

Going beyond Axisymmetry: 2.5D Vector Electromagnetics

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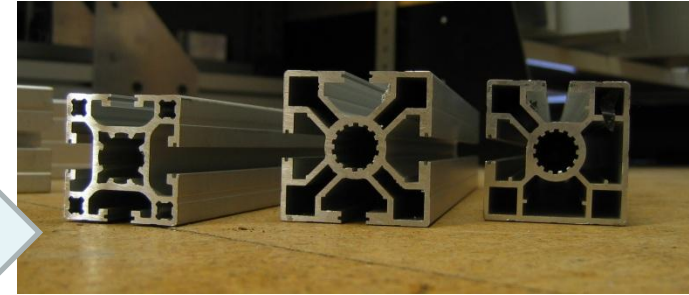
Talk Outline

- 1) Conventional Axisymmetric Modeling
 - Electro- and Magnetostatics
 - Fluid Dynamics
- 2) Going 2.5D: General Recipe
- 3) Vector 2.5D Electromagnetics
 - Multiscale Problems in Plasmonics
 - Optical Scattering and Cloaking Problems



Low-Dimensional Modeling

- The real world is 3D, but symmetries are abundant
- Specific cases lead to solutions where fields are independent of 1 coordinate
- 2D Modeling variations:
 - **Translational** symmetry: $d/dz=0 \Rightarrow$ conventional 2D simulations in Cartesian coordinates
 - **Rotational** symmetry: $d/d\phi=0 \Rightarrow$ axisymmetry modeling in cylindrical coordinates
 - Invent your own coordinate system that fits the symmetry of your particular structure
- Prerequisite to all low-dimensional modeling: the structure must possess some continuous symmetry
 - If a real-world system doesn't, gain understanding with a symmetrized structure



Extruded shapes

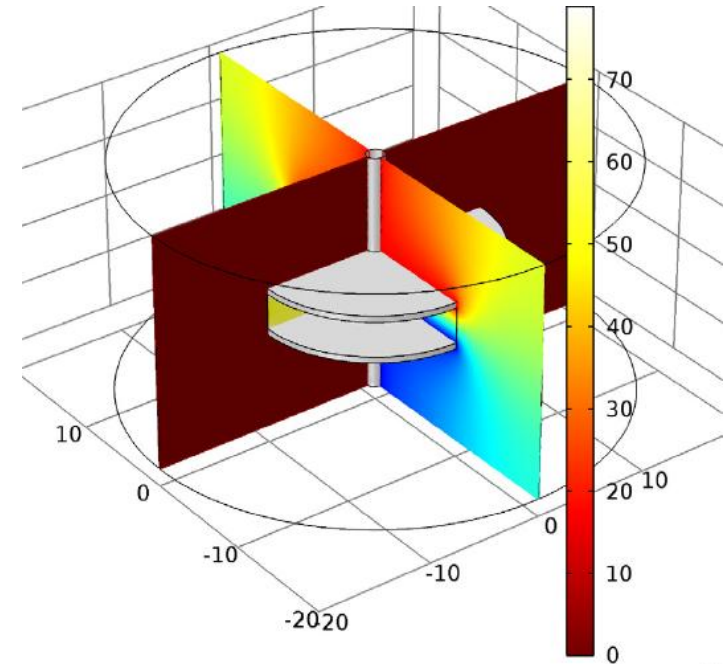


Revolved shapes



2D Axisymmetry Modeling Examples

- Various types of physics:
- **Electro- and magnetostatics, scalar potential**



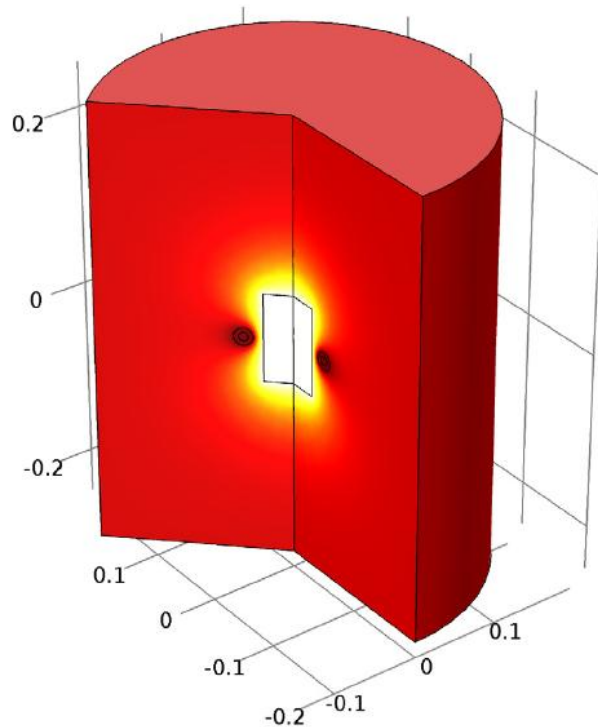
Circular capacitor

*Graphics taken from
COMSOL Model
Library*

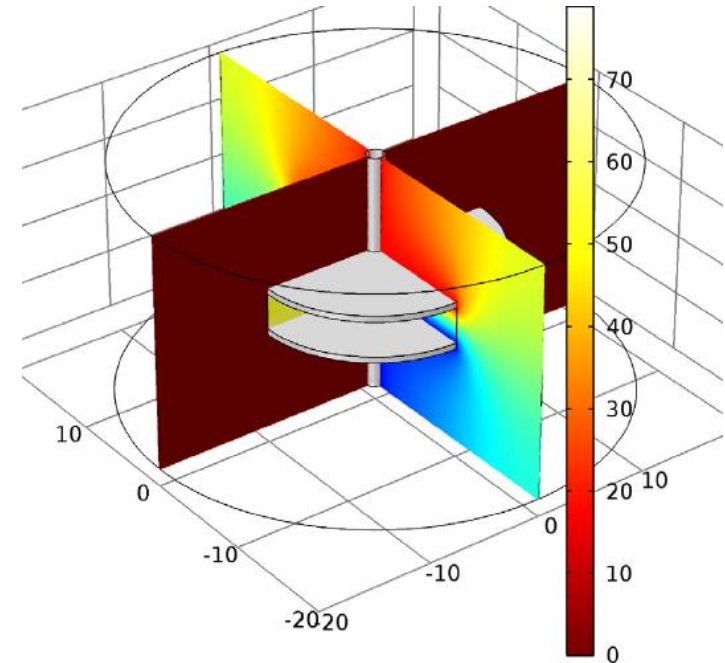


2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- **Conductive heat transfer**



Inductive heating of a metal cylinder



Circular capacitor

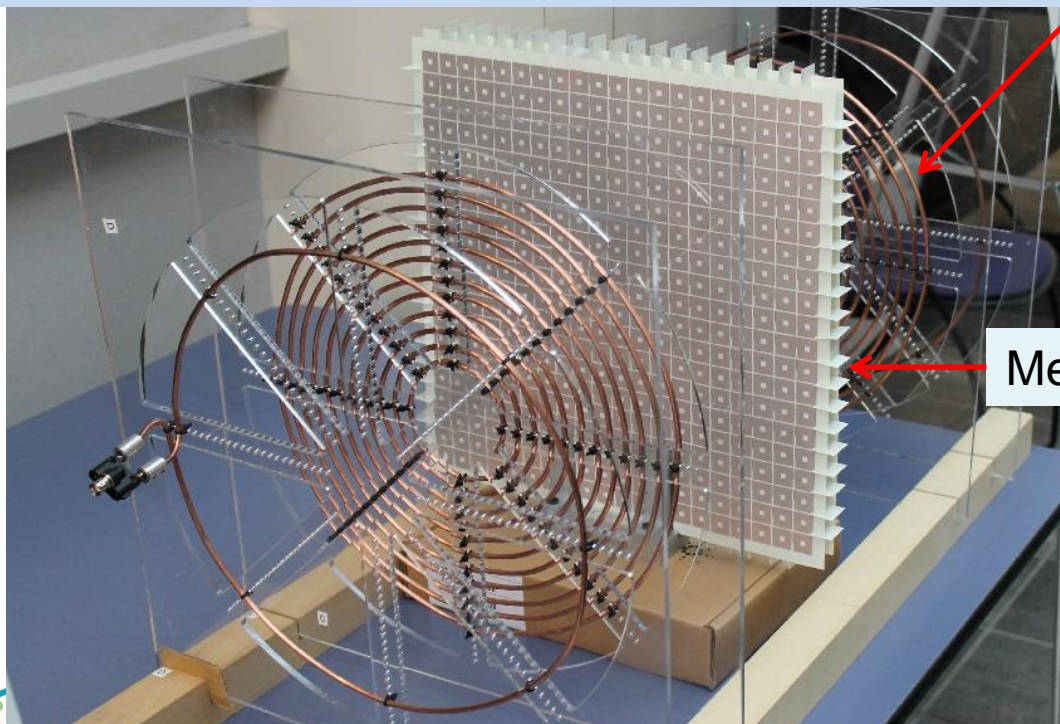
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COMSOL Model
Library*



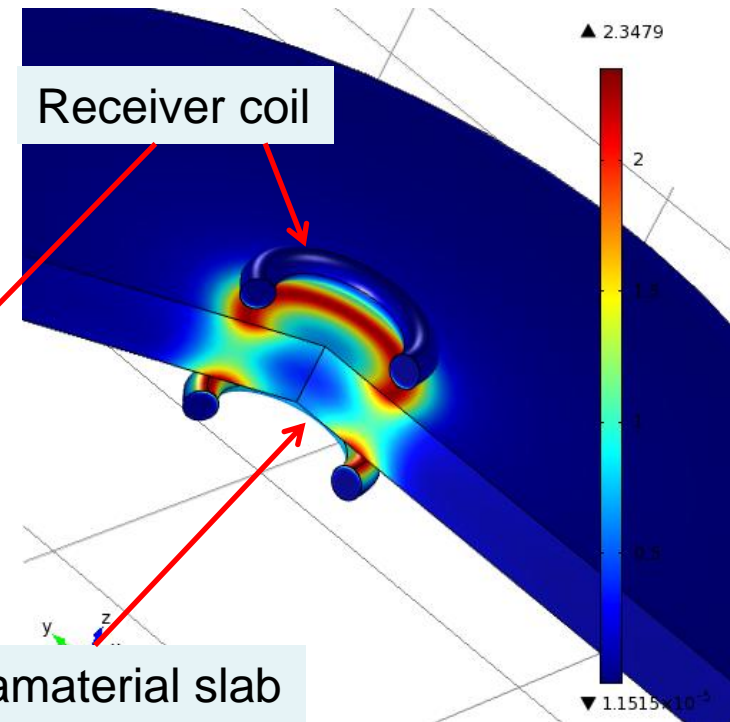
2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- Conductive heat transfer
- **Quasi-magnetostatics, vector potential**

Real system: wireless power transfer, 13.56MHz



Photograph courtesy Toyota Research Institute of North America, Jan. 2012



Toy model: circular coils, disk-shaped lens
Journal ref: J. Appl. Phys. 111, 064902 (2012)

2D Axisymmetry Modeling Examples

- Various types of physics:
- Electro- and magnetostatics, scalar potential
- Conductive heat transfer
- Quasi-magnetostatics, vector potential
- Fluid dynamics

Real system: submarine

INVISIBLE WARRIORS

COMPLETE CLOAKING A material that contains acoustic and fluid cloaking systems to control, trap, and scatter light, an acoustic wave or a fluid flow, can make an object invisible to a wide range of waves. It works by scattering waves in a way that they don't appear to have interacted with the object at all.

SILENT SOUNDS Defense systems can detect stealth submarines using two methods: sonar, which bounces sound waves off a craft, and radar, which can detect subtle disturbances on or below the ocean's surface that can indicate a sub at depth. New scientists are developing acoustic and fluid cloaking methods that could detect both. The acoustic cloaking uses metamaterials—artificially constructed objects to display certain natural properties, such as magnetism or refraction—to bend sound waves around the craft, making it undetectable to sonar. A fluid cloak would alter the flow of water over a craft, concealing any wake or turbulence and eliminating the noise that sonar can detect. Both kinds of cloaking could be used to hide a submarine. The Office of Naval Research is currently running an ongoing program, but it is likely aware that other nations may be working on them—the Navy being a top priority.

ZERO SOUND A fluid cloak would consist of hundreds of small water jets, similar to those found in porpoises, that would scatter a sound wave. The jets would accelerate water as control the sound flow and cancel out the waves to the left (sonar and radar). With no net change in speed, the water would flow around the sub unnoticed, providing no disturbance.

SUBTLE CLOAKINGS A suit traveling at depth can still be tracked by location. The light on the water surface would be an object moving in a straight line. A fluid cloak could be designed to scatter the light from a V-shaped hole (sonar) instead of a flat surface, which can also be detected on the surface. Fluid cloaking would scatter both the sound waves and the light waves.

BETTER SOUNDPROOFING Scientists can engineer metamaterials that bend sound waves around an object by controlling the material's density and elasticity. Waves speed up when they hit the cloak and slow down when they hit the object. So far, scientists have created simple shapes such as cones and spheres. But cloaking may mean complex shapes could be cloaked as well.

Invisibility Comes Within Reach

THE SCIENCE OF STEALTH has long been a matter of hiding your arse, obscure engineers and scientists say, in the deep sea.

new developing materials that could hide anything in plain sight. Instead of bending light inward, the water and glass do. These optical metamaterials bend it outward, bending photons around an object like waves around a stone. The more edges of metamaterials are smooth, the more effective they are.

Light 1000 to 700 nanometers. Light cannot pass unimpeded through any space smaller than its own wavelength, so it gets trapped in the grid. Optical photons can be steered, manipulated or, in this case, hurled around an object and returned to their original course. An object cloaked by a perfectly opaque

limit first physicists were able to produce only paper-thin metamaterials sheets just large enough to cloak objects the size of a bacterium. Last June, John Rogers, a materials scientist at the University of Illinois, unveiled a metamaterial of plastic. "We can now bring the objects of this stuff" he says, "though the sheets will be practical." The sheets are 500 nanometers thick. For now, objects about the size of a bacterium would just appear as thin sheets, since the design is still imperfect. "Being an invisibility cloak would be a huge problem to face," Rogers says. "It's a starting point in the first place is the bigger challenge."

—BILLY WOODEN

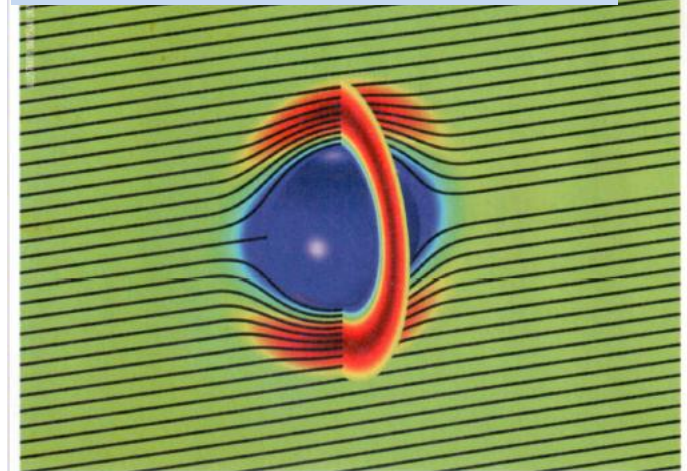
POPULAR SCIENCE • JANUARY 2012

Popular Science mag., Jan. 2012

DUN LAAGJE VERWIJDERT ZOG ACHTER VOORWERP ONDERWATERSTEALTH

Een voorwerp in een stromende vloeistof kan onzichtbaar worden gemaakt met een dun poreus laagje. Dat manipuleert de stroming om

Toy model: spherical cloak



Simulatiebeeld van de waterroom rond een bol. De kleuren geven de stroomsnelheid weer: groen staat voor de uniforme stroom, rood voor een hogere snelheid en blauw betekent snelheid nul.

Onderzoeker dr. Yaroslav Urzhumov van Duke University in de Verenigde Staten heeft op theoretische basis de situatie bestudeerd van een bol in een homogene waterstroming. Op de bol zit een laag poreus metaalmateriaal dat een negatieve druk vlak voor de bol creëert door water naar binnen te zuigen en naar achteren te leiden. Daar wordt het water weer geloosd, met precies dezelfde druk die het had voordat het de structuur inging. Hierdoor lijkt het van buitenaf bekeken net of de bol er helemaal niet is. Op de illustratie blijkt dat uit het feit dat de zwarte stroomlijnen vóór en na de bol er elkaars verlengde liggen. Het concept is vergelijkbaar met elektronische metamaterialen, die licht op zo'n manier om een voorwerp heen buigen dat ze theoretisch als onzichtbaarheidsmantel kunnen werken.

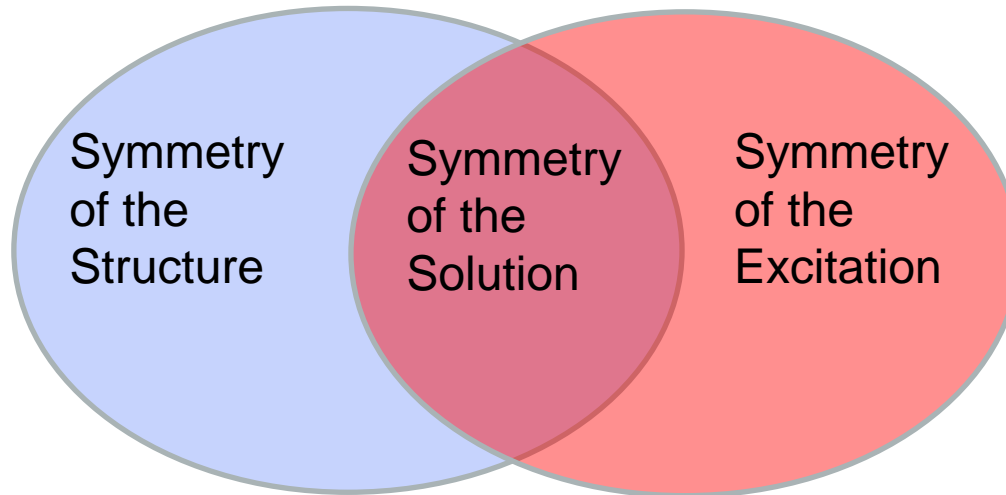
De bevindingen van Urzhumov zijn voorlopig alleen theoretisch van aard, maar de Amerikaanse Marine heeft al belangstelling getoond. "De eerste toetsing van dit principe lijken met kleine onbemande stealth onderzeeërs te staven

De Ingenieur mag., Sept. 2011

www.duke.edu

2D Axisymmetry Limitations

- Even for a rotationally symmetric structure, not every solution is axially symmetric!



- $G_{\text{solution}} = G_{\text{structure}} \cap G_{\text{excitation}}$ ($G = \text{symmetry group}$)

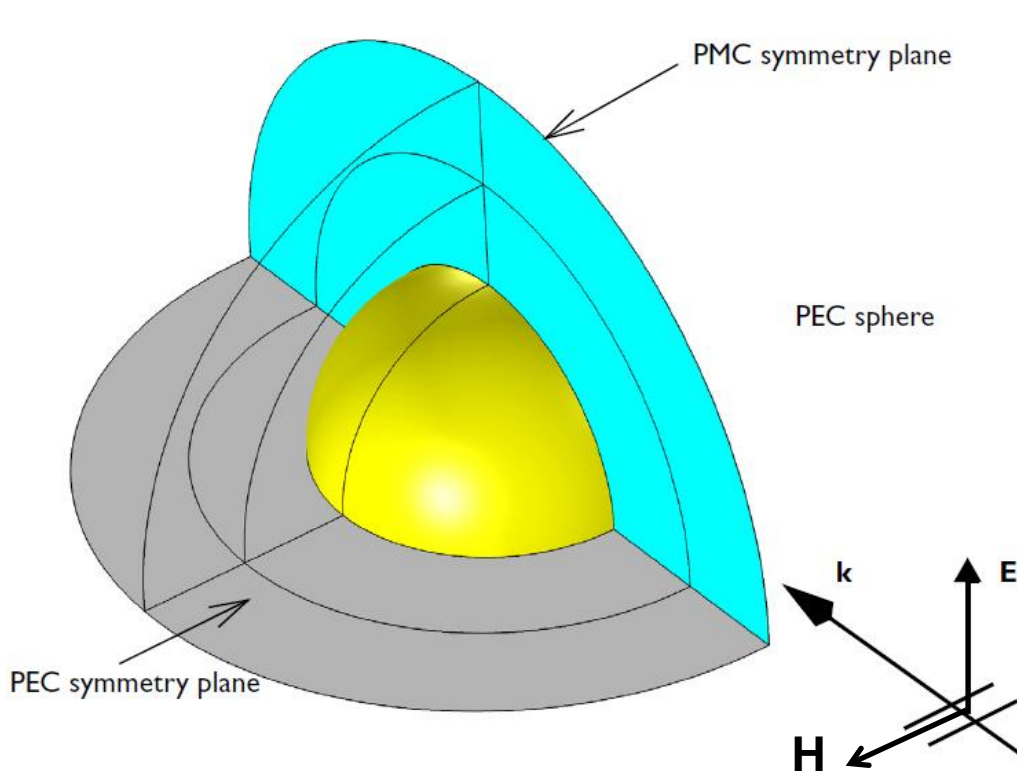


2D Axisymmetry Limitations

- Example: Mie scattering of a plane wave

Symmetry of the
Structure:
Spherical

Symmetry of the
Solution: C_{2v}



Graphics courtesy
COMSOL

Symmetry of
the **Excitation**:
 C_{2v}
(two symmetry
planes)

- Can reduce modeling to a quarter, but not to a 2D slice!



General Recipe for Generalized Axisymmetric Modeling



Decompose

- Each term is char
- $E_{\nu,\mu\lambda}(r,z,\phi) = \sum_{m'} \dots$
- Expand excitation
- Plane wave excita
- Each term is char
- $E_{\nu,\mu\lambda}(r,z,\phi) = \sum_{m'} \dots$
- Expand excitation
- Plane wave excita
- Each term is char



Solve

- Each cylindrical wave propagates independently of others
- For each wave, field dependence upon ϕ is prescribed: $\exp(i m \phi)$
- Write the equations in cylindrical coordinates, replace $d/d\phi \rightarrow i m$
- Do so with boundary conditions if needed (Important: **Scattering** boundary condition)
- Overall factor $\exp(i m \phi)$ cancels out, **2D equation in (r,z) coordinates left**



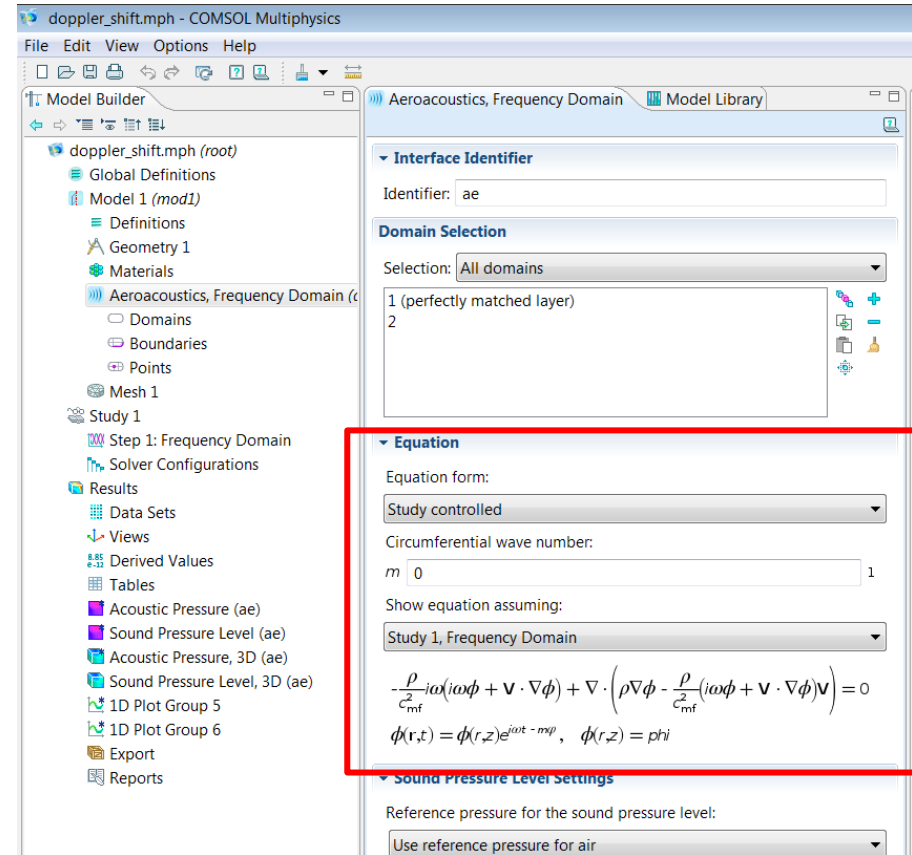
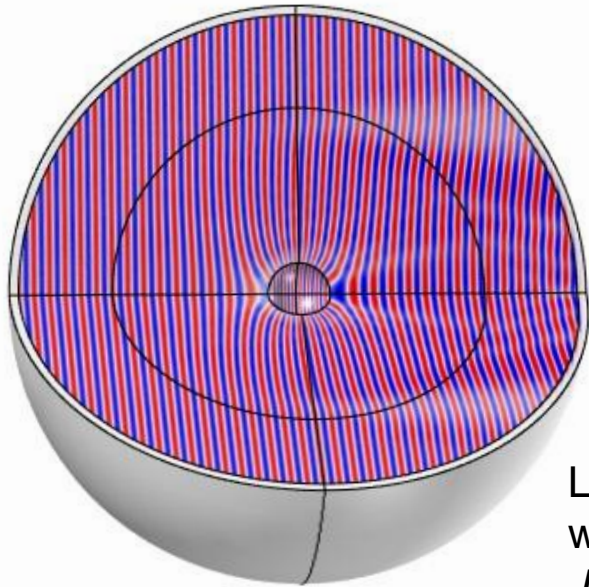
Compose

- Postprocessing: to plot solution for a given value of ϕ , multiply each partial wave with $\exp(i m \phi)$ and sum over all m 's
- Use COMSOL's built-in revolved plots for **3D visualization**



2.5D Axisymmetry for Scalar Waves

- Has been essentially available in the Acoustics Module as part of *pressure acoustics* and *aeroacoustics* interfaces in COMSOL 3.0+
- Feature called “*circumferential wave number (m)*”

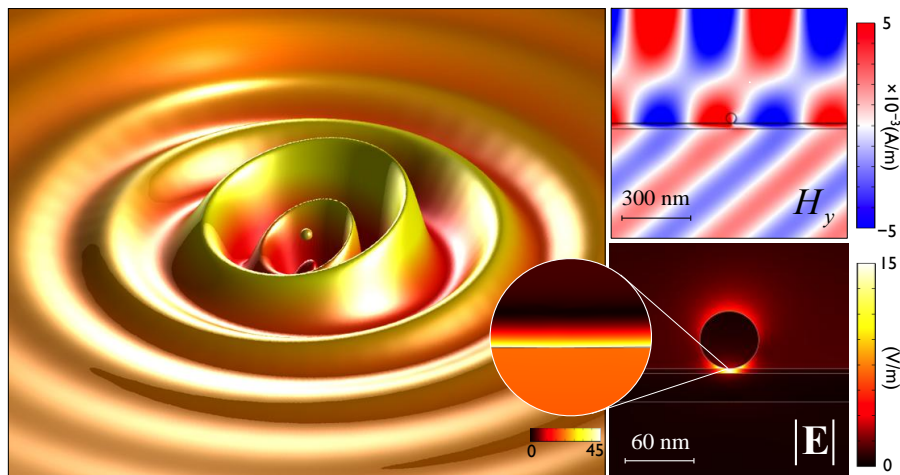


Left: acoustic cloaking in fluids (air, water, etc.); modeled with Acoustics Module, COMSOL 4.1.

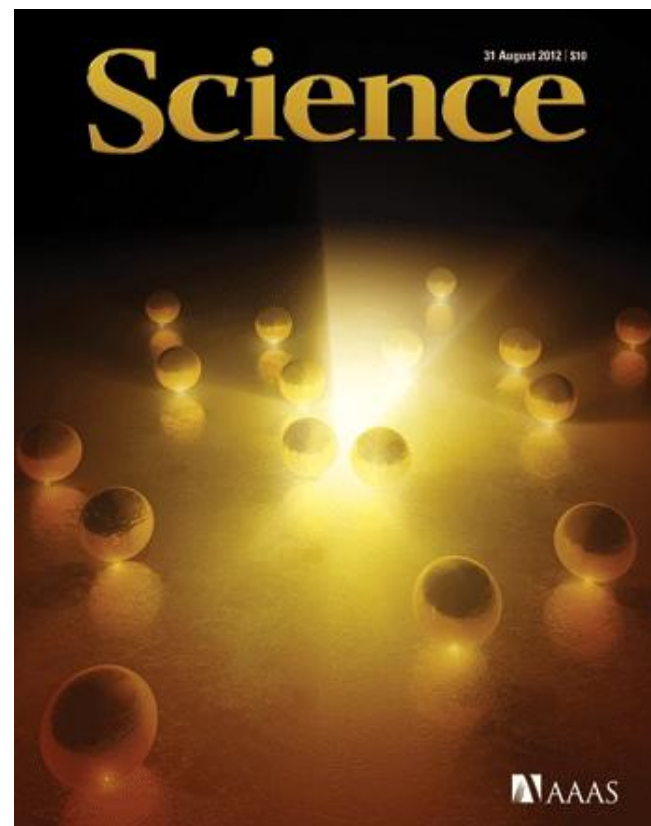
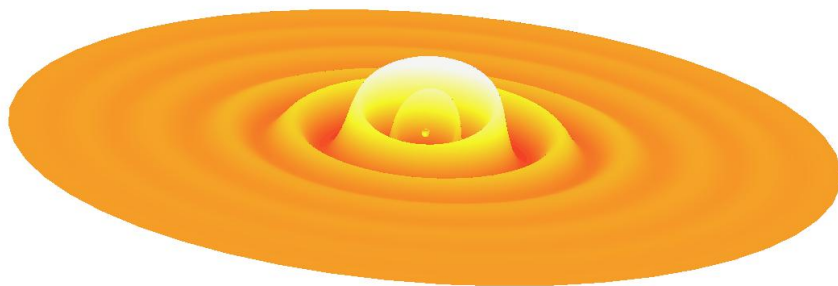
*Journal ref: J. Appl. Phys. 111, 053105 (2012);
arXiv:1203.5831v1*



2.5D Vector Electromagnetics for Scattering Problems: Nonlocal Plasmonics



See the Supplementing materials of this paper for details of the implementation

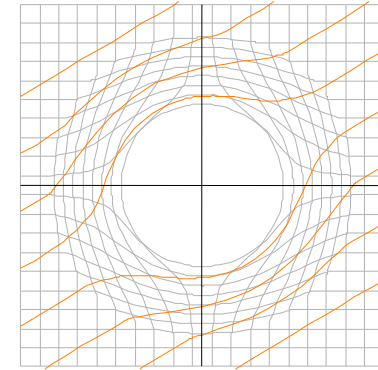


C. Ciraci et al., Science **337**,
1072 (August 31, 2012)
DOI: 10.1126/science.1224823

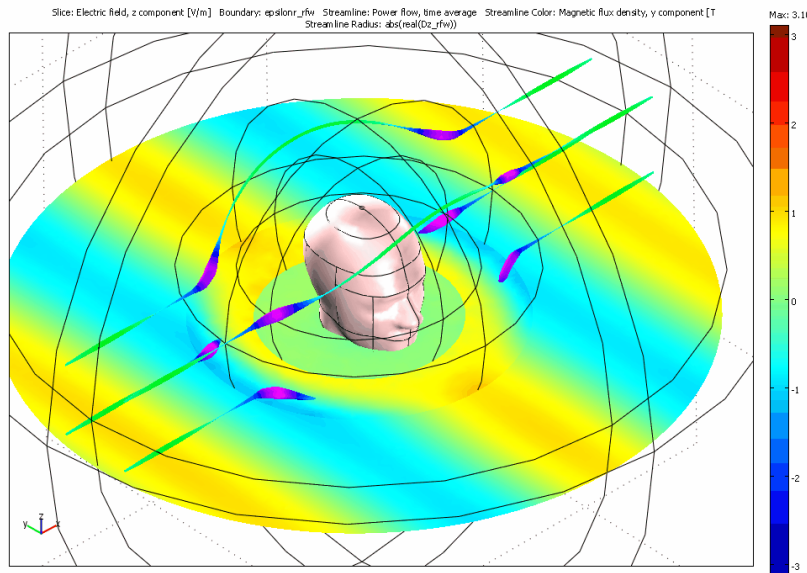


Another Scattering Problem: Optical Cloaking

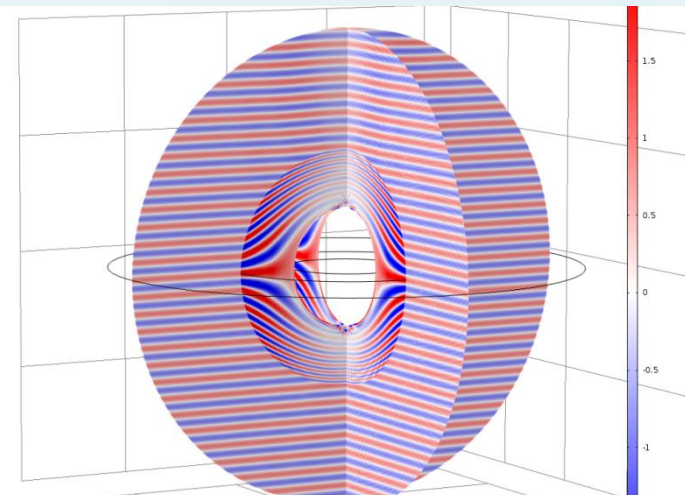
- Novel scattering problem, soon to become canonical: *scattering cancellation (invisibility, cloaking)*
- The only known exact solution is based on so-called Transformation Optics
- A scatterer coated with a special inhomogeneous “metamaterial” miraculously appears as a zero-diameter point



Above: Transformation Optics concept and ray tracing of a cloak



Full 3D model of a cloak: the domain is only 2 wavelengths in diameter

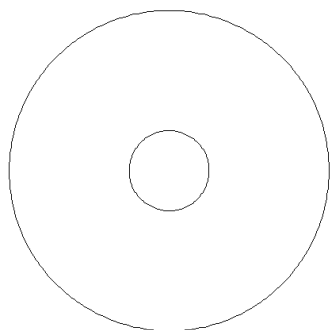


2.5D Model in COMSOL 4.2: some 20 wavelengths; a x1000 increase in volume



Ideal vs. Conformal Cloaks from *Science* 312, 2006

Pendry, Schurig, Smith, p.1780



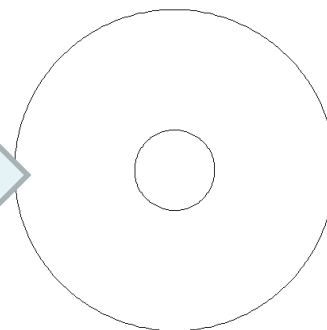
$$n_{\phi} = \frac{b}{b-a} \frac{r-a}{r}, \quad n_r = \frac{b}{b-a}$$

- Compresses object to a point
- **Anisotropic** refractive index
- **Isotropic** distribution in space
- Omnidirectional cloaking
- Finite cloak volume

Leonhardt, p.1777

Apparent width of a flat sheet
of length d from angle θ :

$$\sigma = d |\sin \theta|$$

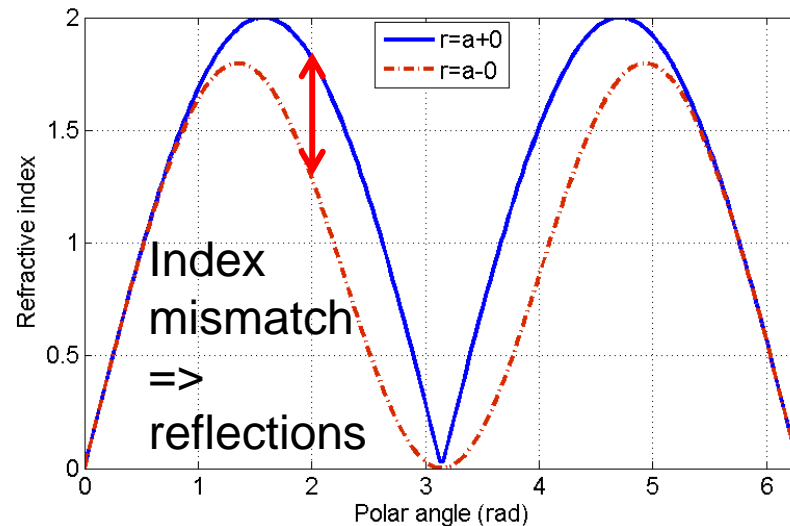
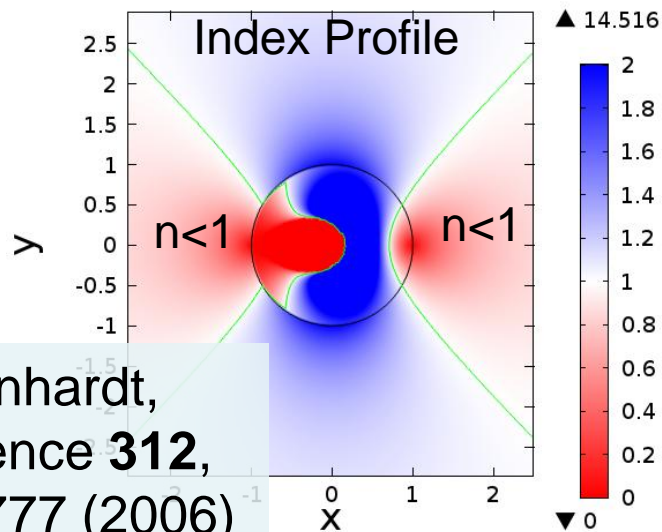


$$n_{\phi} = n_r = \sqrt{1 + \frac{a^4 - 2a^2 r^2 \cos 2\phi}{r^4}}$$

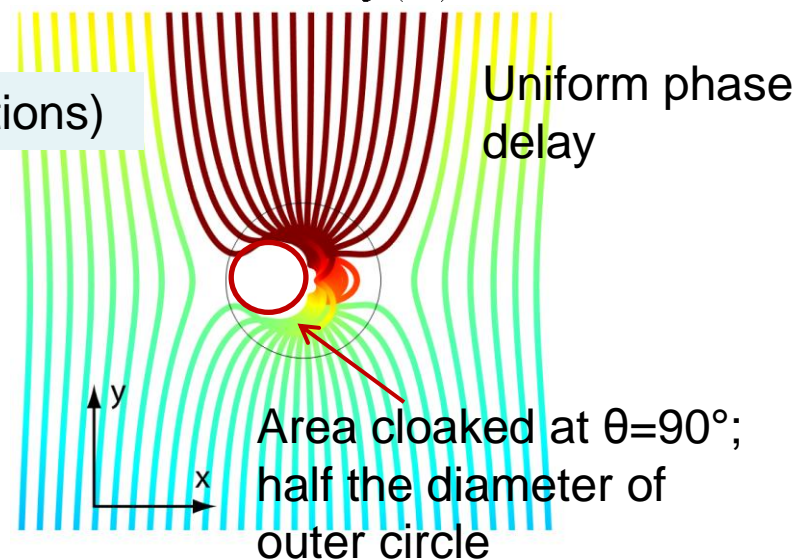
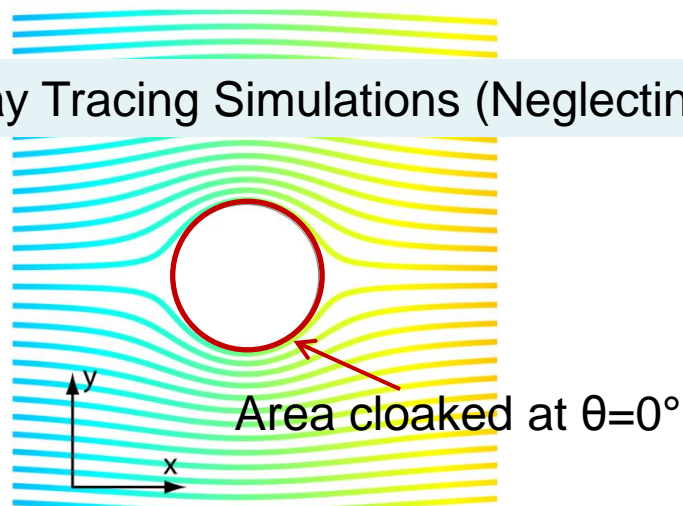
- Flattens object to a sheet
- **Isotropic** refractive index
- **Anisotropic** distribution in space
- Limited angle range
- Infinite cloak volume



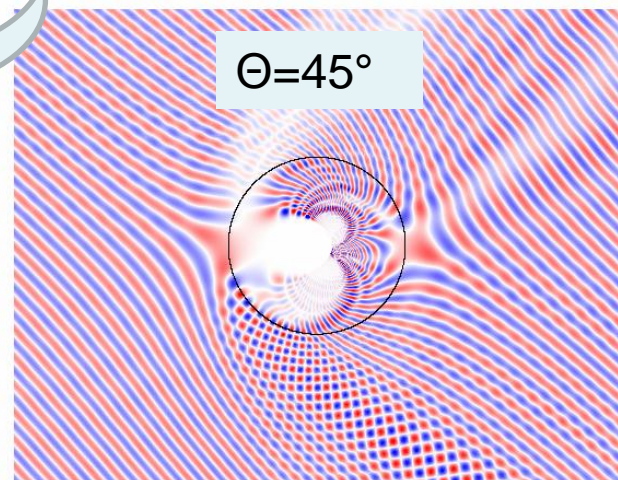
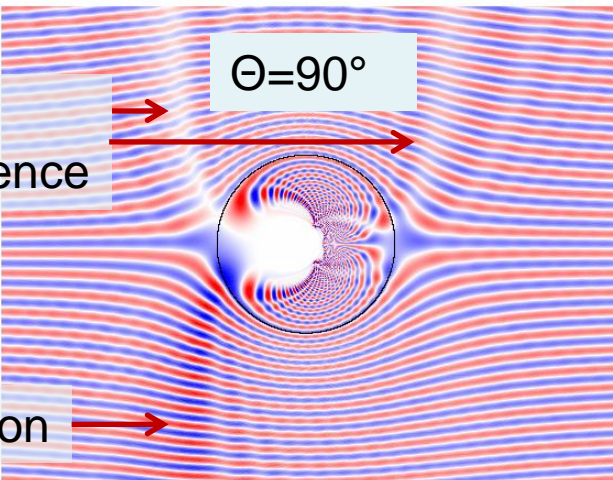
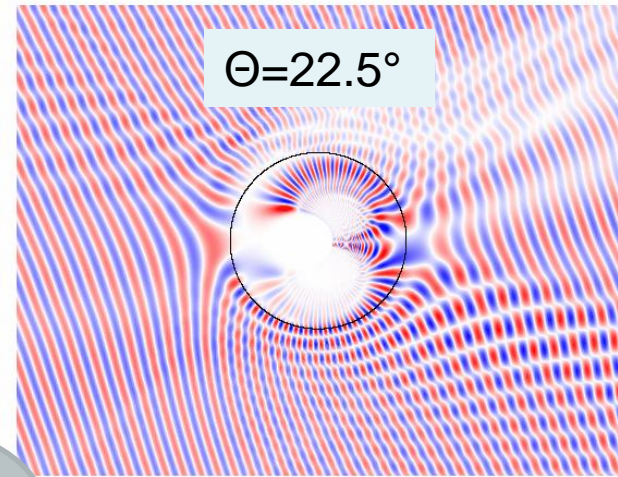
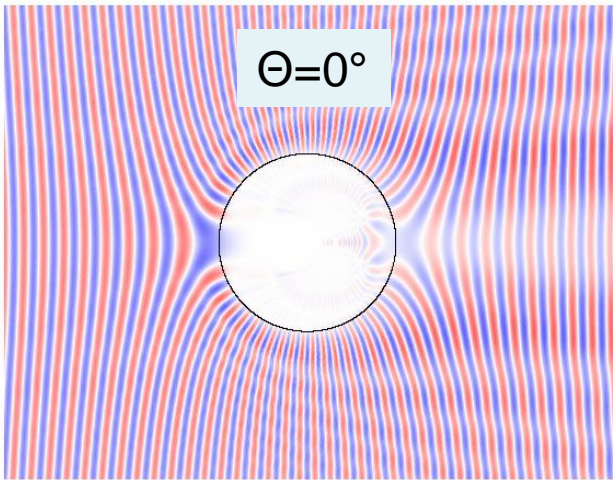
Conformal Cloaking: Directional by Nature



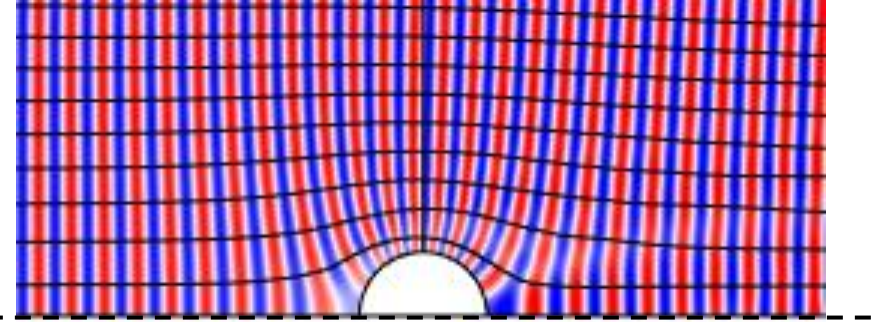
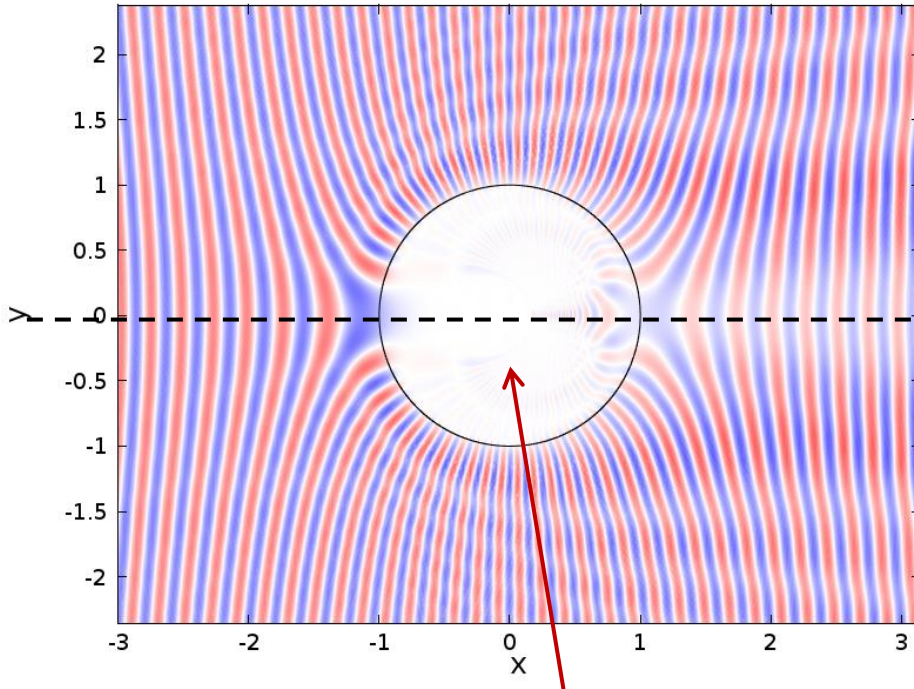
Ray Tracing Simulations (Neglecting Reflections)



Conformal Cloaking: Directional by Nature



Conformal Cloak at its Best Angle



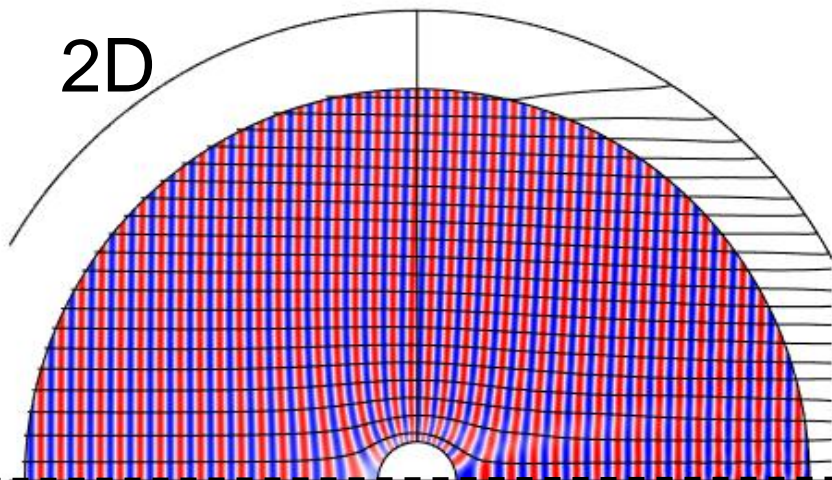
- The circle is filled with refractive index
- Fields leak into the region where they aren't supposed to be according to ray tracing

- The circle is perfectly shielded with a Neumann boundary condition
- *Symmetry plane* introduced to reduce modeling time; may or may not be a *physical reflector*
- Compare to carpet/ground-plane cloaks of Pendry, Smith et al.

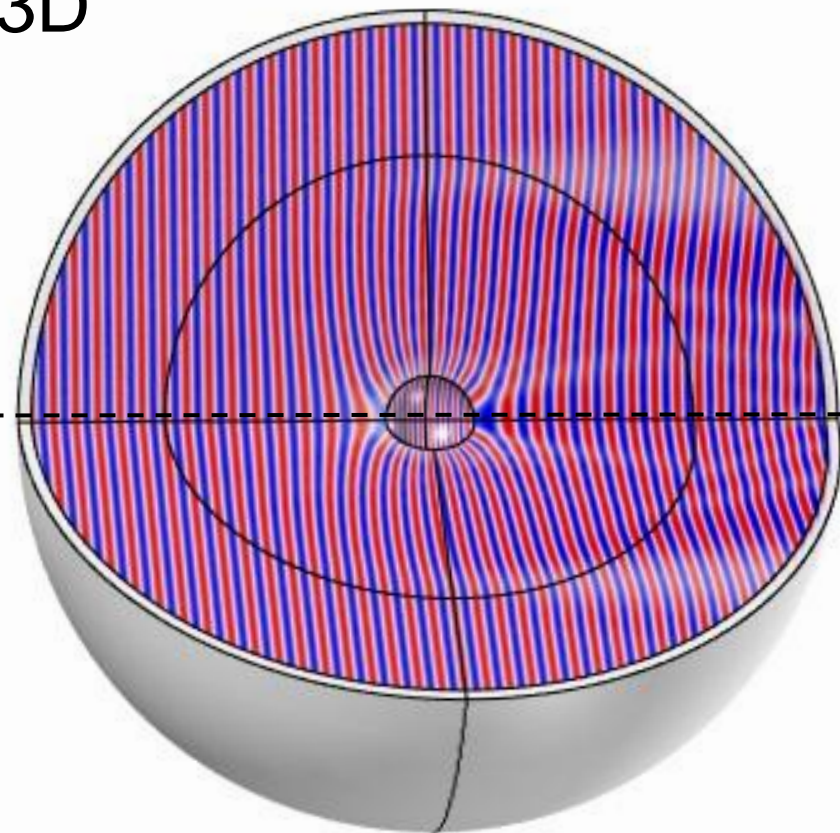


Revolved Conformal Cloak

2D



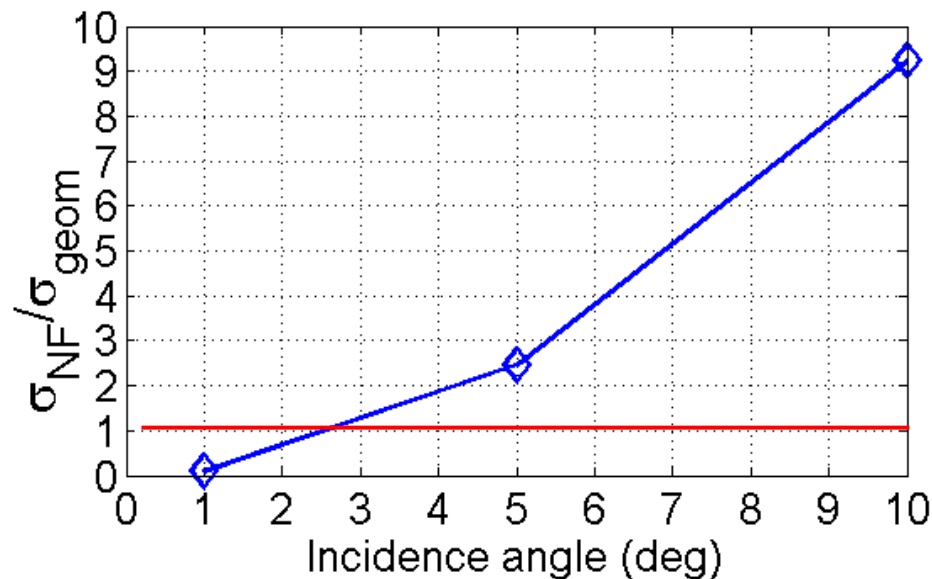
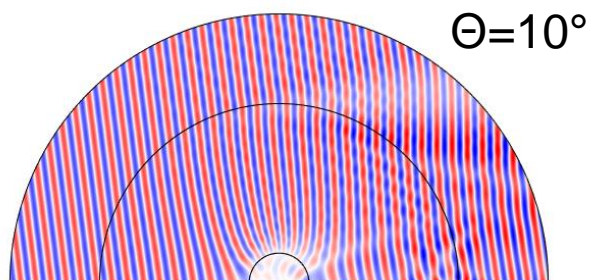
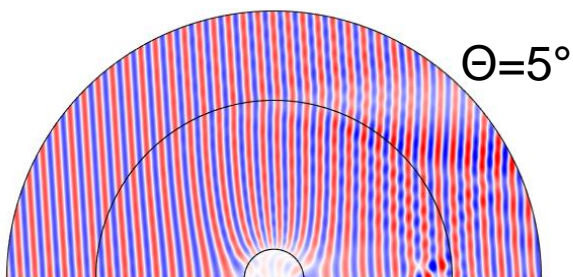
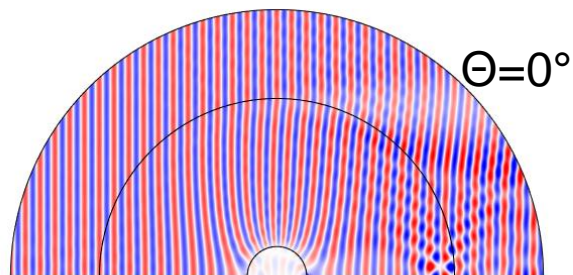
3D



- Change Cartesian coordinates (x,y,z) to cylindrical, modify Helmholtz equation
- Transform the symmetry plane into a revolution axis
- Voila: a three-dimensional conformal map-based design, despite conformal maps being non-existent in 3D



Plane Waves with Arbitrary Propagation Directions



Measure of visibility:

$$\sigma_{NF} = \int dS \left(\frac{|\vec{E}|^2}{|E_0|^2} - 1 \right)$$

2.5D model: easily 40 wavelengths per domain, arbitrary incidence and polarization



Conclusions

- 2D Axisymmetry is abundant in engineering and nature
- To take full advantage of rotational symmetry, one should use 2.5D modeling
- Applications in acoustics, optics, microwave engineering, ...
- Particularly useful for wave scattering problems, including the problem of cloaking



October 3, 2012

Background Image: *Discover Magazine*,
Special issue "Invisible Planet",
July/August 2012

Acknowledgements and Your Questions

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