Bridging the Complexity Gap in Modern Engineering Education with COMSOL Multiphysics

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Outline

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- Teaching Model I: Heat Transfer in an Iron Rod
- Teaching Model II: Bottle in a Fridge
- Conclusions

Introduction

- The industry needs employees, who are well-trained in simulation techniques.

- Graduate Students should be able:
  - ... to handle numerical (and commercial multiphysics) tools in order to set up reliable models.
  - ... to interpret the numerical results comprehensively.
  - ... to estimate the influence of the used approximations on the obtained results.
  - ... to draw the right conclusions for model optimizations.

Introduction

- Therefore the students need:
  - ... a broad understanding of the underlying mathematical and numerical methods, and
  - ... a “feeling” for physics.

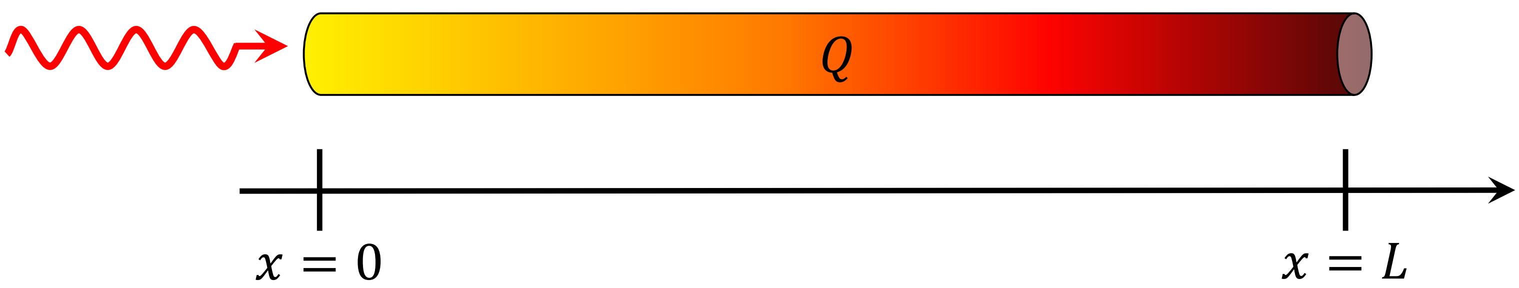
- COMSOL Multiphysics can be used in different phases of a engineering study program such as:
  - Numerical Methods
  - Finite Element Method
  - Multiphysics Simulation
  - etc.

Teaching Model I: HT in an Iron Rod

- Simple example problem, which can be solved analytically:
  - 1D Heat Transfer
  - Internal heat source (e.g. from Joule heating)
  - Flux boundary condition (Neumann)
  - Constraint boundary condition (Dirichlet)

$$q_0 = -k \left.\frac{dT}{dx}\right|_{x=0}$$

$$T(x = L) = T_0$$



Teaching Model I: HT in an Iron Rod

- Corresponding differential equation:

$$-\frac{d}{dx}\left(k(x)\frac{d}{dx}T(x)\right) = Q$$

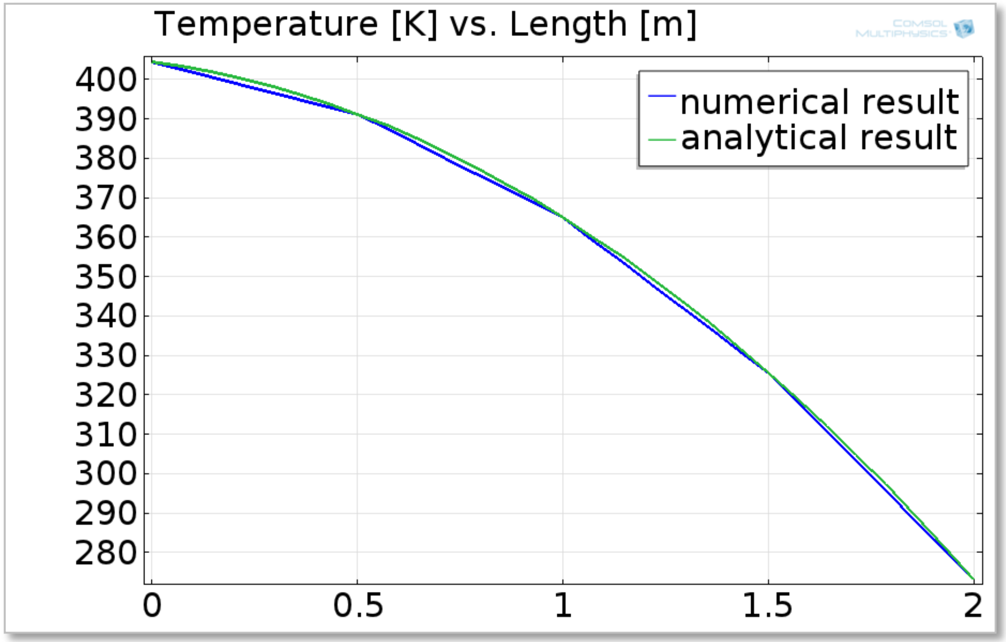
- Analytical result:

$$T(x) = \frac{QL^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{q_0 L}{k}\left(1 - \frac{x}{L}\right) + T_0$$

- The methodical concepts of the FEM are discussed in detail, all necessary steps to find the „numerical“ solution are done by hand!
- The influence of different boundary conditions and input parameters become visible!

Teaching Model I: HT in an Iron Rod

- By comparing numerical and analytical results step by step even complex methodical and physical concepts are much easier to comprehend!



Comparison between the numerical (4 elements and linear shape functions) and the analytical results for $k = 76{,}2\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}$, $L = 2\ \mathrm{m}$, $Q = 4000\ \frac{\mathrm{W}}{\mathrm{m^3}}$, $q_0 = 1000\ \frac{\mathrm{W}}{\mathrm{m^2}}$, and $T_0 = 273{,}15\ \mathrm{K}$.

Teaching Model I: HT in an Iron Rod

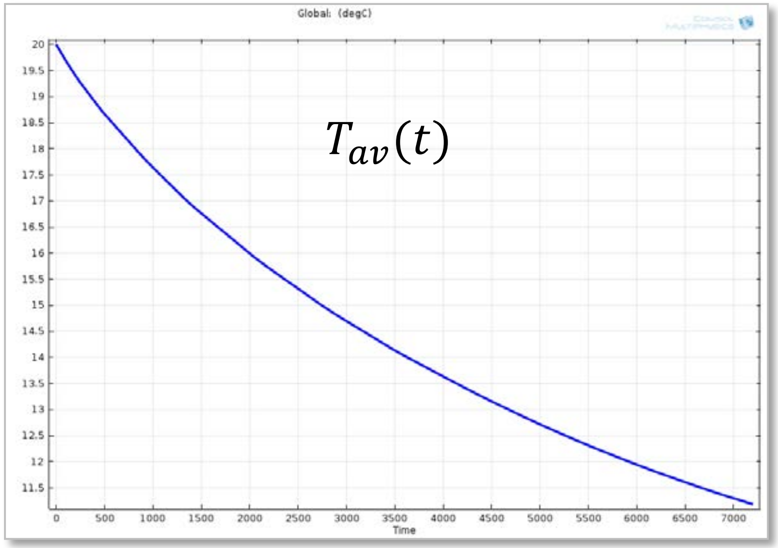
- Model extensions as well as model improvements by using a finer discretization and higher order shape functions can be demonstrated in the GUI of COMSOL Multiphysics in a very comfortable way.
- Btw: In this special case the analytical result can be exactly reproduced by the numerical calculation with only one element and quadratic shape functions.

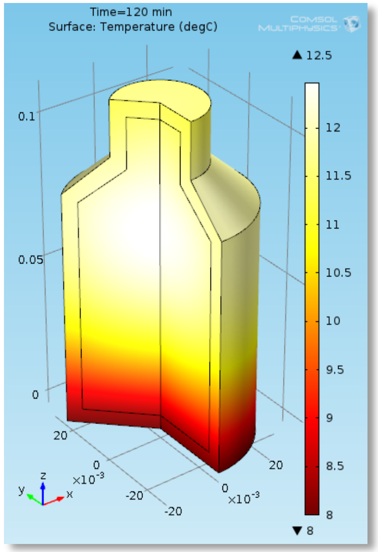
$$\begin{pmatrix} 152{,}4 & -152{,}4 & 0 & 0 & 0 \\ -152{,}4 & 304{,}8 & -152{,}4 & 0 & 0 \\ 0 & -152{,}4 & 304{,}8 & -152{,}4 & 0 \\ 0 & 0 & -152{,}4 & 304{,}8 & -152{,}4 \\ 0 & 0 & 0 & -152{,}4 & 152{,}4 \end{pmatrix} \quad \begin{pmatrix} 177{,}8 & -203{,}2 & 25{,}4 & 0 & 0 \\ -203{,}2 & 406{,}4 & -203{,}2 & 0 & 0 \\ 25{,}4 & -203{,}2 & 355{,}6 & -203{,}2 & 25{,}4 \\ 0 & 0 & -203{,}2 & 406{,}4 & -203{,}2 \\ 0 & 0 & 25{,}4 & -203{,}2 & 177{,}8 \end{pmatrix}$$

Stiffness matrix for (a) 4 elements and linear shape functions,
(b) 2 elements and quadratic shape functions

Teaching Model II: Bottle in a Fridge

- Question: How long does it take until a beer bottle is cooled down in a fridge to an average temperature of $12\,°C$?
- Simple example problem:
  - 2D axisymmetric HT in Solids
  - Initial conditions:
    Glass, Water at $T = 20\,°C$ / Air in Fridge at $T_{ext} = 8\,°C$





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Teaching Model II: Bottle in a Fridge

Quality of the result: How to estimate the influence of the used approximations on the obtained results?

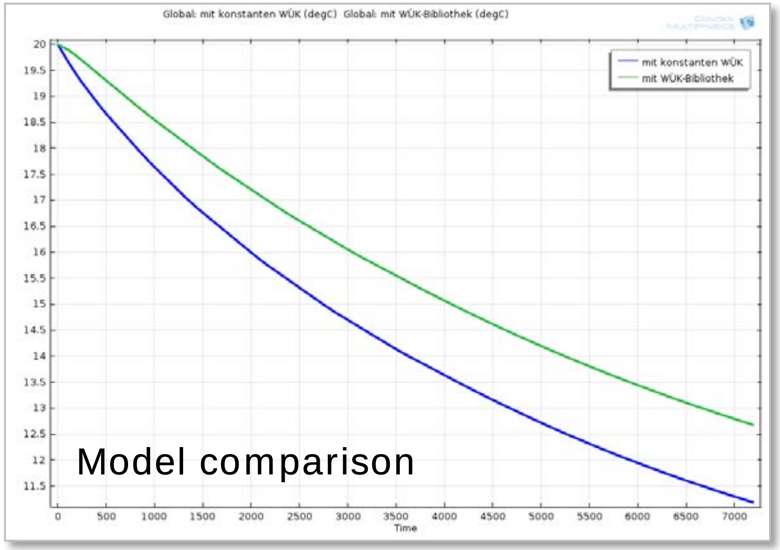
Model extensions / model improvements

- Idealized geometry
  - Influence of geometrical details (bottle cap, fillets, etc.)?
  - Additional domains (air, cap, fridge)?
- Input parameters
  - Are all parameters known exactly?
  - Are the material properties constant or temperature-dependent?
- Meshing
  - mesh dependence study

Teaching Model II: Bottle in a Fridge

Model extensions / model improvements

- Idealized boundary conditions
  - Where does the modeling domain end?
  - Use of heat transfer coefficients from library (natural convection) better than using a constant value $h = 5\ \frac{\mathrm{W}}{\mathrm{m^2\,K}}$.
  - Constraint $T = 8\,°\mathrm{C}$ at the bottom represents an infinite heat sink
    → better use a large heat transfer coefficient here as well.

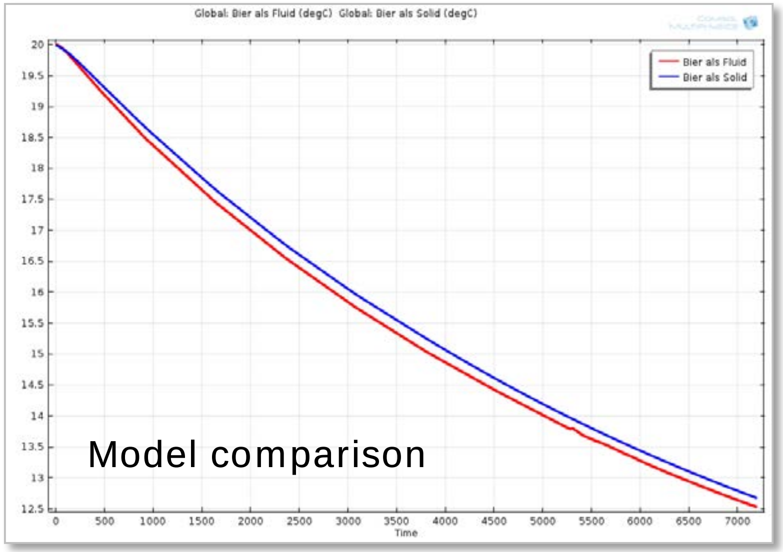


Model comparison

Teaching Model II: Bottle in a Fridge

Model extensions / model improvements

- Is it neccessary to consider fluid convection inside?
  - Computational effort is big (#dof, more complex, finer mesh, more time steps, Pressure point constraint, etc.)



Model comparison

Conclusions

- COMSOL Multiphysics serves as an excellent tool to teach complex mathematics and demonstrate the practical use of simulations.

- The handling of the software is easy to learn and the student experiences quite soon the feeling of success.

- On the basis of simple example models the generalization to more complex situations is straight forward and easy comprehensible.

- The students finally are enabled to set up rather complicated multiphysics models during the accompanying practical lab courses!