surface. This is known as a *magnetic insulation* condition, in analogy to electric or thermal insulation. Since the surface normal of a boundary in a 2D model is in-plane, and since we chose the magnetic vector potential to be out-of-plane only, in effect the value of  $\mathbf{A}$  is zero on all outer borders. This gives you a complete set of equations and therefore, a unique solution.

In order to excite the system, an external electric field  $\mathbf{E}_{\text{ext}}$  is applied in the cable's central conductors. This field comes from an outside source not included in the model, presumably a power plant or a wind turbine.

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4. Notice that, with the electric field being defined solely in terms of the magnetic vector potential, we have been excluding the electric scalar potential, that is;  $V = 0$ . This choice is known as the *Weyl* or *Hamiltonian* gauge. For more information about different gauges and *gauge fixing*, see the AC/DC Module users guide.

## MODELING APPROACH

The tutorial starts with the basics; just by exciting a current in the phases (by means of a **Coil** feature). As a result, strong eddy currents start to flow in the screens and the armor. This configuration is effectively the same as solid bonding; the preferred choice for submarine cable systems<sup>5</sup>.

In an attempt to reduce these losses, the effect of armor twisting is included in the model by means of a **Coil group**. In a third step, the coil features used to excite the phase currents, are set to **Homogenized multi-turn**. This allows for investigating the case where the current in the central conductors is evenly distributed (due to the use of twisted, insulated strands).

The inductance per phase in mH/km is compared to the cable's official specifications. For the AC resistance per phase, in mΩ/km, the DC resistance is used as a frame of reference:

$$R_{dc} = \frac{1}{\sigma_{cu}A}, \quad (10)$$

where  $\sigma_{cu}$  and  $A$  refer to the copper conductivity and the total effective cross sectional surface area of a phase respectively. The DC resistance serves as a lower bound for the AC resistance; it represents a condition where all losses due to inductive effects have been successfully eliminated.

## ON ARMOR TWIST

Without any precautions, the total net out-of-plane current per armor wire will be quite large (even for a well-balanced cable). This is why in practice the armor is twisted, with a different pitch than the central conductors have. Consequently, every armor wire will sense every side of the cable at some point along its length (cyclic even). The first armor wire shown in the cross section ([Figure 1](#)), will be the second one a bit farther down the cable; they are indistinguishable.

Therefore, all wires will have to carry the same total induced current. In a well-balanced cable, this current will be zero. The length of the cable cannot be used as an argument to expect variations in current here, as the cable is much shorter than the wavelength:

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}}, \quad (11)$$

where  $L$  and  $C$  refer to the cable's inductance and capacitance per unit length. The wavelength is about  $2.6 \cdot 10^3$  km, the cable is assumed to be 10 km.

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5. For more information on bonding types, see the *Bonding Capacitive* and *Bonding Inductive* tutorials.

## ON COIL DOMAINS

The **Coil** feature plays a central role in this model, it's one of the most important features included in the **Magnetic Fields** interface (or the AC/DC Module for that matter). The feature can be used as an active or passive element; it allows you to excite the system and to determine lumped parameters like inductance or resistance. Furthermore, it allows you to connect active or passive elements in your model to one another, to an external circuit, or to other models. The feature supports two common conductor models; **Single conductor** and **Homogenized multi-turn**.

### *Single Conductor Model*

In case of the single conductor model, the domain behaves like a single (solid) conductor; currents are free to flow as dictated by Maxwell–Ampère's law. Excitation is done by means of an external electric field  $\mathbf{E}_{\text{ext}}$ . When the conductor is large enough with respect to the skin depth, skin and proximity effects will appear. This is natural behavior.

The skin and proximity effects will redistribute the cross sectional current density; the current accumulates at the surface. This current accumulation leads to an increase in losses. After all, part of the conductor isn't optimally used. This is why in practice, most coils use insulated wires (turns) that are *electrically thin* (thin with respect to the skin depth). In order to be able to carry enough current, not the thickness of the turn is increased, but the amount of turns (leading to the term multi-turn). Moreover, the turns are placed in series<sup>6</sup>, so each turn will carry the same amount of current. With the turns being too thin to be *electrically visible*, a seemingly homogeneous current distribution is achieved in a predefined direction (the wire direction).

### *Homogenized Multi-Turn Model*

With the homogenized multi-turn setting, the coil domain models a bundle of turns (multi-turn coil) or strands (Litz wire), as an *effective material* with strongly anisotropic electrical properties and a homogenized current distribution: No current will flow unless the coil is connected to something, or short circuited. The obvious advantage, is that the individual wires do not have to be resolved by geometry or mesh. In both cases (single conductor and homogenized multi-turn), lumped parameters are determined by integrating current densities and electric fields in the proper directions. The resulting currents and voltages are compared to derive properties like impedance, inductance, resistance, and more.

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6. Note that Litz wires form an exception. In case of a Litz wire, the different strands (or turns) are placed in parallel instead of series. A similar current for each strand is achieved by twisting the strands in such a fashion that each individual strand occupies every part of the total cross section at some point along the wire.

### Coil Groups

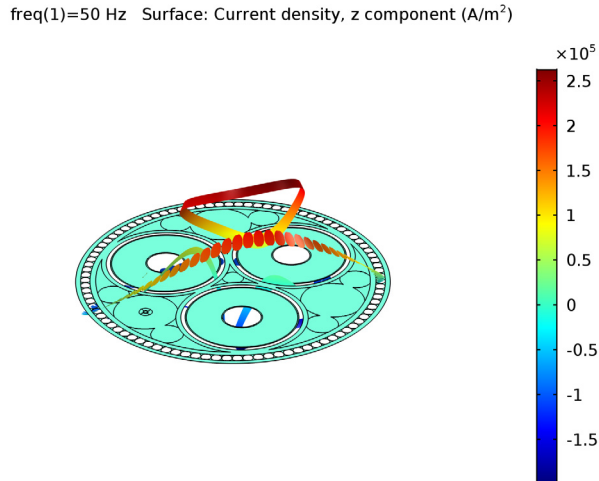
The **Coil group** option is specifically for 2D. When enabled, the coil feature considers every connected cluster of domains in the feature selection a different instance of the same entity crossing the 2D plane multiple times. Say, you have two separate circles in your 2D model. If you add both to the selection of your coil domain with the coil group setting enabled, what you're modeling is the same coil that passes your 2D plane twice. An obvious example is a 2D axisymmetric model with a whole lot of circles representing the same helix as it revolves around the axis a number of times.

In practice, this means the different domains are put in series. More advanced functionality is available in order to create combinations of parallel and series connections and to reverse the current in certain domains (for more details, check the AC/DC module users guide).

### Results and Discussion

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Initially, the losses are rather high: 9.7 kW/km, 10 kW/km, and 47 kW/km for the armor, screens, and phases respectively. The currents induced in the armor and the screens are free to flow; the total net current per screen and per armor wire oscillates way above and below the zero point, see [Figure 2](#).

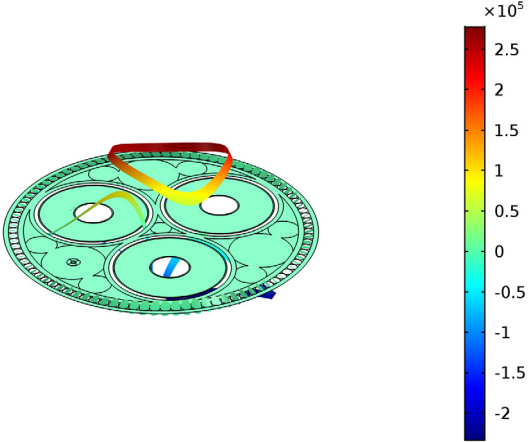


*Figure 2: The real part of the out-of-plane current density in the screens and armor at phase  $\varphi = 0^\circ$ . The animated version is available as [Ref. 2](#).*



When the armor twist is applied, the armor currents are suppressed (see [Figure 3](#)). The total losses go down by about 11%. This results in a loss of 8.8 W/km, 12 kW/km, and 47 kW/km for the armor, screens and phases respectively.

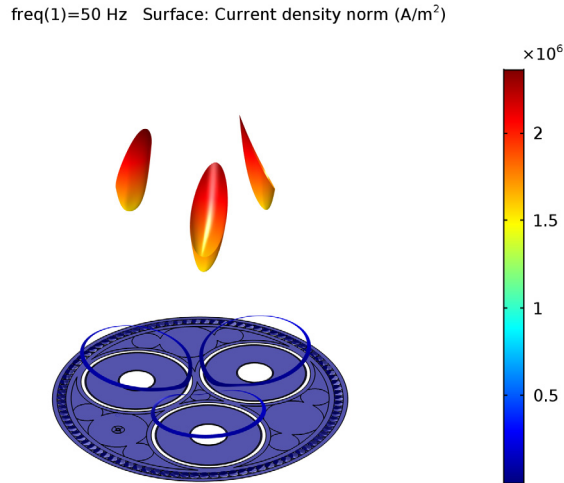
freq(1)=50 Hz Surface: Current density, z component (A/m<sup>2</sup>)



*Figure 3: The real part of the out-of-plane current density at phase  $\varphi = 0^\circ$ , with armor twist applied. The animated version is available as [Ref. 4](#).*

Interestingly, the phase and screen losses went up (about 160 W and 2.1 kW respectively). This waterbed effect is related to a redistribution of the magnetic energy, as a response to the new current distribution; the fields find a new energetic optimum.

As long as the coil domains representing the phases are set to **Single conductor**, there is a skin effect and a proximity effect with a stronger current density near the cable's center, see [Figure 4](#).



*Figure 4: The current density norm (phase independent), with Single conductor setting applied in the phases.*

When the phases are set to **Homogenized multi-turn** however, their current distribution is homogenized. As a consequence, phase losses go down significantly, to 43 kW/km, whereas armor and screen losses go up slightly, to 9.1 W/km and 12 kW/km respectively.

When comparing the model to the cable's official specifications, for the inductance we get a good match: 0.42 mH/km. Finally, we have 43 mΩ/km for the AC resistance and 34 mΩ/km for the DC resistance, with an AC/DC resistance ratio<sup>7</sup>  $\eta$ , of about 1.3.

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<sup>7</sup> Together with the analytically determined DC resistance, this ratio is used in the *Thermal Effects* tutorial to provide an approximate value for the AC resistance.

## ON ACCURACY

The results look very promising. We should not forget however, this 2D model is an approximation. To start with, losses due to in-plane currents have been neglected altogether. In a perfectly straight cable these would be zero, but real cables are twisted. The exact consequences of this are difficult to quantify, but 3D twist models supply some additional insight; depending on the twist angle, you may expect at least 5-10% additional losses.

Secondly, the homogenized current distribution in the phase should be taken with a grain of salt. Putting insulation between the strands makes them thinner, twisting them makes them longer: inductive losses may go down, but the DC resistance will go up. In practice, the used material will be somewhere in-between a solid conductor and a perfectly stranded, twisted one. Lastly, we haven't had a look at thermal effects yet. This topic will be treated in the *Thermal Effects* tutorial.

## References

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1. Video file submarine\_cable\_i\_free\_eddy\_01, available for download at <https://www.comsol.com/model/43431>.
2. Video file submarine\_cable\_j\_free\_eddy\_02, available for download at <https://www.comsol.com/model/43431>.
3. Video file submarine\_cable\_k\_armor\_twist\_01, available for download at <https://www.comsol.com/model/43431>.
4. Video file submarine\_cable\_l\_armor\_twist\_02, available for download at <https://www.comsol.com/model/43431>.

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**Application Library path:** ACDC\_Module/Tutorials,\_Cables/  
submarine\_cable\_04\_inductive\_effects

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## Modeling Instructions

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### ROOT

As this tutorial will focus on inductive effects, geometry, materials, and mesh have been prepared in the file submarine\_cable\_01\_introduction.mph. You can start by opening this file (*and saving it under a different name*).

- 1 From the **File** menu, choose **Open**.
- 2 Browse to the model's Application Libraries folder and double-click the file `submarine_cable_01_introduction.mph`.

## GLOBAL DEFINITIONS

Some **Parameters** have been prepared for running the model, and verifying afterwards.

### *Parameters*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `submarine_cable_c_elec_parameters.txt`.

The added parameters are the same as the ones added in the *Capacitive Effects* tutorial.  $f_0$ ,  $V_0$  and  $I_0$  are pretty straightforward, where  $1/\sqrt{3}$  and  $\sqrt{2}$  convert from phase-to-phase to phase-to-ground, and from root mean square (RMS) to peak value respectively.  $S_{con}$ ,  $S_{pbs}$  are material properties, and  $R_{con}$ ,  $R_{pbs}$  are analytically determined DC resistances per phase. They are given by [Equation 10](#), and it's screen equivalent.

The AC resistance will have to be determined by solving the model. Due to eddy currents, the AC resistance will be higher than the DC one; the DC resistance serves as a lower bound. Consequently, the ratio between the two  $\eta$  shows how successful losses due to inductive effects have been eliminated.

The last three;  $Ex_{lpe}$ ,  $C_{pha}$ , and  $I_{cpha}$ , are related to capacitive effects. They have been discussed and investigated in the previous tutorials of this series. Now that the parameters are in place, let's start by adding the physics and a study.

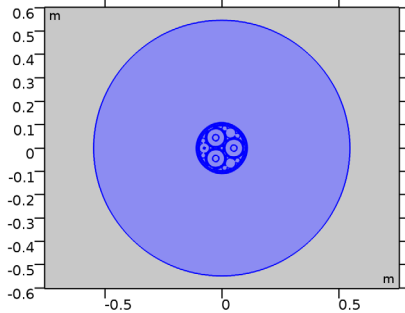
## ADD PHYSICS

- 1 On the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.
- 2 Go to the **Add Physics** window.
- 3 In the tree, select **AC/DC>Magnetic Fields (mf)**.
- 4 Click **Add to Component** in the window toolbar.

## MAGNETIC FIELDS (MF)

- 1 On the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.
- 2 In the **Settings** window for **Magnetic Fields**, locate the **Domain Selection** section.

- 3 From the **Selection** list, choose **Electromagnetic domains**.
- 4 Click the **Zoom to Selection** button on the **Graphics** toolbar.



### ADD STUDY

- 1 On the **Home** toolbar, click **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies**.
- 4 In the **Select Study** tree, select **Preset Studies>Frequency Domain**.
- 5 Click **Add Study**.
- 6 On the **Home** toolbar, click **Add Study** to close the **Add Study** window.

### STUDY 1

#### *Step 1: Frequency Domain*

- 1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 2 In the **Frequencies** text field, type  $f_0$ .

### MATERIALS

Now, you'll see COMSOL starts detecting missing material properties. The properties that should be added are listed in the following table. Please check all of them for the correct value, even the ones that are already filled in. *A quick option is to copy-paste the values directly from this \*.pdf file to COMSOL.*

I In the **Model Builder** window, under **Component 1 (comp1)>Materials**, check the following properties:

Material	sigma	epsilon <sub>r</sub>	mur
Air	1e-14 [S/m]	1	1
Sandy soil, saturated	1 [S/m]	28	1
Polyethylene	1e-18 [S/m]	2.25	1
Polypropylene	1e-18 [S/m]	2.36	1
Semi-conductive compound	2 [S/m]	2.25	1
Cross-linked polyethylene (XLPE)	1e-18 [S/m]	Ex1pe	1
Silica glass	1e-14 [S/m]	2.09	1
Copper	Ncon*Scon	1	1
Pb - Lead	Spbs	1	1
High-strength alloy steel	4.032e6 [S/m]	1	1

Notice that in the end, many of these properties will be either ignored, overridden or proven to be insignificant. This is similar to what happens in the *Capacitive Effects* tutorial, although here, it is the conductors that will prevail.

Furthermore, for the copper in the central conductors the parameter *Ncon* is used. *Ncon* is the ratio between the conductor's true cross sectional surface area *Acon*, and the cross section used in the geometry;  $\pi * (Dcon/2)^2$ . This ratio is below unity since these conductors are not actually solid but consist of a group of compacted strands, with some insulation or gaps in between.

Instead of modeling the strands individually, we choose to have an *effective material* to replace both the copper and the gaps, having a conductivity of *Ncon* times the conductivity of copper. Notice that this does not apply to coil domains using the **Homogenized multi-turn** setting, as those have their own settings for the conductivity and the cross-sectional surface area. We will get back to that later.

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### *Modeling Instructions — Solid Bonding*

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#### **MAGNETIC FIELDS (MF)**

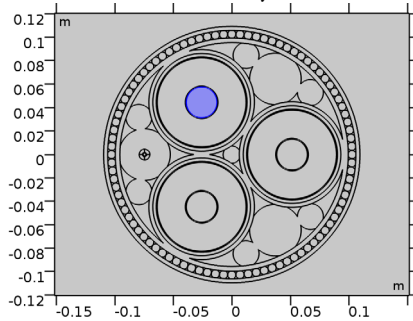
Now that the materials have been set and double-checked, let's have a look at the physics. With the default settings applied (out-of-plane vector potential only), the Magnetic Fields interface assumes zero in-plane currents. When it comes to charging currents, this approach is justified (as seen in the *Capacitive Effects* tutorial). In-plane eddy currents are

larger however (as seen in 3D twist models), but still small enough to make this model a good approximation.

Start by applying currents to the three main conductors and compute your first solution. To get a better view, **Zoom In** a couple of times.

#### Coil 1

- 1 On the **Physics** toolbar, click **Domains** and choose **Coil**.
- 2 In the **Settings** window for **Coil**, type Phase 1 in the **Label** text field.
- 3 Select Domain 236 only.



The **Coil** feature contains a lot of sections. For many of these, the default settings are sufficient. Collapse them to have a closer look at the important part; the **Coil** section.

- 4 Click to collapse the **Model inputs** section, the **Material type** section, the **Coordinate system selection** section, the **Conduction current** section, the **Magnetic field** section, and the **Electric field** section.

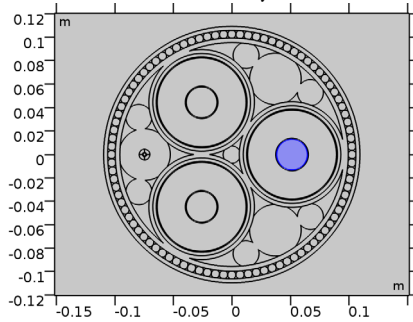
Next, proceed by setting the currents.

- 5 Locate the **Coil** section. In the  $I_{\text{coil}}$  text field, type I0.

#### Coil 2

- 1 On the **Physics** toolbar, click **Domains** and choose **Coil**.
- 2 In the **Settings** window for **Coil**, type Phase 2 in the **Label** text field.

3 Select Domain 253 only.



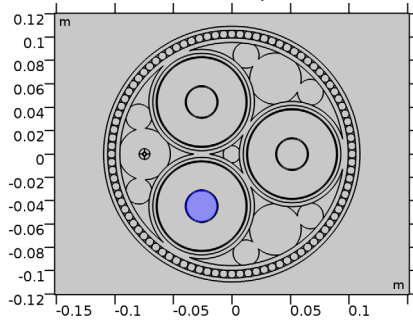
4 Locate the **Coil** section. In the  $I_{\text{coil}}$  text field, type  $I_0 \cdot \exp(-j \cdot 2 \cdot \pi / 3)$ .

*Coil 3*

1 On the **Physics** toolbar, click **Domains** and choose **Coil**.

2 In the **Settings** window for **Coil**, type Phase 3 in the **Label** text field.

3 Select Domain 235 only.



4 Locate the **Coil** section. In the  $I_{\text{coil}}$  text field, type  $I_0 \cdot \exp(+j \cdot 2 \cdot \pi / 3)$ .

Notice that, since we are in the frequency domain here, expressions like  $\exp(-j \cdot 2 \cdot \pi / 3)$  may be used to set a  $120^\circ$  phase shift between the AC currents on the three main conductors.

## STUDY I

On the **Home** toolbar, click **Compute**.

## RESULTS

*Magnetic Flux Density Norm (mf)*

The first thing to notice is that the **Magnetic Flux Density Norm** plot is zoomed-in quite a bit. This is because it is still locked to the camera settings used in the geometry and the



mesh. Let's give it separate view settings and remove the flux lines for now (we will get back to those later).

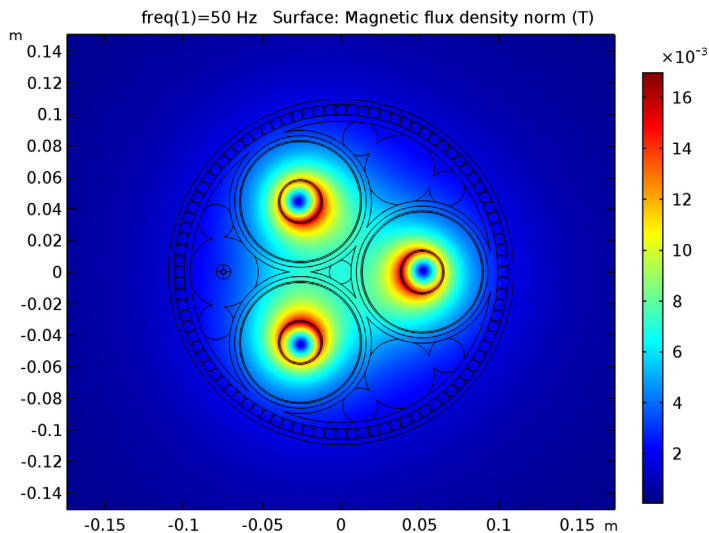
- 1 In the **Model Builder** window, under **Results** click **Magnetic Flux Density Norm (mf)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Plot Settings** section.
- 3 From the **View** list, choose **New view**.

#### *Contour 1*

In the **Model Builder** window, expand the **Magnetic Flux Density Norm (mf)** node.

#### *Surface 1*

- 1 Right-click **Contour 1** and choose **Delete**.
- 2 In the **Settings** window for **Surface**, click to expand the **Quality** section.
- 3 From the **Resolution** list, choose **Finer**.
- 4 On the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**.
- 5 Click the **Zoom Extents** button on the **Graphics** toolbar.
- 6 Click the **Zoom In** button on the **Graphics** toolbar, twice.



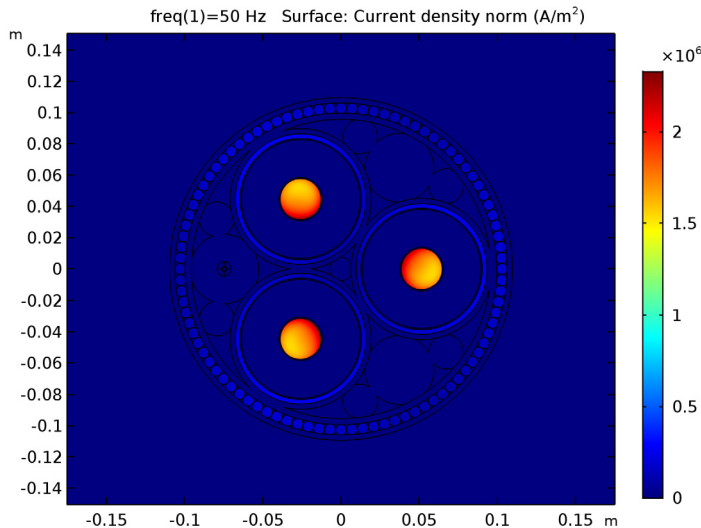
The default plot shows the norm of the magnetic flux density. Since we are in the frequency domain, the magnetic flux density is complex. The norm of this complex vector field is defined as  $|\mathbf{B}| = (|B_x|^2 + |B_y|^2)^{1/2}$ . Consequently, the plot looks symmetric and is phase independent. Now, let's have a look at the current density norm.

### Magnetic Flux Density Norm (mf) 1

- 1 In the **Model Builder** window, under **Results** right-click **Magnetic Flux Density Norm (mf)** and choose **Duplicate**.
- 2 In the **Settings** window for **2D Plot Group**, type Out of Plane Current Density (mf) in the **Label** text field.
- 3 Locate the **Plot Settings** section. From the **View** list, choose **New view**.  
The new view is needed, as we want to decouple the camera settings of this plot from the previous one.

### Surface 1

- 1 In the **Model Builder** window, expand the **Results>Out of Plane Current Density (mf)** node, then click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `mf . normJ`.
- 4 On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.
- 5 Click the **Zoom Extents** button on the **Graphics** toolbar.
- 6 Click the **Zoom In** button on the **Graphics** toolbar, twice.



You've switched from the variable `mf . normB`, to `mf . normJ`. For many quantities, predefined variables are available. You can find them using the buttons in the top-right corner of the **Expression** section, just above the text input field for the expression. There is

some auto-complete functionality too (try hitting Ctrl+Space, when having focus in the text input field).

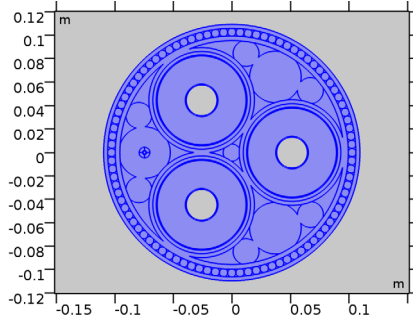
The current in the main conductors dominates. This current is desired (apart from the skin effect). The parasitic currents in the screens and armor however, are not desired. Let's have a closer look by excluding the main conductors from the **Solution**.

#### *Study 1/Solution 1 (sol1)*

In the **Model Builder** window, expand the **Results>Data Sets** node.

#### *Selection*

- 1 Right-click **Study 1/Solution 1 (sol1)** and choose **Duplicate**.
- 2 On the **Results** toolbar, click **Selection**.
- 3 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 4 From the **Geometric entity level** list, choose **Domain**.
- 5 From the **Selection** list, choose **Cable**.
- 6 Deselect **Phase 1**, **Phase 2**, and **Phase 3**.

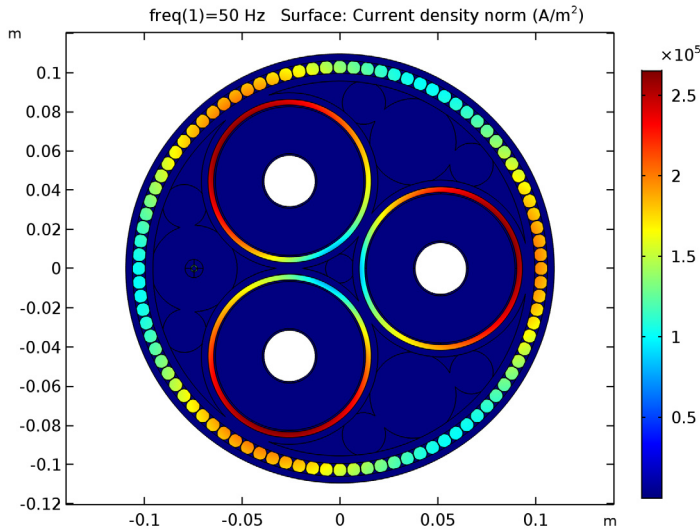


- 7 Select the **Propagate to lower dimensions** check box.

#### *Out of Plane Current Density (mf)*

- 1 In the **Model Builder** window, under **Results** click **Out of Plane Current Density (mf)**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Data set** list, choose **Study 1/Solution 1 (2) (sol1)**.
- 4 On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.

5 Click the **Zoom Extents** button on the **Graphics** toolbar.



These are the currents. The corresponding losses are of the resistive type (as opposed to magnetic or dielectric hysteresis losses). This kind of loss, also known as *Ohmic heating* or *Joule heating*, follows from Ohm's law. In the frequency domain, it is given by  $|I|^2 R/2$ , or, in differential form:  $|\mathbf{J}|^2/2\sigma$ .

Notice that, as the sea bed surrounding the cable has been excluded from the solution, the size of the plotted cross section has become smaller. It is for this reason, that we preferred to have separate camera settings for this plot.

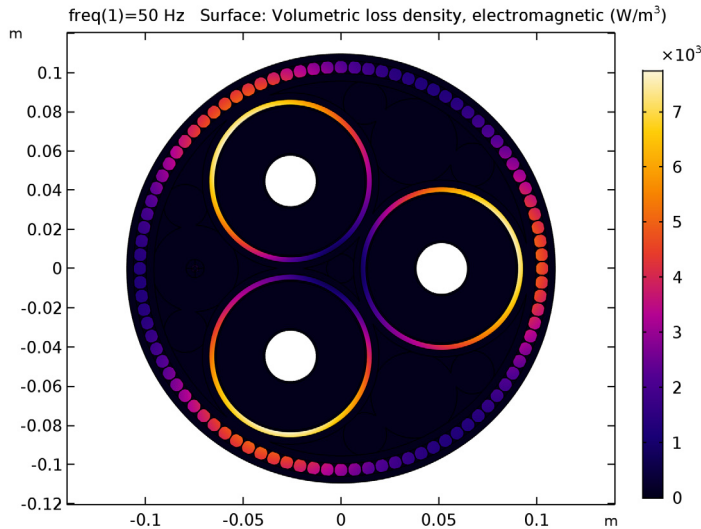
#### *Out of Plane Current Density (mf) I*

- 1 Right-click **Results>Out of Plane Current Density (mf)** and choose **Duplicate**.
- 2 In the **Settings** window for **2D Plot Group**, type **Volumetric Loss Density (mf)** in the **Label** text field.

#### *Surface I*

- 1 In the **Model Builder** window, expand the **Results>Volumetric Loss Density (mf)** node, then click **Surface I**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type **mf.Qh**.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **HeatCamera**.
- 5 On the **Volumetric Loss Density (mf)** toolbar, click **Plot**.

6 Click the **Zoom Extents** button on the **Graphics** toolbar.



These losses are undesired. In a well designed cable special steps have been taken in order to reduce them. Before reproducing these steps however, let's investigate the phase dependency of the fields and currents by creating some nice animations.

For this, we make the **Magnetic Flux Density Norm** plot phase dependent using the norm of the instantaneous vector  $||\text{Re}(\mathbf{B})|| = (\text{Re}(B_x)^2 + \text{Re}(B_y)^2)^{1/2}$ . As opposed to the norm we used previously, this one is phase dependent: as  $B_x$  and  $B_y$  rotate on the complex plane,  $\text{Re}(B_x)$  and  $\text{Re}(B_y)$  will oscillate. A contour plot of  $A_z$  is added for the corresponding field lines.

7 In the **Model Builder** window, click **Surface 1**.

8 In the **Settings** window for **Surface**, locate the **Expression** section.

9 In the **Expression** text field, type `sqrt(real(mf.Bx)^2+real(mf.By)^2)`.

10 Select the **Description** check box.

11 In the associated text field, type `Magnetic flux density norm (instantaneous)`.

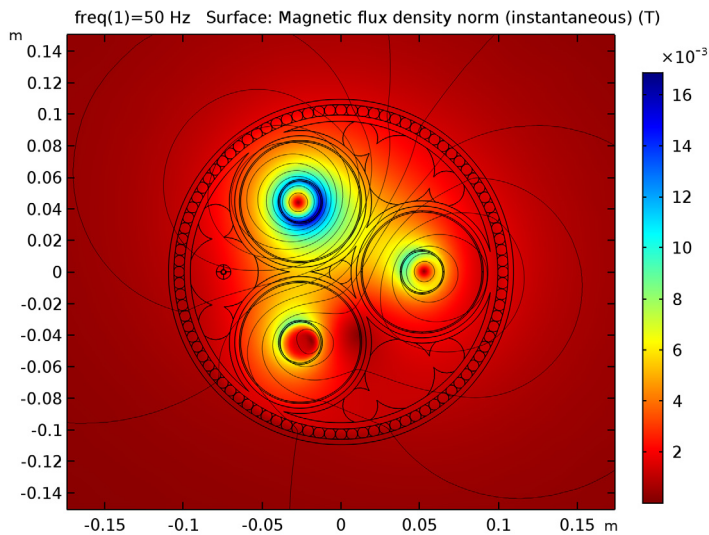
12 Locate the **Coloring and Style** section. Select the **Reverse color table** check box.

#### *Contour 1*

1 In the **Model Builder** window, under **Results** right-click **Magnetic Flux Density Norm (mf)** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

- 3 In the **Expression** text field, type  $A_z$ .
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 6 From the **Color** list, choose **Black**.
- 7 Clear the **Color legend** check box.
- 8 Click to expand the **Quality** section. From the **Resolution** list, choose **Finer**.
- 9 On the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**.



#### Animation 1

- 1 On the **Results** toolbar, click **Animation** and choose **Player**.
- 2 In the **Settings** window for **Animation**, locate the **Animation Editing** section.
- 3 From the **Sequence type** list, choose **Dynamic data extension**.
- 4 Locate the **Frames** section. In the **Number of frames** text field, type 60.
- 5 Locate the **Playing** section. Select the **Repeat** check box.
- 6 Right-click **Animation 1** and choose **Play** (see the animation from [Ref. 1](#)).

*Hint; if you feel generating the animation takes too long on your machine, change the quality/resolution setting of the corresponding surface and contour plot from finer, too coarse.*

Creating field lines by means of a contour plot of  $A_z$  is valid in this case, because of the relation  $\mathbf{B} = \nabla \times \mathbf{A}$ , combined with the fact that  $\mathbf{A}$  is strictly out of plane here. As opposed

to the standard, general purpose stream line plots (as used for heat flux or fluid flow), this contour plot gives you a physically accurate depiction of the magnetic flux lines. That is, it shows perfect loops and the contour density is directly proportional to the instantaneous magnetic flux density. In case of stream lines, it's only the *direction* that is accurate.

Furthermore, notice that *plotting itself* takes the real part of  $A_z$ . This is because plotting an actual complex 2D field would require a 3D plot. In a way, that is what the animation does; it uses time as a third dimension to show the phase information contained in the solution.

As to the shape of the field lines, you might have noticed they bend when crossing the screen and the armor. This is an additional hint the parasitic currents are significant.

Let's see how the currents behave as a function of phase.

- 7 Click the **Stop** button on the **Graphics** toolbar.

#### *Surface 1*

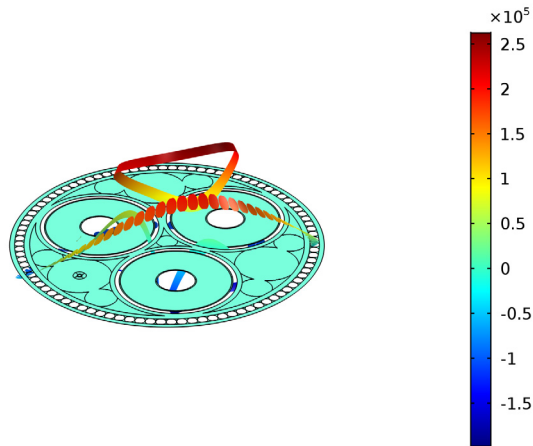
- 1 In the **Model Builder** window, under **Results>Out of Plane Current Density (mf)** click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type  $mf \cdot Jz$ .

#### *Height Expression 1*

- 1 Right-click **Results>Out of Plane Current Density (mf)>Surface 1** and choose **Height Expression**.
- 2 On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.

3 Click the **Go to Default View** button on the **Graphics** toolbar.

freq(1)=50 Hz Surface: Current density, z component (A/m<sup>2</sup>)



#### Animation 2

- 1 On the **Results** toolbar, click **Animation** and choose **Player**.
- 2 In the **Settings** window for **Animation**, locate the **Scene** section.
- 3 From the **Subject** list, choose **Out of Plane Current Density (mf)**.
- 4 Locate the **Animation Editing** section. From the **Sequence type** list, choose **Dynamic data extension**.
- 5 Locate the **Frames** section. In the **Number of frames** text field, type 60.
- 6 Locate the **Playing** section. Select the **Repeat** check box.
- 7 Right-click **Animation 2** and choose **Play** (see the animation from [Ref. 2](#)).

So the currents in the armor and the screen are oscillating way above and below the zero point. This is why in practice the armor is twisted, with a different pitch than the central conductors have. As a result all wires will carry the same total induced current (see section [On Armor Twist](#)). In a well-balanced cable, this current will be zero.

Notice that by using cross bonding or single-point bonding, a similar effect can be achieved for the screens. This will be further investigated in the *Bonding Inductive* tutorial. In the next part we will limit ourselves to the armor twist however, as cross bonding and single-point bonding are rarely used for submarine applications (for *terrestrial* use however, this is a different matter).

But first, let's quantify the losses so we can make a good comparison later on.



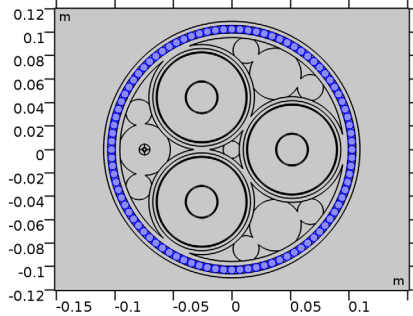
8 Click the **Stop** button on the **Graphics** toolbar.

### Surface Integration 1

1 On the **Results** toolbar, click **More Derived Values** and choose **Integration> Surface Integration**.

2 In the **Settings** window for **Surface Integration**, locate the **Selection** section.

3 From the **Selection** list, choose **Armor wires**.



4 Locate the **Expressions** section. In the table, enter the following settings:

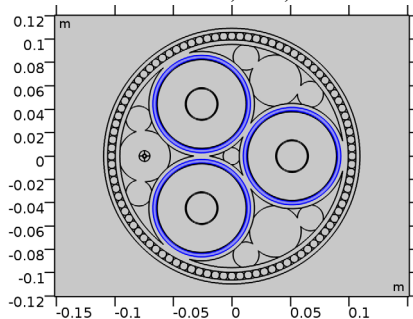
Expression	Unit	Description
mf . Qh	W/km	Armor losses

5 Click **Evaluate**.

### Surface Integration 2

1 On the **Results** toolbar, click **More Derived Values** and choose **Integration> Surface Integration**.

2 Select Domains 230, 231, and 247 only.



3 In the **Settings** window for **Surface Integration**, locate the **Expressions** section.

4 In the table, enter the following settings:

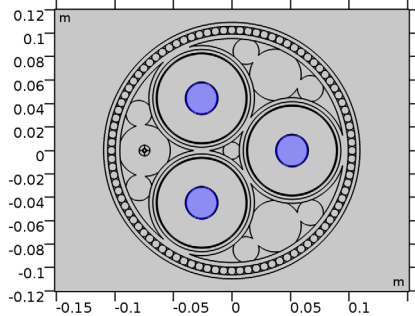
Expression	Unit	Description
mf . Qh	W/km	Screen losses

5 Click **Evaluate**.

### Surface Integration 3

1 On the **Results** toolbar, click **More Derived Values** and choose **Integration> Surface Integration**.

2 Select Domains 235, 236, and 253 only.



3 In the **Settings** window for **Surface Integration**, locate the **Expressions** section.

4 In the table, enter the following settings:

Expression	Unit	Description
mf . Qh	W/km	Phase losses

5 Click **Evaluate**.

### TABLE

1 Go to the **Table** window.

The losses per kilometer should be about 9.7 kW, 10 kW, and 47 kW for the armor, screens, and phases respectively.

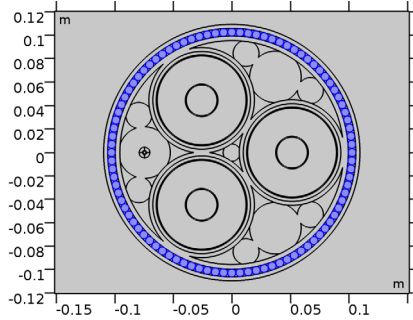
## Modeling Instructions — Armor Twist

### MAGNETIC FIELDS (MF)

Now, apply a **Coil group** to model the effect of the armor twist.

### Coil 4

- 1 On the **Physics** toolbar, click **Domains** and choose **Coil**.
- 2 In the **Settings** window for **Coil**, type **Armor** in the **Label** text field.
- 3 Locate the **Domain Selection** section. From the **Selection** list, choose **Armor wires**.



- 4 Locate the **Coil** section. Select the **Coil group** check box.
- 5 From the **Coil excitation** list, choose **Voltage**.
- 6 In the  $V_{\text{coil}}$  text field, type  $0[\text{V}]$ .

With the **Coil group** setting enabled, COMSOL considers the selected domains different instances of the same entity crossing the 2D plane multiple times; it puts the domains in series (see section [On Coil Domains](#)). Additionally, you have set the excitation voltage to  $0[\text{V}]$ , effectively cross-circuiting the coil group. Now, the currents may flow freely, as long as the total current is the same in all selected domains. *Hint; you can investigate the resulting behavior further by de-balancing the cable.* Let's **Compute**.

### STUDY 1

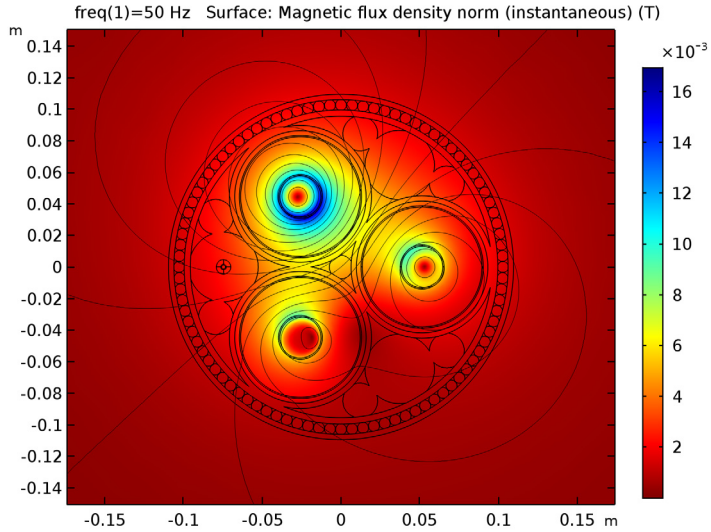
On the **Home** toolbar, click **Compute**.

### RESULTS

*Magnetic Flux Density Norm (mf)*

- 1 In the **Model Builder** window, under **Results** click **Magnetic Flux Density Norm (mf)**.

**2** On the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**.



Notice how the field lines flow straight through the armor this time (for the corresponding animation, see [Ref. 3](#)).

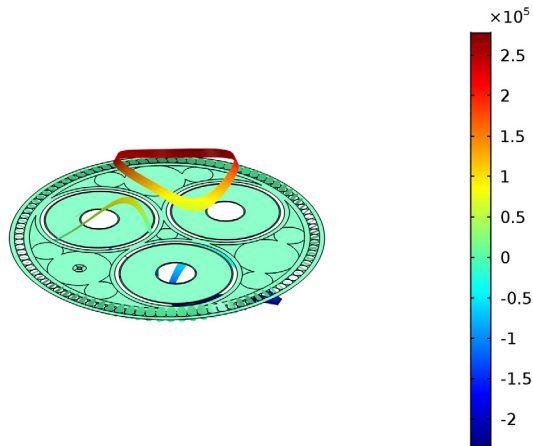
*Out of Plane Current Density (mf)*

**1** In the **Model Builder** window, under **Results** click **Out of Plane Current Density (mf)**.

**2** On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.

3 Click the **Go to Default View** button on the **Graphics** toolbar.

freq(1)=50 Hz Surface: Current density, z component (A/m<sup>2</sup>)



#### Animation 2

1 In the **Model Builder** window, under **Results>Export** right-click **Animation 2** and choose **Play** (see the animation from [Ref. 4](#)).

The currents in the armor have been reduced significantly. Although they still oscillate locally, the total net current per armor wire is now zero (since it is a well-balanced cable).

For those interested in different bonding types, *feel free to investigate what happens when you add an additional coil group to the screens*. Let's see how the armor twist affects the losses.

2 Click the **Stop** button on the **Graphics** toolbar.

#### Surface Integration 1

1 In the **Model Builder** window, under **Results>Derived Values** click **Surface Integration 1**.

2 In the **Settings** window for **Surface Integration**, click **Evaluate**.

#### Surface Integration 2, Surface Integration 3

Repeat these steps for **Surface Integration 2** and **Surface Integration 3**.

## TABLE

1 Go to the **Table** window.

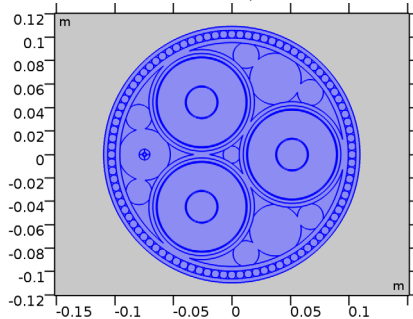
The losses per kilometer should be about 8.8 W, 12 kW and 47 kW for the armor, screens, and phases. As a response to the large reduction in losses for the armor, the phase and screen losses went up (about 160 W and 2.1 kW respectively). This waterbed effect is related to a redistribution of the magnetic energy. The total losses have been reduced by about 11%.

This is a great improvement, but we can still gain another 4 kW by having a further look at the main conductors.

## RESULTS

### Selection

- 1 In the **Model Builder** window, under **Results>Data Sets>Study 1/Solution 1 (2) (sol1)** click **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Selection** list, choose **Cable**.



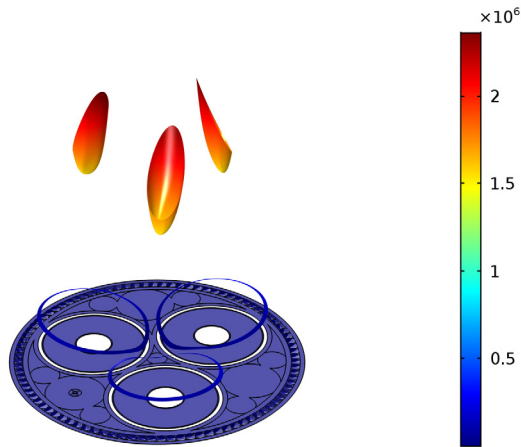
- 4 Select the **Propagate to lower dimensions** check box.

### Surface 1

- 1 In the **Model Builder** window, under **Results>Out of Plane Current Density (mf)** click **Surface 1**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type  $mf \cdot \text{norm}J$ .
- 4 On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.

5 Click the **Go to Default View** button on the **Graphics** toolbar.

freq(1)=50 Hz Surface: Current density norm (A/m<sup>2</sup>)



As you can see, the current in the main conductors is not homogeneous. There is a skin effect and a proximity effect, with a stronger current density near the cable's center. We have modeled solid conductors here (apart from the correction term;  $N_{con}$ ).

In practice however, the main conductors consist of twisted strands. Depending on the kind of insulation (if any) between the strands, the current density may be partly homogenized. For the sake of argument, let's assume this twisting technique performs well (as is the case for *Litz wire* at the proper frequency).

### *Modeling Instructions — Phase Twist*

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#### **MAGNETIC FIELDS (MF)**

##### *Phase I*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Magnetic Fields (mf)** click **Phase I**.
- 2 In the **Settings** window for **Coil**, locate the **Coil** section.
- 3 From the **Conductor model** list, choose **Homogenized multi-turn**.
- 4 Locate the **Homogenized Multi-Turn Conductor** section. In the  $N$  text field, type 1.
- 5 In the  $\sigma_{coil}$  text field, type  $S_{con}$ .

6 In the  $a_{\text{coil}}$  text field, type Acon.

*Phase 2, Phase 3*

Repeat these steps for **Phase 2**, and **Phase 3**.

We have used the **Homogenized Multi-Turn** setting here. This is ideal for coils with multiple thin turns, or Litz wires with strands much thinner than the skin depth. With this setting enabled, the turn or strand bundle will behave like a strongly anisotropic continuum with a homogenized cross sectional current density (see section [On Coil Domains](#)).

Since the strands are electrically placed in parallel, we set the number of turns to 1, and the wire cross section to Acon; the sum of all strands. Let's **Compute** the final result.

### STUDY I

On the **Home** toolbar, click **Compute**.

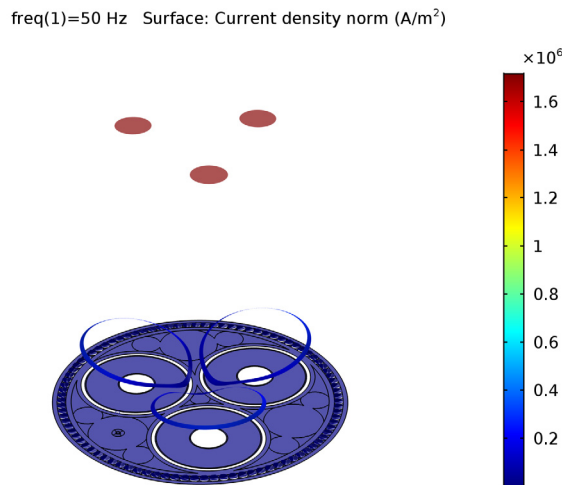
### RESULTS

*Out of Plane Current Density (mf)*

1 In the **Model Builder** window, under **Results** click **Out of Plane Current Density (mf)**.

2 On the **Out of Plane Current Density (mf)** toolbar, click **Plot**.

3 Click the **Go to Default View** button on the **Graphics** toolbar.



The current distribution is homogeneous now, with the total current through the main conductors still the same ( $I_0$  for each phase).



### Surface Integration 1

- 1 In the **Model Builder** window, under **Results>Derived Values** click **Surface Integration 1**.
- 2 In the **Settings** window for **Surface Integration**, click **Evaluate**.

### Surface Integration 2, Surface Integration 3

Repeat these steps for **Surface Integration 2** and **Surface Integration 3**.

## TABLE

- 1 Go to the **Table** window.

The losses per kilometer should be about 9.1 W, 12 kW and 43 kW for the armor, screens and phases respectively.

Finally, let's have a look at the lumped parameters and compare to the cable's official specifications.

## RESULTS

### Global Evaluation 1

- 1 On the **Results** toolbar, click **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
mf.RCoil_1/1[m]	mΩ/km	Phase 1 AC resistance
mf.RCoil_2/1[m]	mΩ/km	Phase 2 AC resistance
mf.RCoil_3/1[m]	mΩ/km	Phase 3 AC resistance
Rcon	mΩ/km	DC resistance, 20°C (Analytic)

- 4 Click **Evaluate**.

## TABLE

- 1 Go to the **Table** window.

The phase resistance per kilometer should be about 43 mΩ and 34 mΩ for the AC and the DC resistance respectively. Notice that the AC resistance is fairly close to the DC one. As mentioned when introducing the parameters at the beginning of this tutorial, the DC resistance serves as a lower bound; all attempts we have done to get the losses down, resulted in the AC resistance approaching the DC one, *feel free to check this*.

## RESULTS

### Global Evaluation 2

- 1 On the **Results** toolbar, click **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
mf.LCoil_1/1[m]	mH/km	Phase 1 inductance
mf.LCoil_2/1[m]	mH/km	Phase 2 inductance
mf.LCoil_3/1[m]	mH/km	Phase 3 inductance

- 4 Click **Evaluate**.

### TABLE

- 1 Go to the **Table** window.

The inductance per kilometer should be about 0.42 mH. This is close to the official specifications, so this model seems to give a pretty accurate insight in the cables inductive performance.

Since the coil group puts the domains in series (as opposed to forcing the total net current per domain to zero directly), the armor will show natural behavior in the sense that the losses will increase when the cable becomes de-balanced, *feel free to check this*.

Although these results look very promising, there are a couple of loose ends, still. The homogenized current in the central conductors is one of many approximations, so is the solid conductor with conductivity  $N_{con} * S_{con}$  (see section [On Accuracy](#)). Some might notice the current in the fiber's armor should be put to zero, although the corresponding losses are negligible. Furthermore, we have the charge currents flowing through the screen, resulting in some additional losses (about 0.38 kW per phase, as determined in the *Bonding Capacitive* tutorial).

More significant however, is the effect of *heat*. Thermal effects give rise to increased resistivity and consequently, increased losses. The cable we have modeled here is 20°C, whereas any real cable would operate at a temperature of about 70 to 90°C. The next tutorial in this series will include a detailed thermal analysis.

You have now completed this tutorial, subsequent tutorials will refer to the resulting file as submarine cable 04 inductive effects.mph.