Modeling of Anisotropic Laminated Magnetic Cores using Homogenization Approaches

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Abstract: A specific issue in transformer modeling using the finite element method is the consideration of electric sheets or other laminated core materials which are commonly used to reduce eddy currents. Instead of explicitly modeling the geometry of every single core sheet material homogenization procedures can be applied. They substitute the laminated core structure for a virtual bulk material which has the same outer dimensions and nearly the same electromagnetic behavior. In our study, we have implemented a selection of different homogenization approaches in an inductor model in COMSOL Multiphysics. Simulation results thereby obtained are compared to those from models with explicitly modeled core structure and also to experimental test results. This reveals the accuracy and the application limits of the investigated homogenization approaches.

Keywords: transformer model, laminated core, electric sheets, homogenization procedure, nonlinear ferromagnetic material.

1. Introduction

3D-modeling and simulation of inductors and transformers with finite element methods is a challenge due to the involved nonlinearities and coupling effects between different physical domains. Primarily, there are the non-linear magnetic characteristics of the core materials including ferromagnetic hysteresis and eddy currents. With a non-linear magnetic material or a non-harmonic excitation, simulations need to be performed in transient rather than in time-harmonic mode.

A specific issue in transformer modeling is the proper consideration of laminated core materials which are used to reduce eddy currents and thus to minimize dynamic hysteresis losses. Normally, the thickness of the individual sheets is small as compared to the core thickness. Therefore, it would be impractical to explicitly model a large number of sheets with a high dimensional aspect ratio, as this would lead to a large number of elements and hence to unacceptable computational costs. Therefore, several material homogenization approaches for laminated cores have been proposed in the past. They replace the laminated structure with a single domain of an electrically and magnetically orthotropic material which exhibits similar macroscopic behavior in a certain range of conditions. Thus, the computational effort is significantly reduced.

In a former study, we compared several homogenization approaches for laminated toroidal cores in a 3D transformer model [1] by simulating the influence of the core eddy currents on the dynamic hysteresis losses. The example revealed significant differences between the results of the approaches. However, they could not be directly compared to test results as there was a remarkable static hysteresis which was not included in our models. In this paper we now present improved models together with their experimental validation. Figure 1 depicts an example of the toroidal magnetic cores which were investigated both experimentally and in simulations.

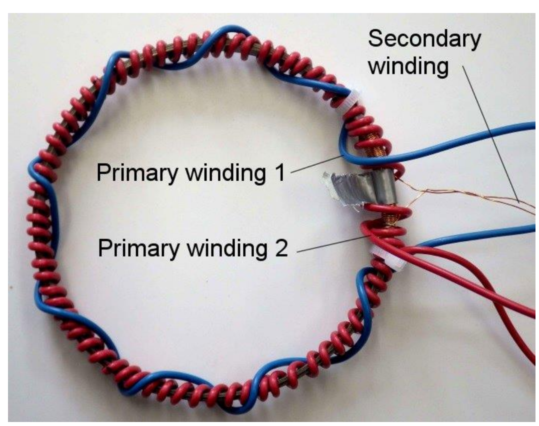

Figure 1. Permalloy tape-wound core for static and dynamic hysteresis measurements.

2. Transient Inductor Model

For an accuracy analysis of the dynamic performance of models that base on material homogenization approaches the results obtained from the simulations must be compared to real test results. However, this would require quite a large number of different samples and measurements if the comparison shall cover a wide range of several model parameters. Therefore, we decided to perform the analysis, at least partially, by comparing to simulation results of models with explicitly modeled lamination. Even though this approach will be limited to low model complexities, it should provide a more accurate reference for the homogenization models. Of course, the explicit models must also be validated by test results, but only within limited parameter ranges.

For the evaluation hysteresis loops $B(H)$ of the core material were simulated for both types of models (explicitly laminated and homogenized). In both cases, the geometrical symmetry was exploited to reduce the model size (see Figure 2). We used the *mf* mode in combination with a time-dependent study which is required from the non-linear static magnetic material characteristic.

A primary current i_1 is fed in as an edge (line) current generating the magnetic field $H(t)$ in the core material according to Ampère's circuital law. For a closed core with a highly permeable material, a mean magnetic length L_c and a number N_1 of primary turns, $H(t)$ is given as:

$$H(t) = \frac{i_1(t) \cdot N_1}{L_c} \quad (1)$$

According to the law of induction a voltage u_2 is generated across the secondary winding. The mean flux density $B(t)$ in the core corresponds to the integrated secondary voltage u_2 in the time interval $[t_0; t]$ divided by the core cross section area A_c and the number of secondary turns N_2:

$$B(t) = \frac{1}{N_2 \cdot A_c} \int_{t_0}^{t} u_2(\tau) d\tau + B(t_0) \quad (2)$$

Equations (1) and (2) are used to deduce the (static and dynamic) magnetic hysteresis [8, 9] from measurements. In the simulations, B can be directly derived from the numerical solution.

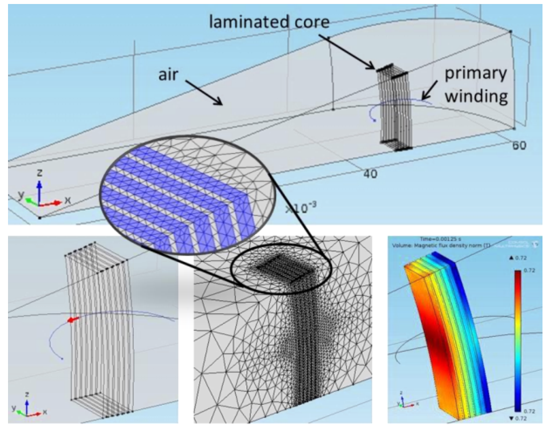

Figure 2. Model of the closed toroidal Permalloy core with an explicitly modeled lamination taking advantage of the $^1/_2 \cdot ^1/_{32}$ symmetry.

Therefore, the secondary winding is not modeled. In the models the electric conductivity of the core material was implemented in a linear orthotropic form according to the respective homogenization approach, while the non-linear magnetic characteristics is based on an orthotropic $\mu_{\text{rel}}(|B|)$ relationship. Thus, it is possible to take the lower effective permeability perpendicular to the lamination plane into account. Both properties are described in a curvilinear coordinate system which is aligned according to the local lamination direction in the curved magnetic core.

3. Homogenization Approaches

As mentioned above, homogenization approaches replace the laminated structure with a single solid domain of an electrically anisotropic material which exhibits a similar macroscopic behavior. This is achieved by adapting the electrical conductivity such that the Ohmic resistance in the homogenized core is equal to that of the current path in the laminated structure (Figure 3). Further, the adapted permeability of the homogenized core considers the thickness of the insulation layers between the sheets by means of a filling factor.

In [1] we have found that the approaches KIWITT [2] and WANG [3] agree best with our test results. Both of them propose an orthotropic electrical conductivity $[\sigma]$ and permeability $[\mu]$ to adapt to the desired behavior (see Table 1).

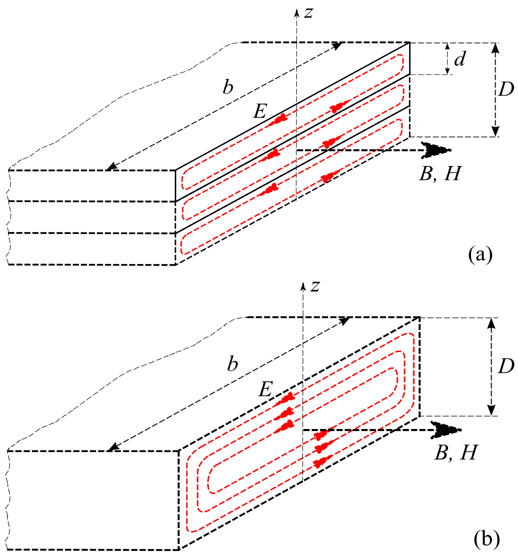

Figure 3. Eddy currents in a core with laminated electric sheets (a) and in a homogenized core (b).

Table 1. Selected homogenization approaches for laminated magnetic core materials; σ_b isotropic conductivity of the basic material, n number of stacked sheets, F lamination filling factor, see Figure 3 for further variables.

	$[\sigma] = \begin{bmatrix} \sigma_x & & \\ & \sigma_y & \\ & & \sigma_z \end{bmatrix}$	$[\mu] = \begin{bmatrix} \mu_x & & \\ & \mu_y & \\ & & \mu_z \end{bmatrix}$
KIWITT [2]	$\sigma_x = \sigma_y = \frac{1}{n^2}\sigma_b$ $\sigma_z = \sigma_b$	$\mu_x = \mu_y = F\mu_b$
WANG [3]	$\sigma_x = \sigma_y = \sigma_b$ $\sigma_z = \left(\frac{d}{b}\right)^2 \sigma_b$	$\mu_x = \mu_y = F\mu_b$ $\mu_z = \frac{1}{\frac{F}{\mu_b} + \frac{1-F}{\mu_0}}$

4. Experimental Setup and Measurements

Since the homogenization models do not consider the static magnetic hysteresis and the measurements do not distinguish between static and dynamic hysteresis, a Permalloy core was taken for experimental model validation as this material has a rather narrow static loop (Figure 4). Therefore, dynamic effects are dominant in the main part of the investigated frequency range, i.e., comparison with both measurement data that include static hysteresis effects and simulation results that do not may allow a validation of the homogenization approaches.

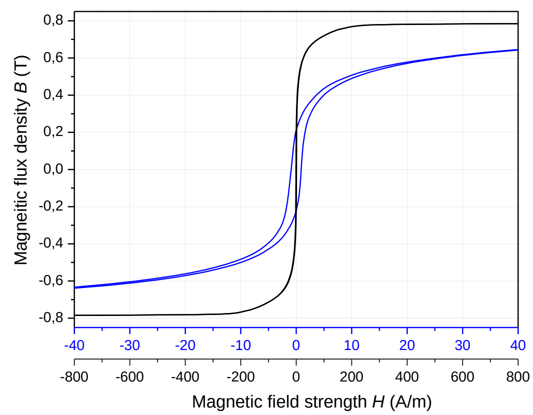

Figure 4. Measured static hysteresis loop of the toroidal tape-wound Permalloy core as shown in Figure 1 (plotted in two different scales).

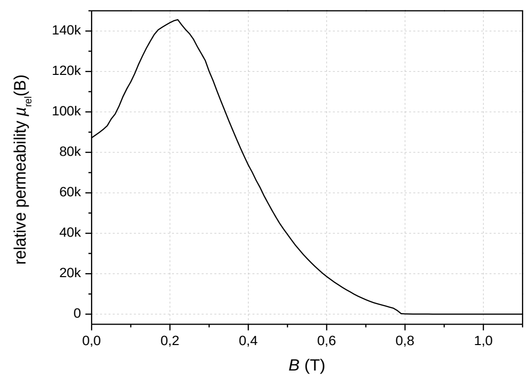

Figure 5. Static $\mu_{\text{rel}}(B)$ characteristics of Permalloy as used in the simulation models (derived from the hysteresis measurement shown in Figure 4).

The implementation of the non-linear and anisotropic material behavior of the magnetic core according to Table 1 requires the $\mu_{\text{rel}}(B)$ form. The permeability used in the simulations was derived from static hysteresis measurements (see Figure 4 and Figure 5). Simulations were performed for two test cases.

5. Test Case 1: Magnetic Flux along the Sheet Layers

This test case was designed according to Figure 1, Figure 2 and Table 2. In the model the helical structure of the tape-wound core was simplified to a stack of concentric circular sheets.

Table 2. Core properties (see Figure 1).

Core width b	6 mm
Mean length of the flux path	284 mm
Core material	Permalloy
Sheet thickness d	250 µm
Number of sheets n	5
Number of primary turns	winding 1: 9 winding 2: 74
Number of secondary turns	65

This is justified as in the helical structure of the tape-wound core the necessary cross-over of the magnetic flux between adjacent layers occurs in large areas, i.e. the corresponding perpendicular flux density is low and negligible as compared to main flux which follows the sheets.

Figure 6 shows a comparison of hysteresis curves from simulations of an explicitly modeled laminated structure to measured curves. It shows that the explicit model is able to describe the main frequency dependency of the hysteresis parameters quite well. Therefore, we will use it as another reference in the evaluation of the homogenization approaches.

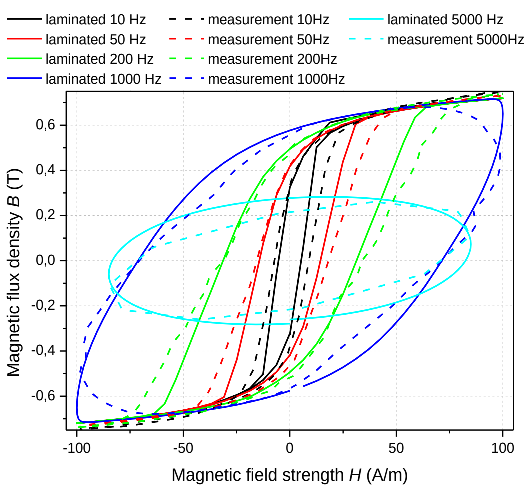

Figure 6. Comparison of dynamic hysteresis loops of the toroidal Permalloy core as obtained from simulations with explicitly modeled lamination (laminated) to measured loops (measurement).

In Figure 7 and Figure 8 dynamic hysteresis curves from the laminated core model are compared to results of the KIWITT- and the WANG-homogenization approaches, respectively. The measured and simulated dynamic loops of the following figures have been created with a sinusoidal excitation current at frequencies of 10, 50, 200, 1000 and 5000 Hz.

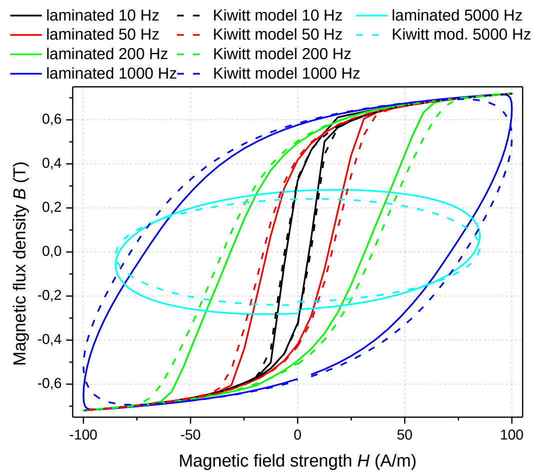

Figure 7. Simulation results as obtained from models with explicitly modeled lamination (laminated) and with homogenized core material according to [2] (KIWITT model).

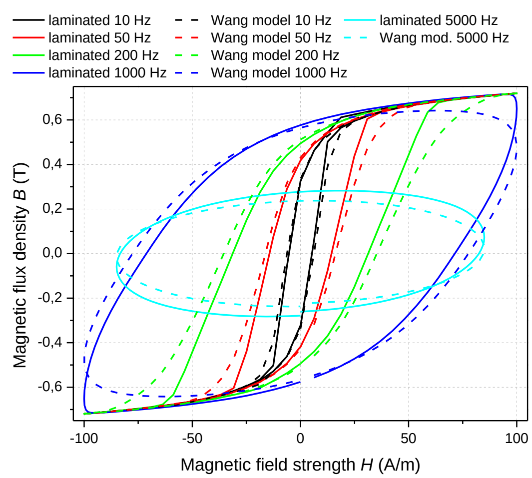

Figure 8. Simulation results as obtained from models with explicitly modeled lamination (laminated) and homogenized core according to [3] (WANG model).

In figures 9 and 10 the coercivity and power loss values extracted from the above hysteresis curves originating from the different models and measurements are compared to each other. Coercivity and losses are plotted as relative values that are normalized with respect to the results of the laminated model.

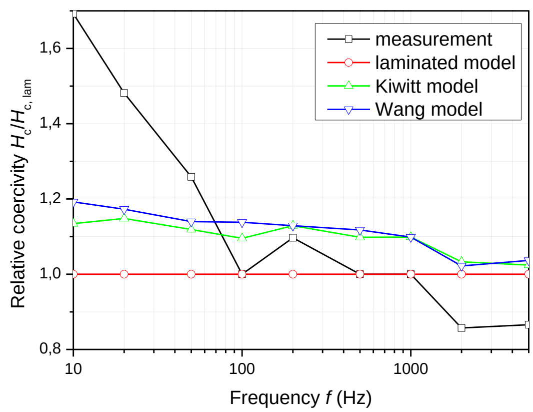

Figure 9. Relative coercivity from measurements and simulations (normalized to the values obtained with explicitly modeled lamination).

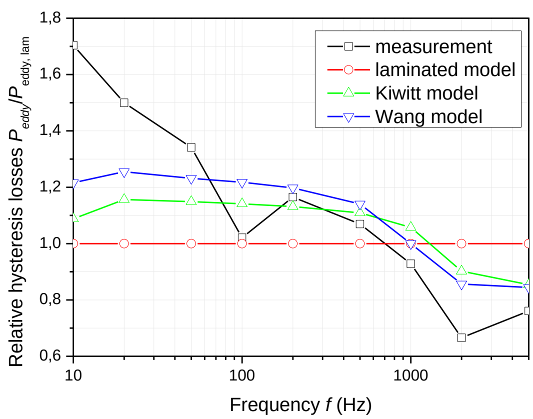

Figure 10. Relative dynamic hysteresis losses from measurements and simulations (normalized to the values obtained with explicitly modeled lamination).

With respect to the coercivity both homogenization approaches are roughly comparable with deviations to the laminated model clearly below 20 % for the most part. When it comes to hysteresis losses the KIWITT model performs about 5-10 % better over a wide frequency range. The higher deviations between laminated model and measurements in the lower frequency range might be caused by the neglection of the static hysteresis in the model and integration inaccuracies in the measurement analysis.

By means of a simplified 2D-axisymmetric model with a linear core material we have further investigated the influence of the sheet dimensions (aspect ratio b/d) for different frequencies (i.e. skin depths). A core model with μ_{rel} = 1,500 and a sheet thickness of d = 0.5 mm was used. These results are shown in Figure 11 and Figure 12.

It turns out that both the KIWITT and the WANG model perform pretty well over a wide range of frequencies and sheet dimensions. Deviations are mostly below 20 % as compared to the results obtained with the explicitly modeled lamination. Overall, the KIWITT model performs slightly better, especially for low sheet aspect ratios.

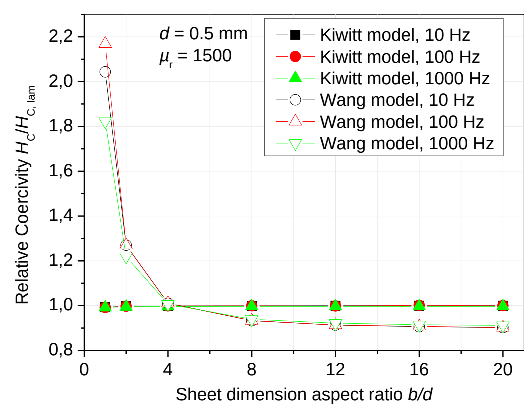

Figure 11. Relative coercivity from homogenized core models (normalized to the values obtained with explicitly modeled lamination) as a function of aspect ratio and frequency.

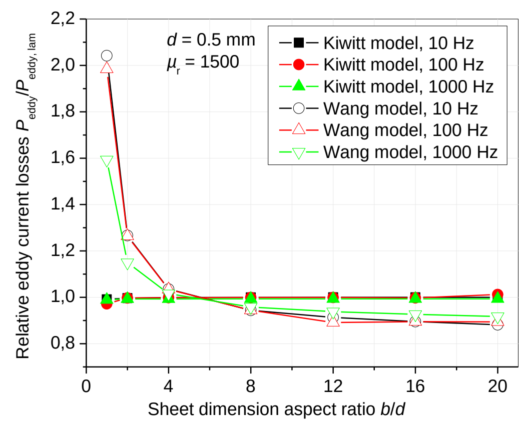

Figure 12. Relative dynamic losses from homogenized core models (normalized to the values obtained with explicitly modeled lamination) as a function of aspect ratio and frequency.

6. Test Case 2: Magnetic Flux has Component Normal to Sheet Layers

This test case was designed for investigating the accuracy of the homogenization approaches in magnetic circuit configurations in which the plane of the lamination differs from the mean magnetic flux direction. This can happen close to large air gaps, e.g. in electromagnetic actuators and machines with moving and rotating parts.

In our 3D test case the magnetic core has its plane of lamination inclined by an angle φ with respect to the direction of the mean magnetic flux density, as illustrated in Figure 13.

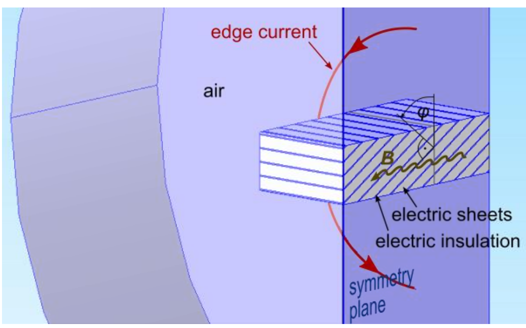

Figure 13. Model geometry with an angle φ between the lamination plane and the direction of the mean magnetic flux density.

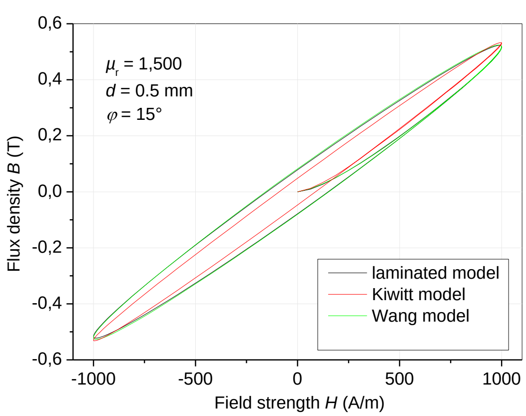

Figure 14. Simulated dynamic hysteresis curves of one explicitly modeled and two homogenized cores with φ = 15°.

We compared models with homogenized against models with laminated cores for inclination angles of $0° \leq \varphi \leq 45°$. An angle of 0° corresponds to test case I in the previous section. The magnetic core material was assumed to have a constant relative permeability of $\mu_r = 1{,}500$. Since the implementation of the orthotropic conductivity is quite different in the KIWITT and in the WANG model (see Table 1), different simulation results were to be expected. Due to the reduction of the in-plane conductivity in the KIWITT model it can be assumed that it will underestimate the dynamic hysteresis caused by in-plane eddy currents which will occur for $\varphi > 0°$. This is confirmed by the simulation results shown in Figure 14 for $\varphi = 15°$, $f = 500$ Hz, $d = 0.5$ mm and also by the φ-dependency shown in Figure 15.

In contrast, the WANG model slightly overestimates the dynamic hysteresis effects for larger φ. This may be caused by the influence of the aspect ratio (cp. Figure 11 and Figure 12) since the aspect ratio of the inclined sheets is seemingly smaller.

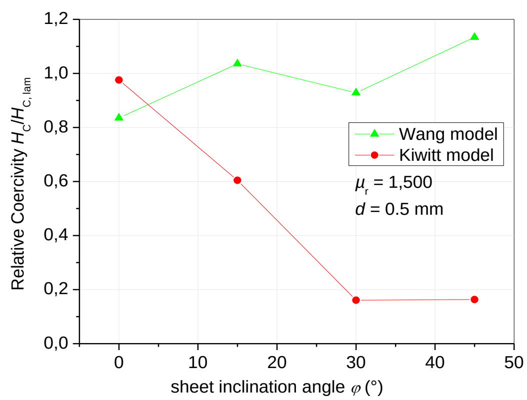

Figure 15. Simulated coercivity of two homogenized cores (normalized with respect to the explicit model) for $\varphi = 0...45°$.

7. Conclusions

In our study we analyzed the performance of different known homogenization approaches for describing the dynamic hysteresis of laminated magnetic cores for use in transient simulations. Simulation results were compared both to those found on explicitly modeled laminations and to experimental results. Since the simulations do not consider static hysteresis, a laminated Permalloy core with small static hysteresis was chosen to facilitate comparability between experimental and simulation results.

The shape of the hysteresis curves, the coercivity, and the dynamic losses have been utilized as comparison criteria. As a result of a former study [1] we have selected the homogenization approaches of KIWITT [2] and WANG [3] for more detailed investigations.

The KIWITT approach simulates best the behavior of laminated cores, provided that the magnetic flux is in parallel to the lamination plane. In this case, the KIWITT approach is reliable within large ranges of frequency and aspect ratio of the sheet geometry.

The WANG model underestimates slightly but systematically both the coercivity and dynamic losses. It can be applied only for sheets with an aspect ratio (width/thickness) of larger than 4. However, in contrast to the KIWITT approach, the WANG model is robust against inclinations between flux density and the lamination plane.

These results suggest that care needs to be taken in the use of the material homogenization approaches if a precise modeling of laminated magnetic structures is required. However, if the known restrictions are taken into account, the number of the finite elements in the simulation models can be significantly reduced, which will result in faster computation and lower simulation costs.

8. References

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