Going beyond Axisymmetry: 2.5D Vector Electromagnetics

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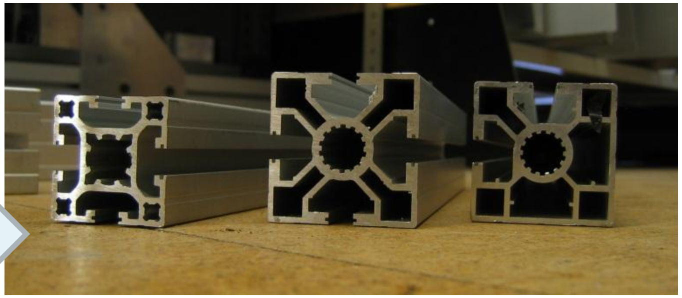
Talk Outline

1) Conventional Axisymmetric Modeling
   - Electro- and Magnetostatics
   - Fluid Dynamics
2) Going 2.5D: General Recipe
3) Vector 2.5D Electromagnetics
   - Multiscale Problems in Plasmonics
   - Optical Scattering and Cloaking Problems

Low-Dimensional Modeling

- The real world is 3D, but symmetries are abundant
- Specific cases lead to solutions where fields are independent of 1 coordinate
- 2D Modeling variations:
  - **Translational** symmetry: d/dz=0 => conventional 2D simulations in Cartesian coordinates
  - **Rotational** symmetry: d/dφ=0 => axisymmetry modeling in cylindrical coordinates
  - Invent your own coordinate system that fits the symmetry of your particular structure
- Prerequisite to all low-dimensional modeling: the structure must possess some continuous symmetry
  - If a real-world system doesn't, gain understanding with a symmetrized structure

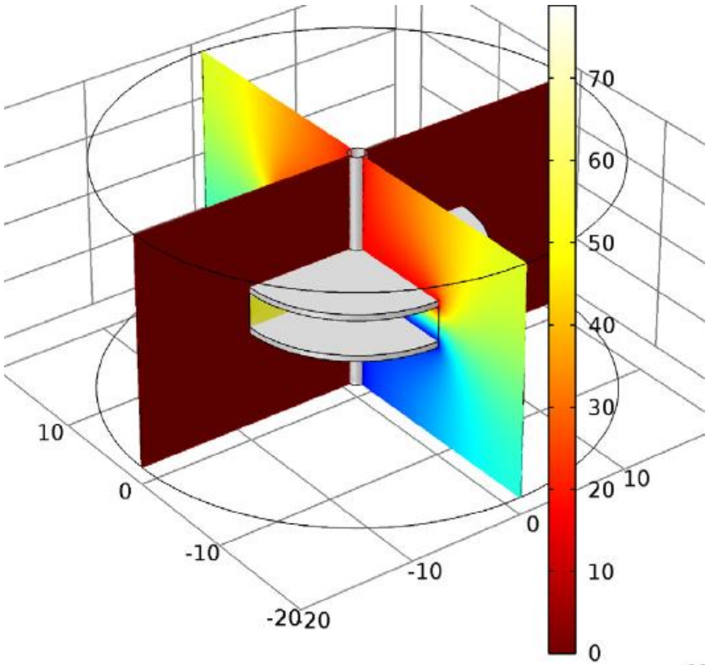


Extruded shapes

Revolved shapes

2D Axisymmetry Modeling Examples

- Various types of physics:
- **Electro- and magnetostatics, scalar potential**

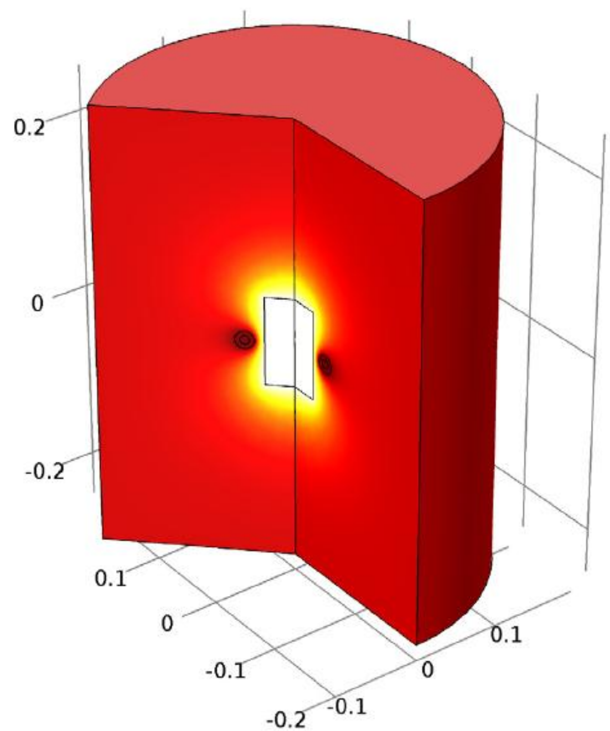


Circular capacitor

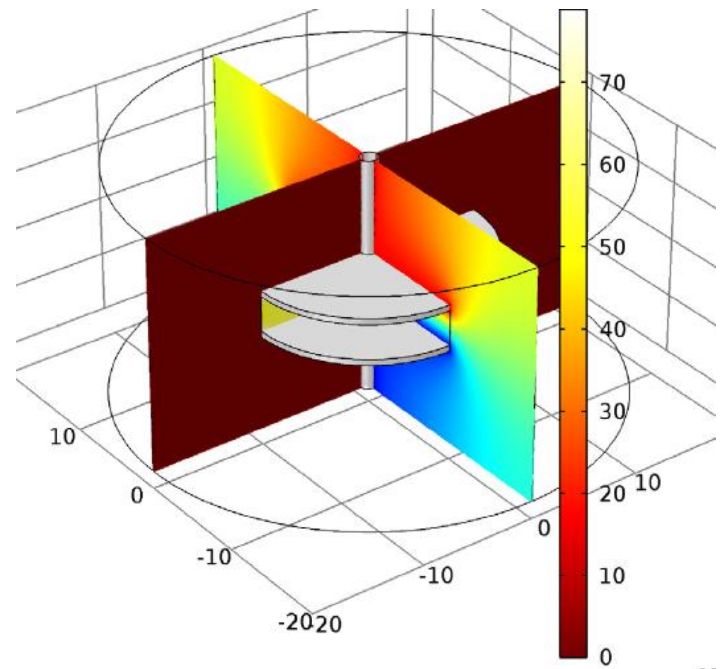
Graphics taken from COMSOL Model Library

2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- **Conductive heat transfer**



Inductive heating of a metal cylinder



Circular capacitor

Graphics taken from COMSOL Model Library

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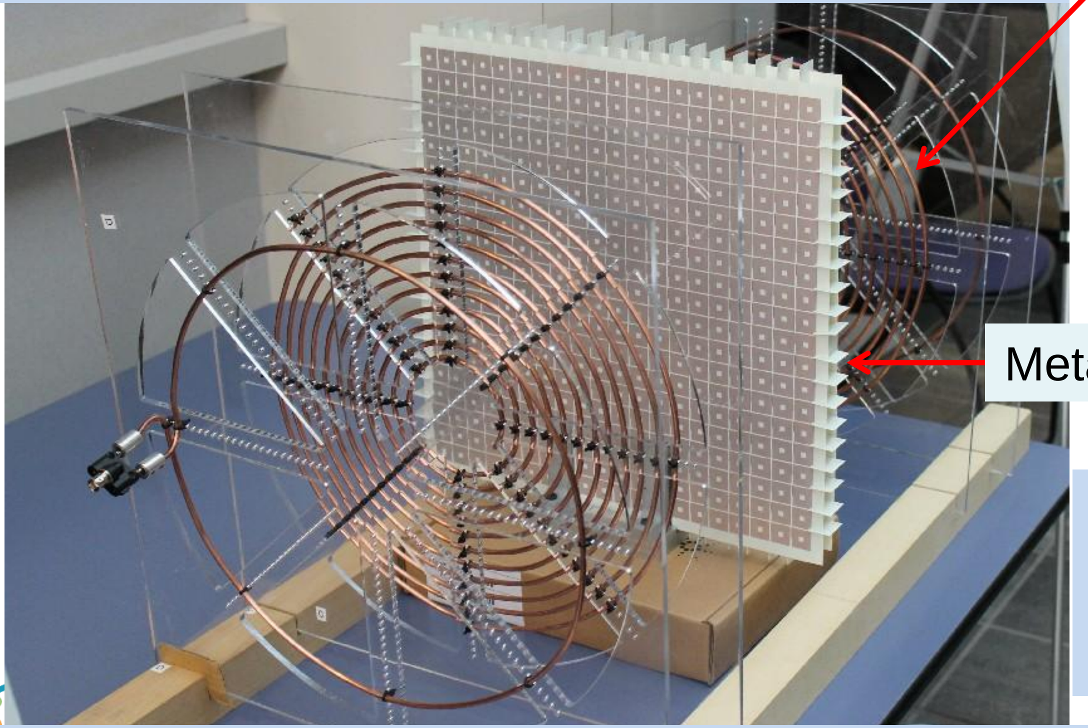
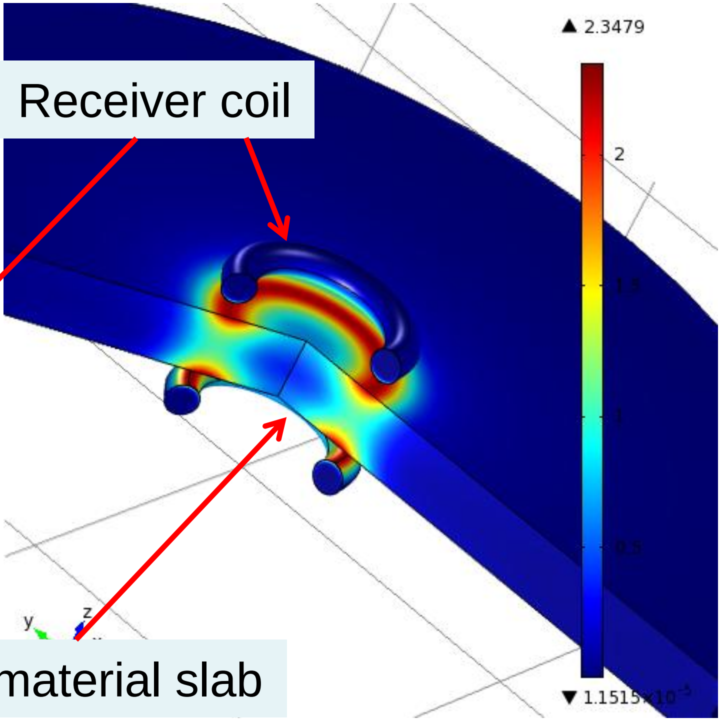
2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- Conductive heat transfer
- **Quasi-magnetostatics, vector potential**

Real system: wireless power transfer, 13.56MHz



Photograph courtesy Toyota Research Institute of North America, Jan. 2012



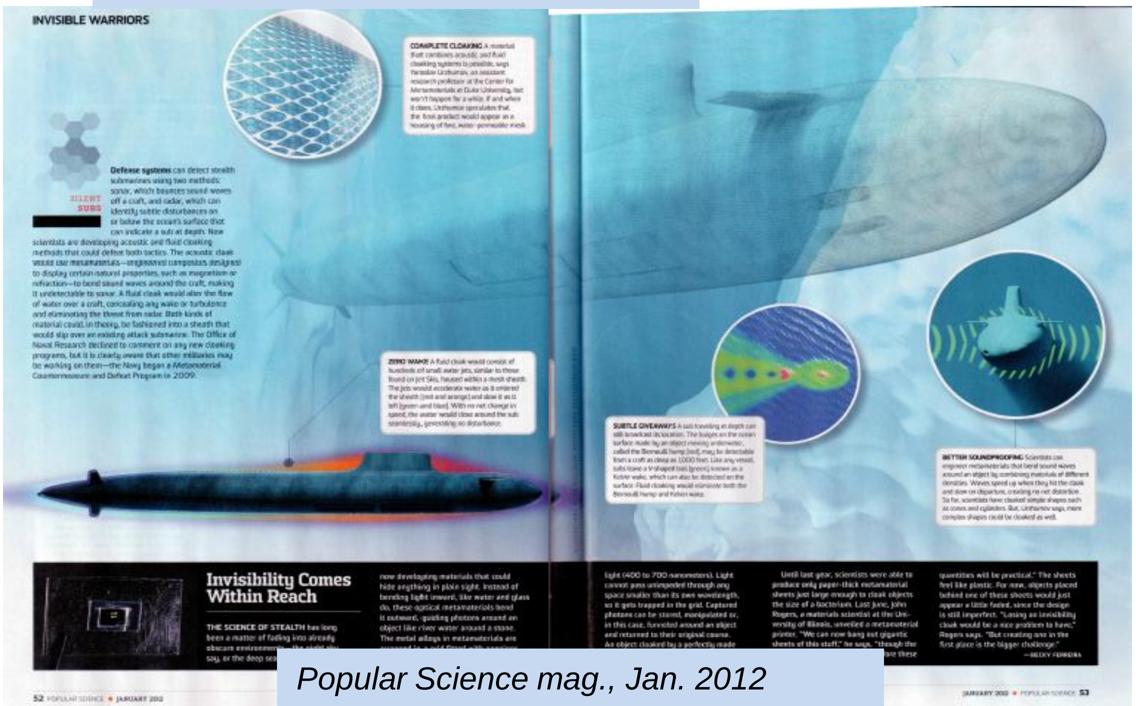
Toy model: circular coils, disk-shaped lens

Journal ref: J. Appl. Phys. 111, 064902 (2012)

2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- Conductive heat transfer
- Quasi-magnetostatics, vector potential
- **Fluid dynamics**

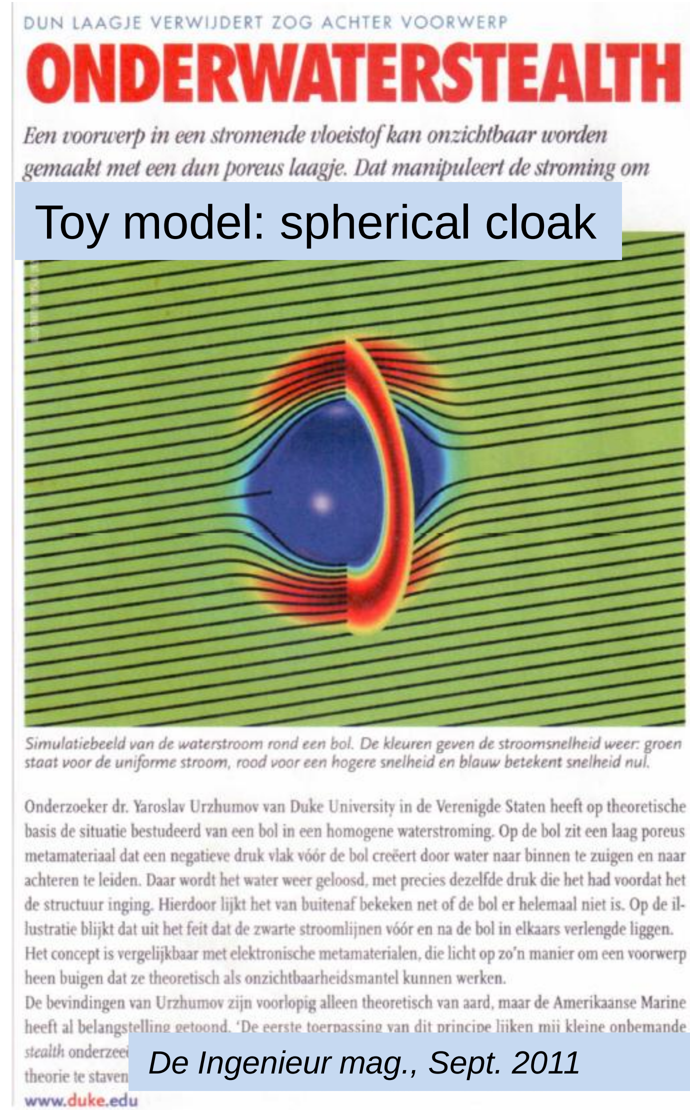
Real system: submarine


INVISIBLE WARRIORS

Invisibility Comes Within Reach

Popular Science mag., Jan. 2012

Toy model: spherical cloak


DUN LAAGJE VERWIJDERT ZOG ACHTER VOORWERP

ONDERWATERSTEALTH

Een voorwerp in een stromende vloeistof kan onzichtbaar worden gemaakt met een dun poreus laagje. Dat manipuleert de stroming om

Simulatiebeeld van de waterstroom rond een bol. De kleuren geven de stroomsnelheid weer: groen staat voor de uniforme stroom, rood voor een hogere snelheid en blauw betekent snelheid nul.

Onderzoeker dr. Yaroslav Urzhumov van Duke University in de Verenigde Staten heeft op theoretische basis de situatie bestudeerd van een bol in een homogene waterstroming. Op de bol zit een laag poreus metamateriaal dat een negatieve druk vlak vóór de bol creëert door water naar binnen te zuigen en naar achteren te leiden. Daar wordt het water weer geloosd, met precies dezelfde druk die het had voordat het de structuur inging. Hierdoor lijkt het van buitenaf bekeken net of de bol er helemaal niet is. Op de illustratie blijkt dat uit het feit dat de zwarte stroomlijnen vóór en na de bol in elkaars verlengde liggen. Het concept is vergelijkbaar met elektronische metamaterialen, die licht op zo'n manier om een voorwerp heen buigen dat ze theoretisch als onzichtbaarheidsmantel kunnen werken.

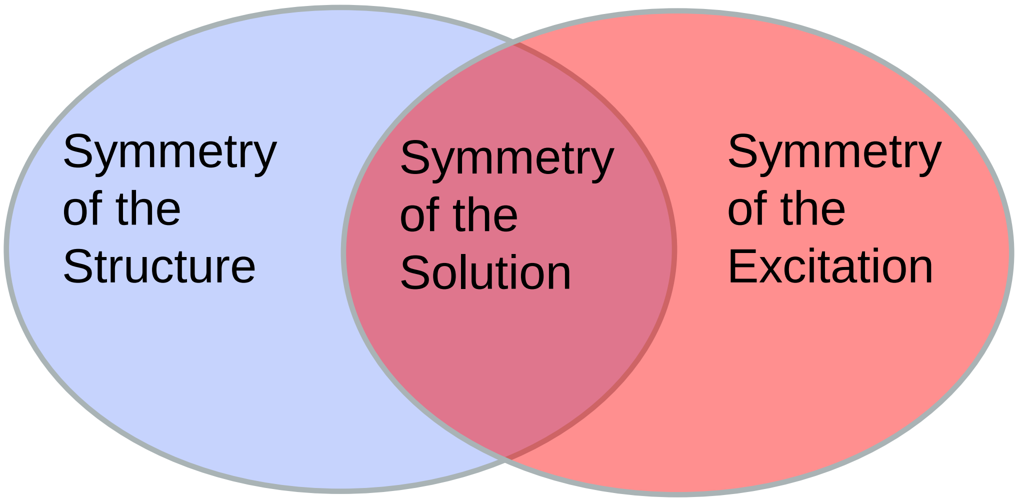
De bevindingen van Urzhumov zijn voorlopig alleen theoretisch van aard, maar de Amerikaanse Marine heeft al belangstelling getoond. 'De eerste toepassing van dit principe lijken mij kleine onbemande stealth onderzee... theorie te staven...

www.duke.edu

De Ingenieur mag., Sept. 2011

2D Axisymmetry Limitations

- Even for a rotationally symmetric structure, not every solution is axially symmetric!



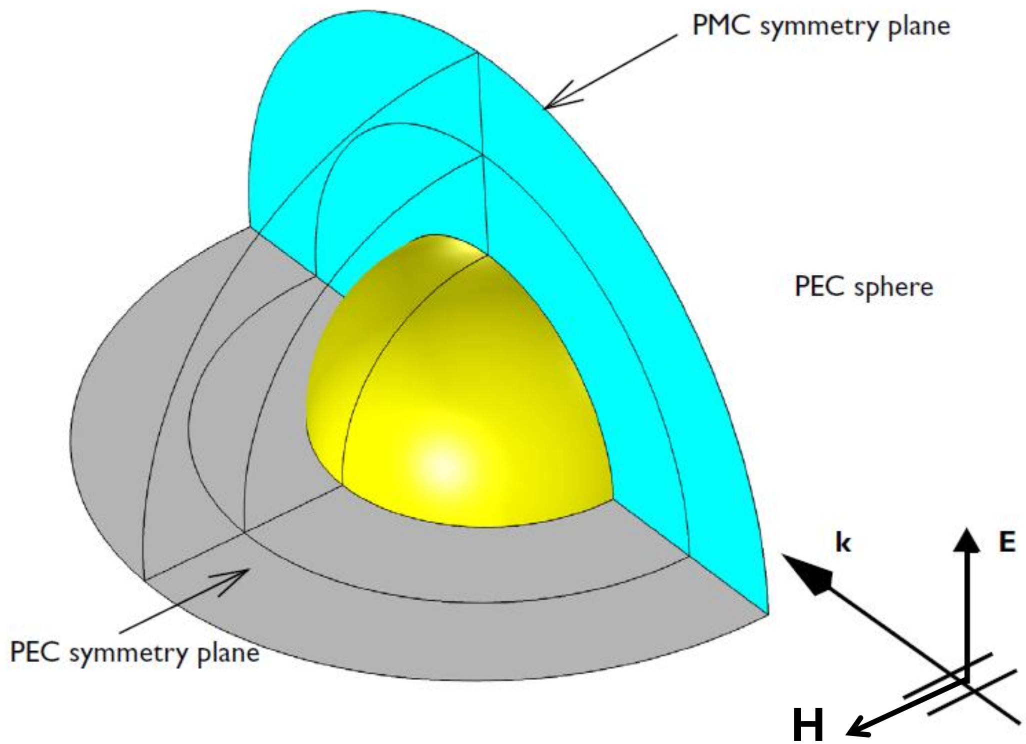
- $G_{solution} = G_{structure} \cap G_{excitation}$ (G = symmetry group)

2D Axisymmetry Limitations

- Example: Mie scattering of a plane wave

Graphics courtesy COMSOL

Symmetry of the Structure: **Spherical**



Symmetry of the Excitation: $\mathbf{C_{2v}}$ (two symmetry planes)

Symmetry of the Solution: $\mathbf{C_{2v}}$

- Can reduce modeling to a quarter, but not to a 2D slice!

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General Recipe for Generalized Axisymmetric Modeling

Decompose

Solve

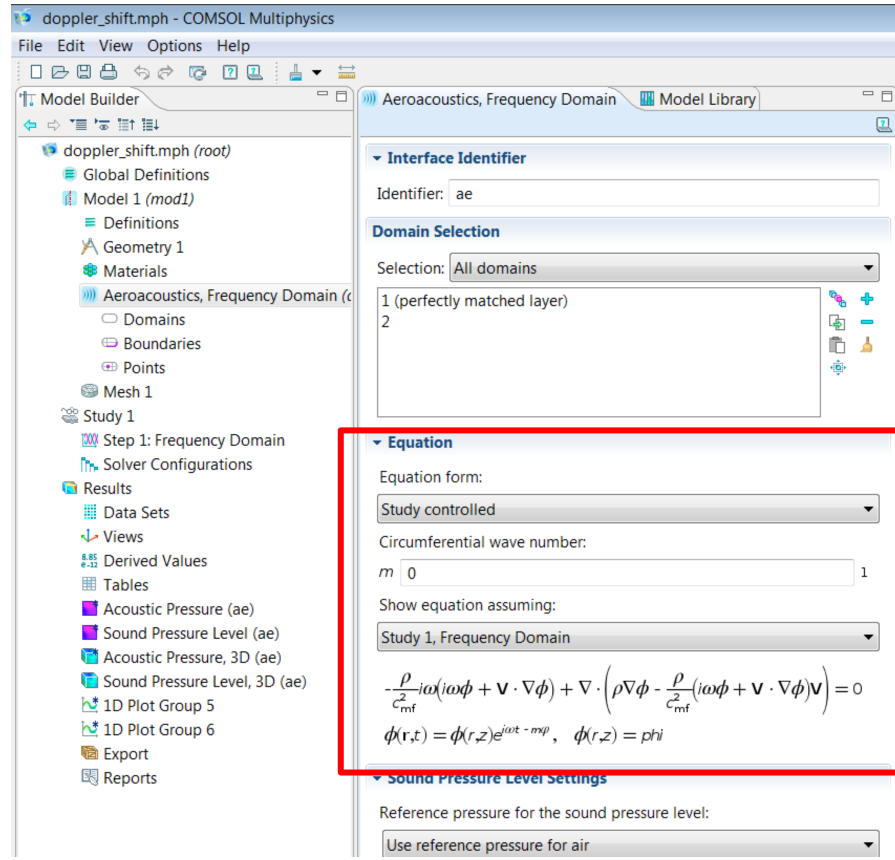
- Each cylindrical wave propagates independently of others
- For each wave, field dependence upon φ is prescribed: **exp(i m φ)**
- Write the equations in cylindrical coordinates, replace **d/dφ -> i m**
- Do so with boundary conditions if needed (Important: **Scattering** boundary condition)
- Overall factor **exp(i m φ)** cancels out, **2D equation in (r,z) coordinates left**

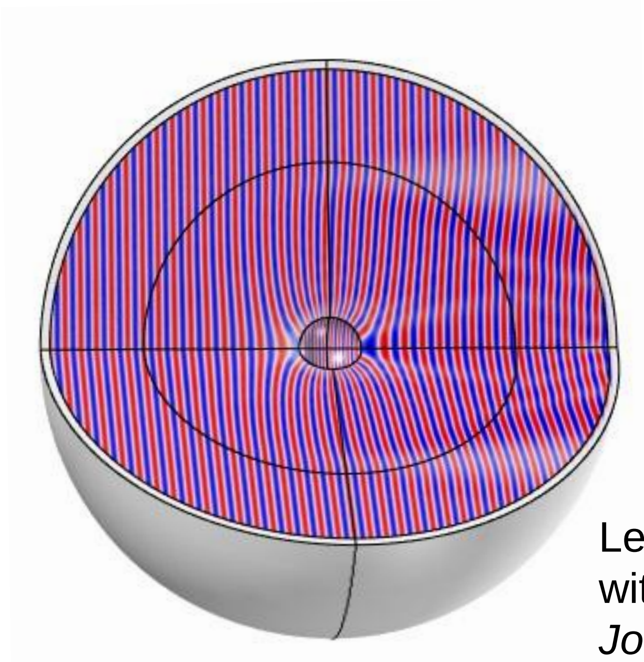
Compose

- Postprocessing: to plot solution for a given value of φ, multiply each partial wave with exp(i m φ) and sum over all m's
- Use COMSOL's built-in revolved plots for **3D visualization**

2.5D Axisymmetry for Scalar Waves

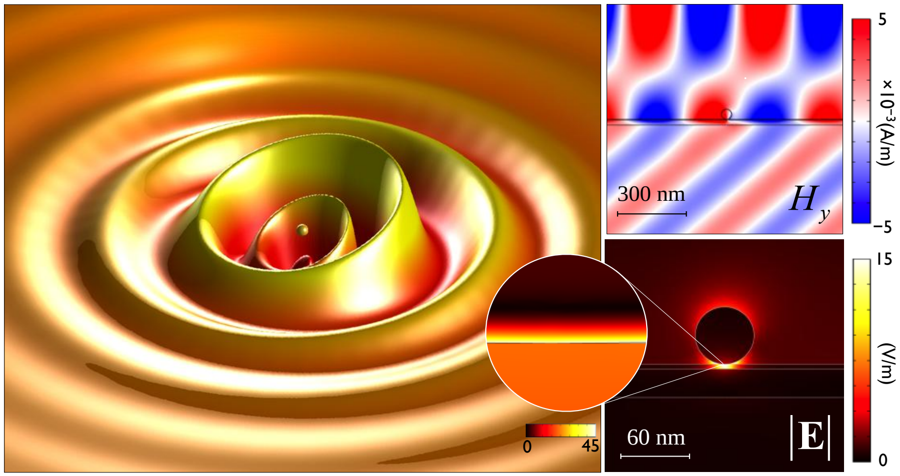
- Has been essentially available in the Acoustics Module as part of *pressure acoustics* and *aeroacoustics* interfaces in COMSOL 3.0+
- Feature called "*circumferential wave number (m)*"



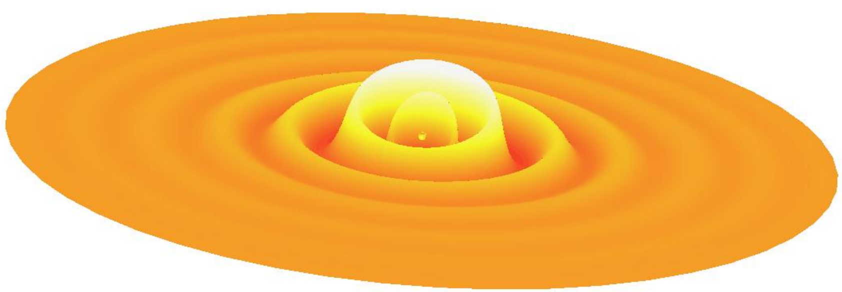


Left: acoustic cloaking in fluids (air, water, etc.); modeled with Acoustics Module, COMSOL 4.1.
Journal ref: J. Appl. Phys. 111, 053105 (2012); arXiv:1203.5831v1

2.5D Vector Electromagnetics for Scattering Problems: Nonlocal Plasmonics



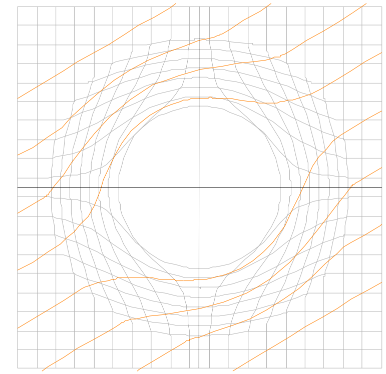
See the Supplementing materials of this paper for details of the implementation



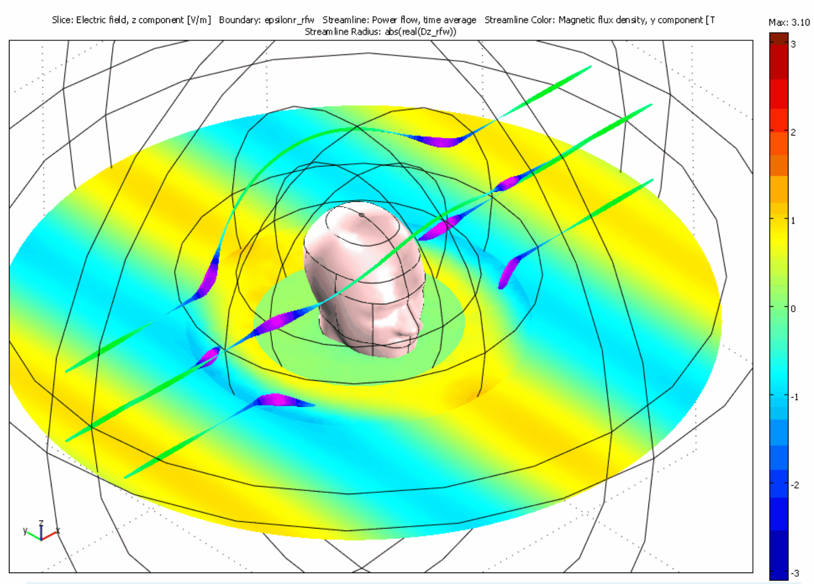
C. Ciraci et al., Science **337**, 1072 (August 31, 2012)
DOI: 10.1126/science.1224823

Another Scattering Problem: Optical Cloaking

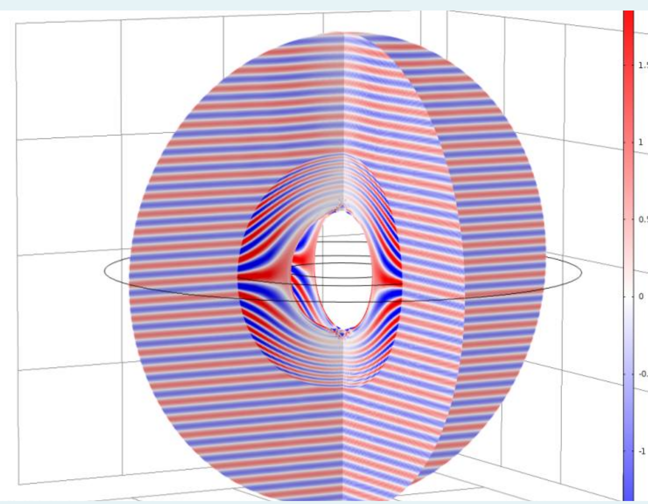
- Novel scattering problem, soon to become canonical: *scattering cancellation (invisibility, cloaking)*
- The only known exact solution is based on so-called Transformation Optics
- A scatterer coated with a special inhomogeneous "metamaterial" miraculously appears as a zero-diameter point



Above: Transformation Optics concept and ray tracing of a cloak



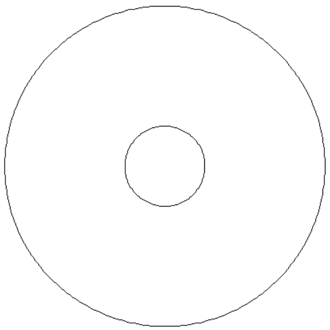
Full 3D model of a cloak: the domain is only 2 wavelengths in diameter



2.5D Model in COMSOL 4.2: some 20 wavelengths; a x1000 increase in volume

Ideal vs. Conformal Cloaks from *Science* 312, 2006

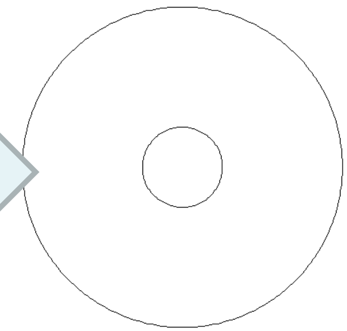
Pendry, Schurig, Smith, p.1780



Apparent width of a flat sheet of length ***d*** from angle ***θ***:

$$\sigma = d|\sin\theta|$$

Leonhardt, p.1777



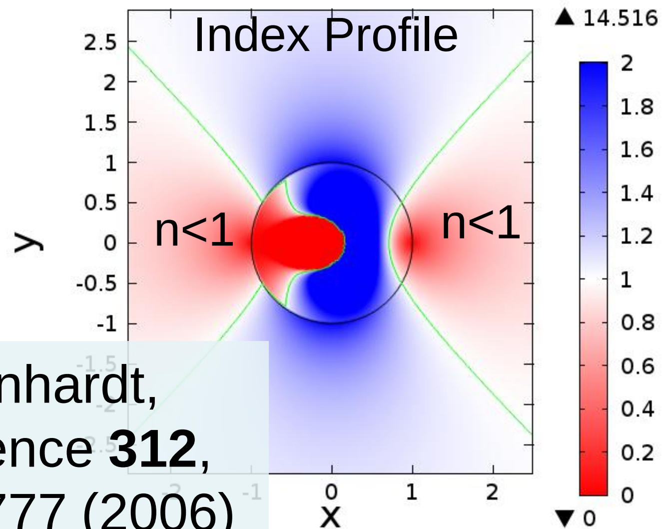
$$n_\varphi = \frac{b}{b-a}\frac{r-a}{r}, \qquad n_r = \frac{b}{b-a}$$

- Compresses object to a point
- Anisotropic refractive index
- Isotropic distribution in space
- Omnidirectional cloaking
- Finite cloak volume

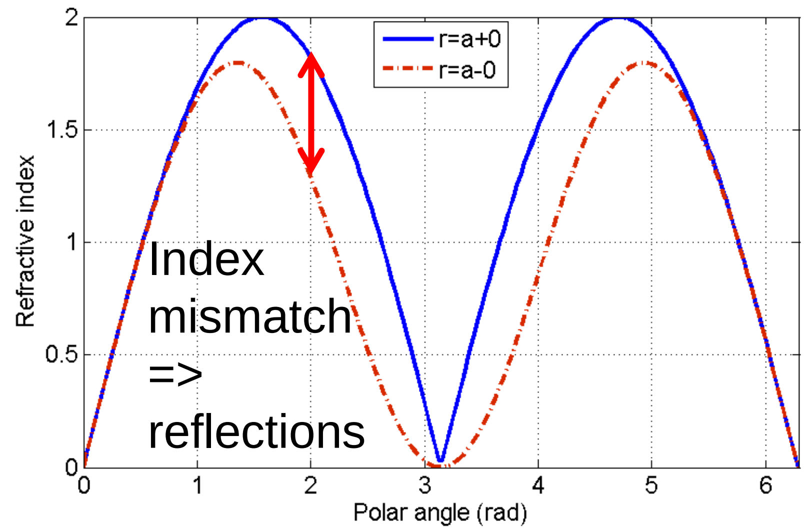
$$n_\varphi = n_r = \sqrt{1 + \frac{a^4 - 2a^2r^2\cos 2\phi}{r^4}}$$

- Flattens object to a sheet
- Isotropic refractive index
- Anisotropic distribution in space
- Limited angle range
- Infinite cloak volume

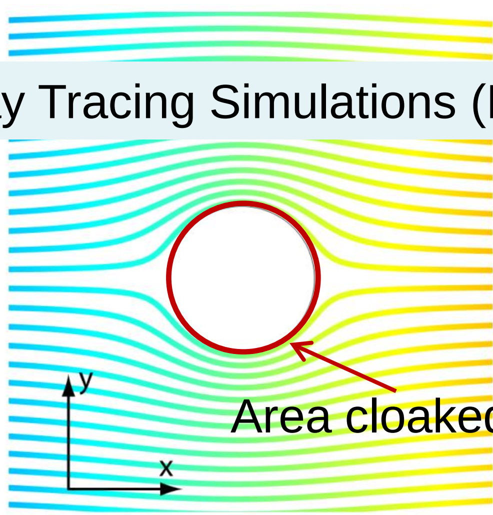
Conformal Cloaking: Directional by Nature

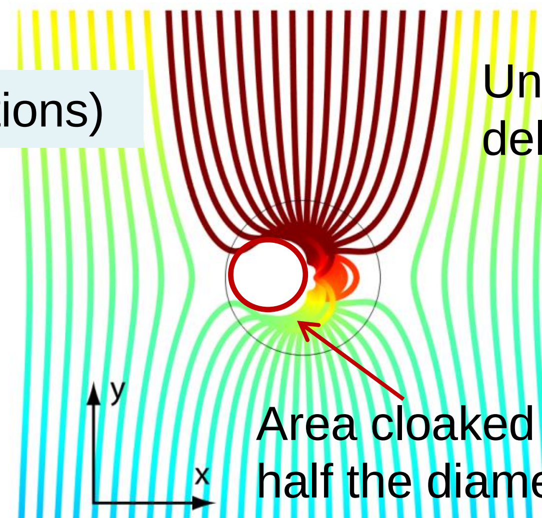


Leonhardt, Science **312**, p.1777 (2006)

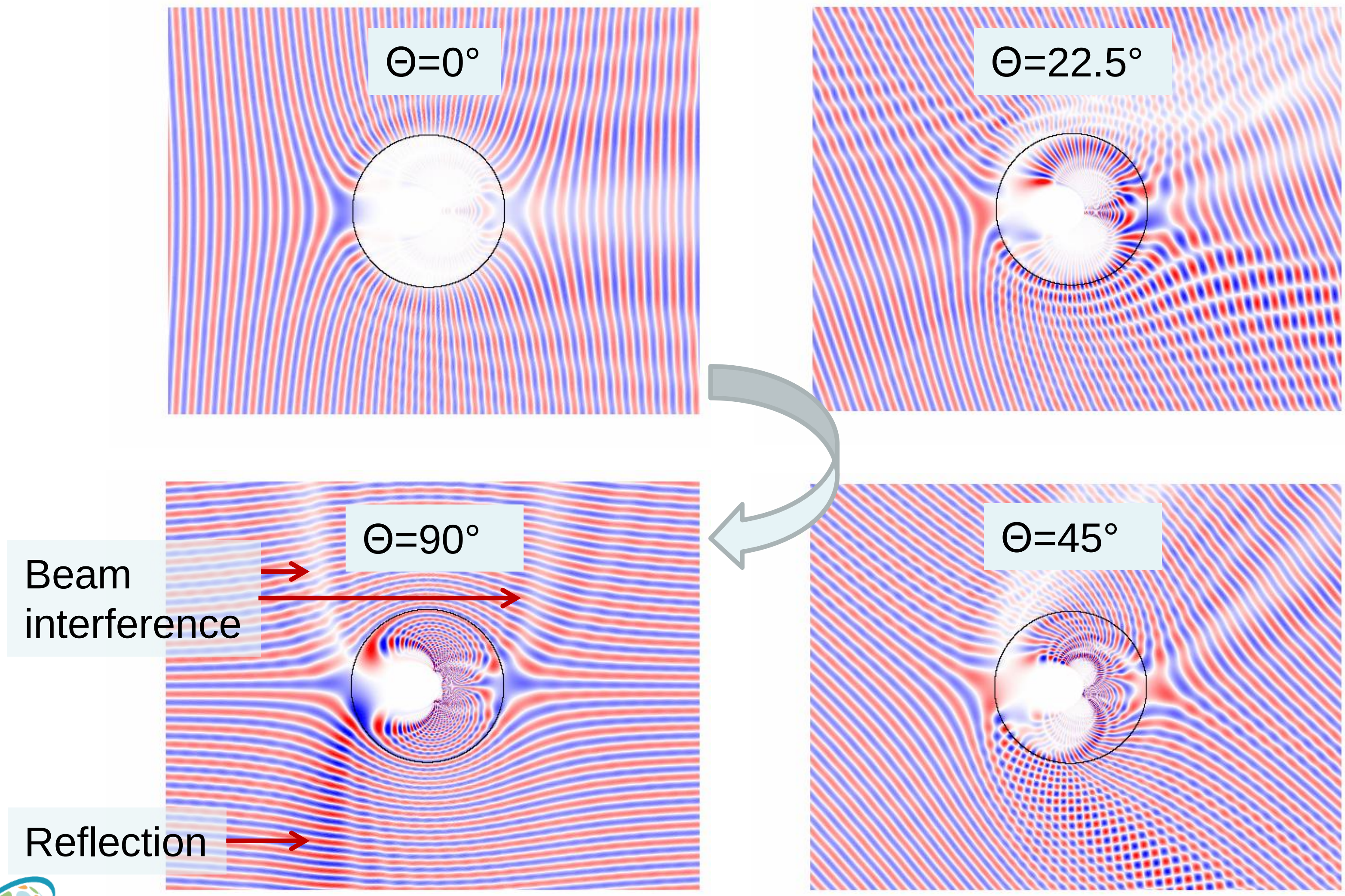


Ray Tracing Simulations (Neglecting Reflections)

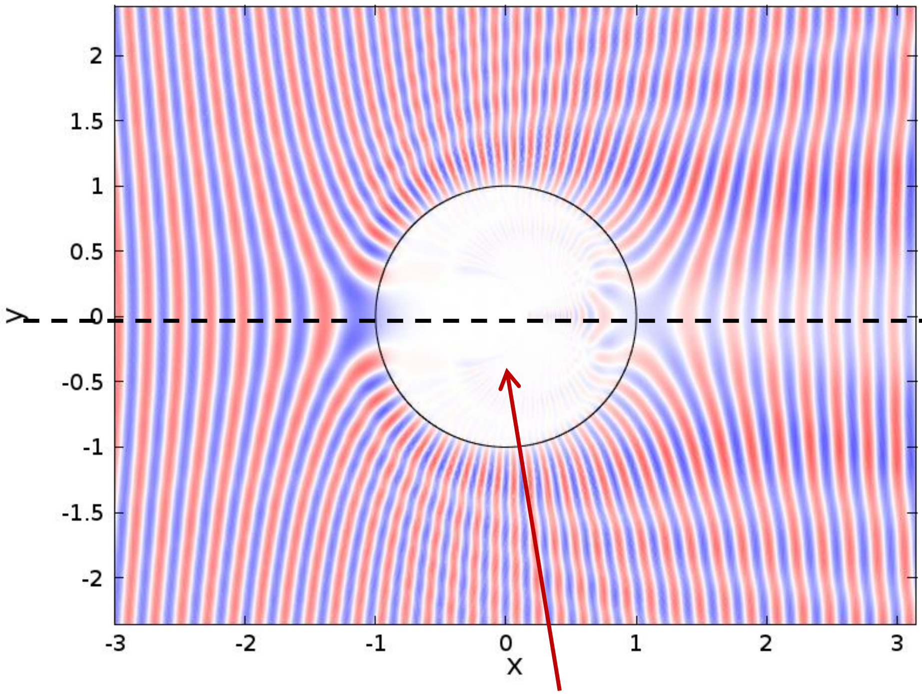


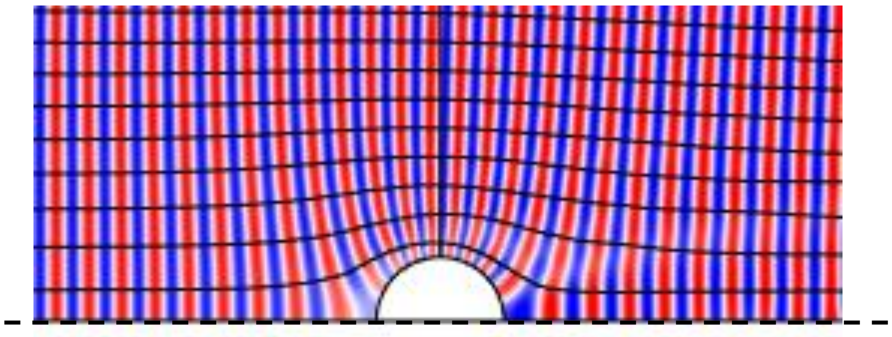


Conformal Cloaking: Directional by Nature



Conformal Cloak at its Best Angle

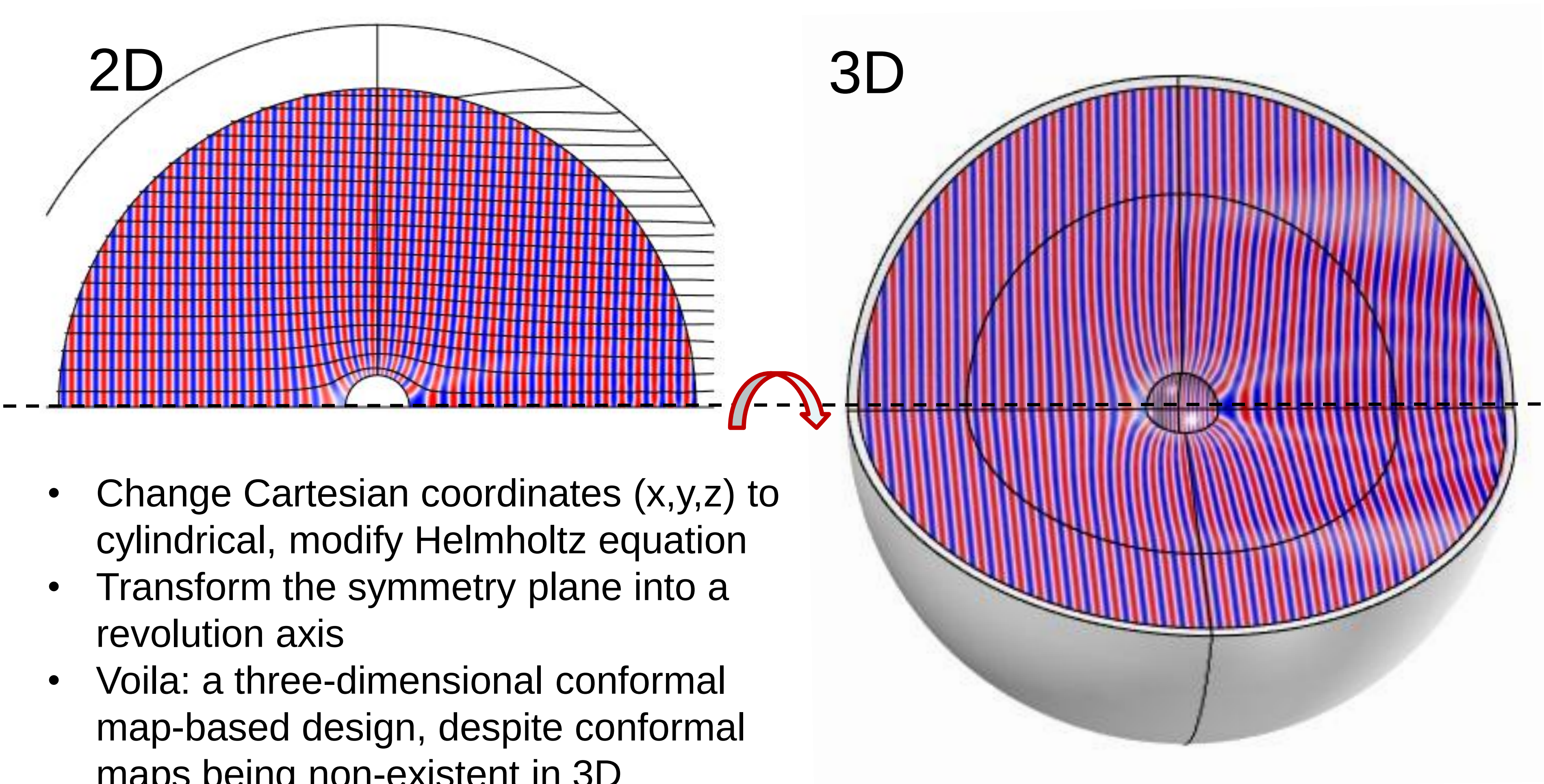




- The circle is filled with refractive index
- Fields leak into the region where they aren't supposed to be according to ray tracing

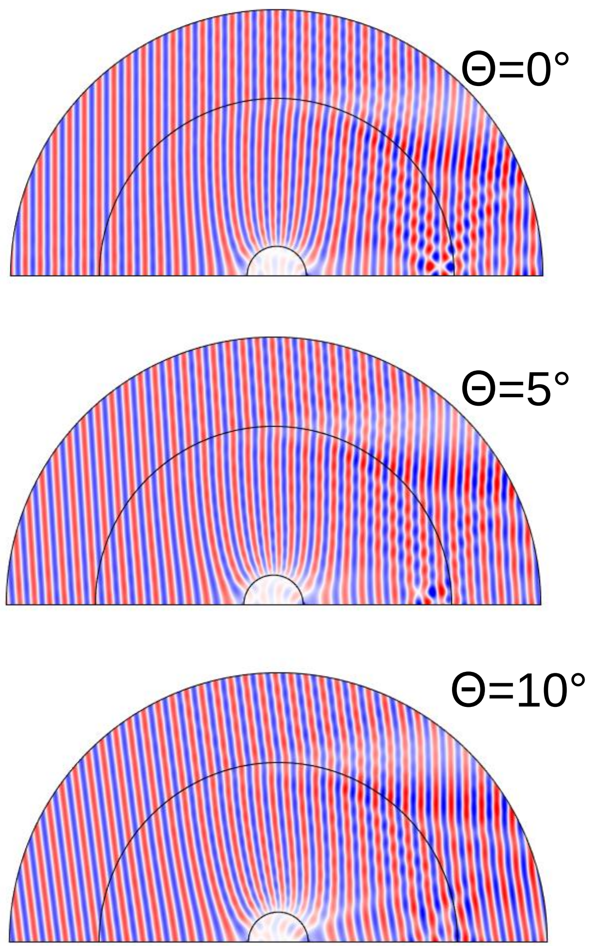
- The circle is perfectly shielded with a Neumann boundary condition
- *Symmetry plane* introduced to reduce modeling time; may or may not be a *physical reflector*
- Compare to carpet/ground-plane cloaks of Pendry, Smith et al.

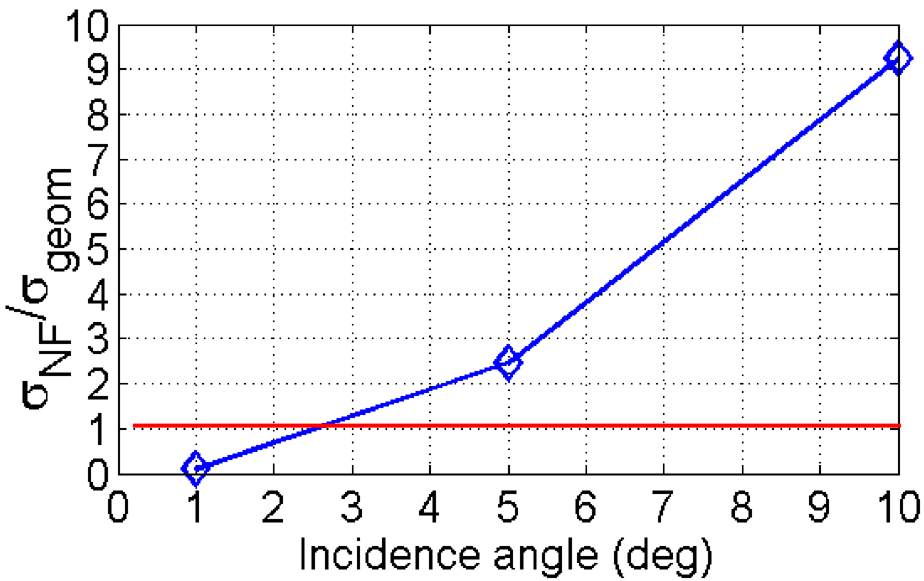
Revolved Conformal Cloak



- Change Cartesian coordinates (x,y,z) to cylindrical, modify Helmholtz equation
- Transform the symmetry plane into a revolution axis
- Voila: a three-dimensional conformal map-based design, despite conformal maps being non-existent in 3D

Plane Waves with Arbitrary Propagation Directions





Measure of visibility:

$$\sigma_{NF} = \int dS \left(\frac{|\vec{E}|^2}{|\vec{E_0}|^2} - 1 \right)$$

2.5D model: easily 40 wavelengths per domain, arbitrary incidence and polarization

Conclusions

- 2D Axisymmetry is abundant in engineering and nature
- To take full advantage of rotational symmetry, one should use 2.5D modeling
- Applications in acoustics, optics, microwave engineering, …
- Particularly useful for wave scattering problems, including the problem of cloaking

Background Image: *Discover Magazine*, Special issue “Invisible Planet”, July/August 2012

Acknowledgements and Your Questions

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Yaroslav Urzhumov is the Technical Coordinator of CMIP.