Modeling Acoustic Modes in a Continuous Loop Piping System

Marderness, Eric[1] and Gutierrez-Miravete, Ernesto[*2],
[1]General Dynamics-Electric Boat, [2]Rensselaer at Hartford
*Corresponding author: RPI, 275 Windsor Street, Hartford, CT 06120; gutiee@rpi.edu

Abstract: This paper discusses the low frequency axial modes of the fluid resonances within a continuous loop piping system. The frequency at which the axial modes occur is a function of the phase velocities and changes in impedance within the fluid loop. The analysis is restricted to axial modes of the lowest order ("plane waves"). Acoustic equations are developed to represent the fluid loop as a special case of a cylindrical Helmholtz resonator. The ends of a straight finite circular cylindrical fluid column are set equal to each other to model a fluid "loop" with an infinite radius of curvature. The Transfer Matrix Method (TMM) and finite element models in COMSOL are implemented to investigate how local differences in elasticity and impedance influence the frequencies and mode shapes of the axial fluid loop modes. It is shown that for a fluid filled loop of uniform cross section, the axial modes occur in pairs at frequencies corresponding to integer multiples of whole wave lengths. The pairs of axial modes in non-uniform loops are shown to shift independently resulting in two axial modes of the same "wavelength" occurring at different frequencies within the same physical system. The mode shapes are shown to have an uneven axial distribution in phase resulting in an apparent "kink" in the mode shape. The independent shifting of the modes is shown to be modally specific and related to the local phase velocities and impedances that result from the dimensions and materials of the system. The "kinks" in the modes shapes are explained and it is shown that the pressure and displacement mode shapes maintain the necessary continuity requirements. The results from theory, previous works on radial and circumferential modes in fluid filled elastic cylinders, and the TMM models are compared to the eigenvalue analysis results from finite element models developed in COMSOL. The finite element model results for the axial loop modes are verified to be reasonable. Physical interpretations of the results are provided.

Keywords: Acoustic Impedance, Acoustic Modes, Continuous Loop Piping System.

1. Introduction

This paper presents an analysis of the low order axial fluid acoustic modes within a system of piping and components that form a continuous loop. The "loop" arrangement of the system allows for axial modes to occur in the fluid column that follow the centerline of the loop and wrap around the entire system form a continuous loop or loop mode. The analysis focuses on predicting the frequencies and mode shapes of the system-wide axial modes and on how changes to the system parameters affect the frequencies and modes shapes of those modes. Figure 1 shows a schematic example piping system arranged in a "loop" that includes three cylindrical cavities representing generic tanks or components.

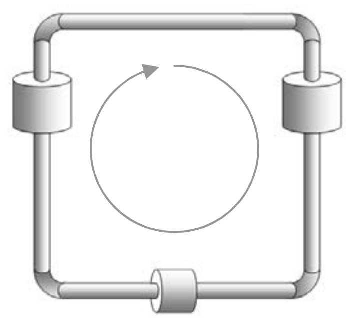

Figure 1. Schematic of an Example Fluid Filled Piping System with Components that form a Continuous Loop of Fluid.

In its simplest form a piping system is a series of inter-connected cylindrical shells intended to transport a fluid from one location to another. The fluid is used to transfer heat, mass, pressure or mechanical energy between the connected locations. In some cases the fluid is the quantity that is being transferred between locations such as in oil pipelines or water distribution systems. In other cases the fluid is only the transport medium being used to transfer thermal or mechanical energy between two points such as the fluids in cooling or hydraulics systems.

Two of the most common examples of an industrial fluid system that involve a loop are hydraulic and heating/coolant systems. These systems consist of varying lengths of pipes and hoses connecting multiple components together. The fluid passes from a starting point along a supply path to a location where the quantity within the fluid is transferred into another process through a component such as a heat exchanger, separator or hydraulic actuator. Once the quantity of interest has been transferred, the fluid is recirculated through a return leg to the original location forming a "continuous" loop of fluid. The original location may also be a component that adds a desired quantity to the fluid such as a boiler, pump or reservoir. These types of industrial systems exist in a wide range of sizes from commercial oil and power plants to cars and hand tools.

The analysis of axial fluid resonances within a system loop of piping is an acoustic problem. These resonances are sometimes simply referred to as "acoustic" resonances as opposed to structural resonances, even though the fluid and structural resonances are not actually independent of each other. Fluid resonances can detrimentally impact the operation of fluid systems and components. The unwanted impacts of the fluid resonances include increased system noise, excessive component fatigue, interference with test measurements and monitoring instrumentation, improper system and potentially system or component failure. When the piping system is elastic and contains a dense fluid (such as commonly used steel or aluminum pipes containing water) the fluid-structure interaction (FSI) between the piping structure and the enclosed fluid can significantly alter the acoustic response of both the fluid and structural portions of the system making it necessary to consider FSI during the system design as discussed in [1]. How the structure of the piping (or components) and the internal fluid interact depends on many system specific factors including the properties of the structural materials, the properties of the internal fluid, the geometry of the system, and the frequency range of interest.

2. System Description, Properties and Assumptions

The example system shown in Figure 1 was specifically developed to support the present study of axial loop resonances. The system is assumed to be a closed system and will be analyzed in a free-free boundary condition. Assuming the loop is in a free-free boundary condition allows the analysis to neglect body forces, neglect the effects of pipe or component foundations and assume uniform conditions in the radial direction of the pipe. The structures of the piping and component walls are assumed to be either rigid or linearly elastic. The internal fluid is a liquid and assumed to be compressible, lossless, inviscid, irrotational and free from bubbles or dissolved particulates. The fluid will also be assumed to be at rest and at a uniform temperature and pressure. The analysis will be restricted to low frequencies where the wave lengths are very long relative to radial dimensions of the system. Limiting the analysis to low frequencies also restricts the analysis to very low wave numbers where at most only 2 axially symmetric radial modes will exist in an elastic fluid filled cylinder and only the 0th order mode exists in a rigid wall fluid filled cylinder. The axial loop resonances investigated are restricted to resonances of the lowest order axially symmetric radial mode or plane waves. The materials of the example system are steel and water.

3. COMSOL Multiphysics Models

Two finite element models of a baseline, uniform loop were constructed using the acoustics module of the COMSOL multi-physics software. The first model was a straight fluid column, with rigid walls, filled with water. The second model was an actual fluid loop with rigid walls, filled with water, and using the dimensions and material properties given in Table 1 and Table 2 in [2]. Additionally, COMSOL models were develop for non-uniform loops. Specifically, loops with a single cavity and with three cavities, as in Figure 1. For purposes of comparison, acoustic analysis was carried out on all the above systems using the Transfer Matrix Method and also Analytical Methods when possible.

4. Results and Discussion

A comparison of results obtained for the mode frequencies of the elastic uniform loop model by the TM method and using COMSOL shows excellent agreement. The frequencies of the axial loop modes occurred at lower values than those calculated for the axial modes within a rigid wall loop of the same fluid and dimensions. The pressure and displacement mode shapes remained sinusoidal. Unlike the high frequency results presented elsewhere, the elasticity alone does not result in a unique shifting or spacing of the low frequency axial modes when implemented uniformly for the entire loop.

Differences in fluid density, phase velocity or cross sectional area result in different acoustic impedances at various locations around a non-uniform loop. In addition, any system properties that affect the fluid density, phase velocity, or cross sectional area will also cause changes to the acoustic impedance. These changes can be gradual such as the one due to a temperature gradient over the length of a heat exchanger, which changes the density and characteristic fluid speed of sound, or sudden such as the change in cross sectional area where a pipe connects to a large tank. System properties that can cause either gradual or sudden changes to the acoustic impedance include, but are not limited to; temperature, pressure, the amount of entrained gases, structural materials, the thickness of piping and component walls, foundations and piping supports, and intricate component internal arrangements.

At a location where a change in impedance occurs, a portion of the acoustic wave can be reflected back from the location while the remainder of the acoustic wave is transmitted through the impedance change. The amount of the acoustic wave that is reflected or transmitted depends on how severe the mismatch in impedance is and how suddenly the change occurs. The analysis of transmitted and reflected waves in a piping system is typically focused on traveling waves and determining how much transmission loss occurs for an incident signal through a system of impedance changes.

Figure 2 shows schematic representations of the TMM and COMSOL models developed for the loop with three cavities and Figure 3 shows the computed first pressure mode shape. The frequencies of the mode pairs calculated from the TMM and FE models for the non-uniform loop models are similar to the theoretical solutions to within a fraction of a percent. The theoretical, TMM, and FE results for the frequencies and modes shapes of the axial loop modes in a simulated loop (a straight pipe with continuity boundary conditions) are similar to the results calculated from a FE model of an actual loop that included the elbows. This result indicates that the simulated loop models are reasonable engineering approximations of the axial loop modes at low frequencies.

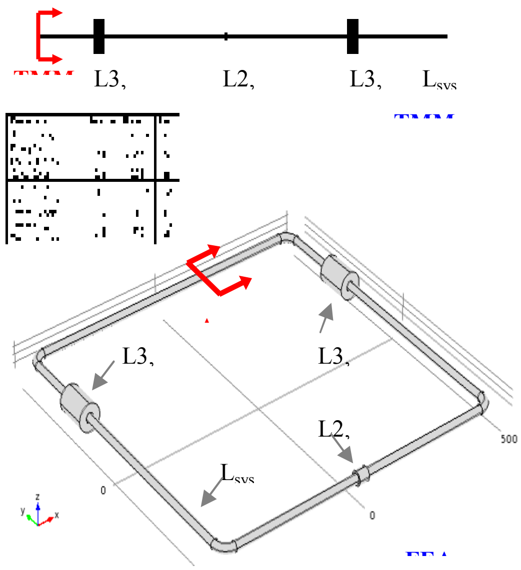

Figure 2 Example System with Three Cavities; TMM and COMSOL FEA models.

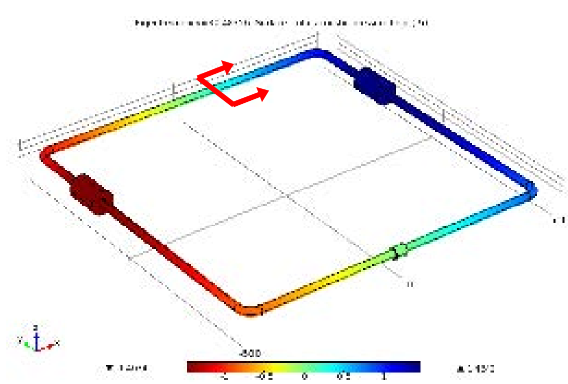

Figure 3 Computed first pressure mode shape.

In summary, the frequencies and modes shapes of the axial loop modes calculated by the FEA models for the example systems are reasonable relative expectations based on acoustic theory. The axial loop modes satisfy the conditions of continuity for pressure and volume velocity. The net change in volume for each mode sums to zero in agreement with the requirements for a closed system and the net change in phase for each mode is equal to integer multiples of 2¶ consistent with whole wave length axial modes.

5. Conclusions

It is important to be able to predict the frequency and mode shape of resonances within a piping system during system design. The frequency of a resonance is necessary to determine potential sources of excitation within the system. The mode shape provides locations where a specific system mode might cause excessive or damaging responses as well as locations that may efficiently excite the mode. Understanding how specific system properties affect the frequencies and mode shapes of the system resonances provides additional information into how design choices will affect the noise performance of the system.

Overall the frequencies and modes shapes of the axial loop modes calculated by the COMSOL FE models were in good agreement with acoustic theory and the results from the TMM models. The whole wavelength modes, mode pairs at different frequencies, non-harmonic frequency spacing of high order axial modes and the kinked pressure mode shapes were all shown to be reasonable and make physical sense. The results presented in this analysis provide a set of physics based expectations for the frequencies and modes shapes of the axial modes in a system that forms a continuous loop of fluid.

The reader is advised to consult reference [2] for detailed explanations, data and an extensive list of references.

6. References

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