Numerical Modeling of the Near-subsurface Temperature Distributions in Presence of Time Varying Air Temperature in the Boundary Condition and Space Varying Temperature for the Initial Condition Using COMSOL Multiphysics

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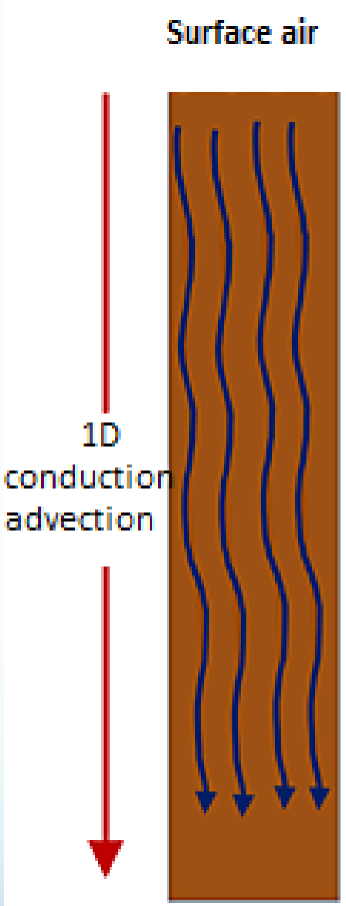
Objective

- Develop a model to describe the distribution of temperatures in the subsurface due to variations in the surface air temperature.
- Develop a numerical solution for which exponential terms are included in both initial and robin type boundary condition that can be apply to simulate for future climates.
- To observe the effect of interaction between the air and the surface temperatures, groundwater velocity on the subsurface thermal structure in changing climate.

Introduction

- Heat is transported into the near subsurface mainly by conduction and advection
- Subsurface temperature distribution is significant in thermo-physical, climatological problems.
- Analysis of Surface air temperatures gives evidences for warming or cooling of the near surface (Wigley et al,) thus perturbs into the subsurface, develops an envelope or signatures of thermal regime.



Physical Scenario

Methodology

- Signatures of the climate change can be inferred by assuming one dimensional transient heat transfer in porous media with groundwater velocity in the subsurface is given by

$$(\rho C_p)_s \frac{\partial T}{\partial t} + (\rho C_p)_f u \frac{\partial T}{\partial z} = K \frac{\partial^2 T}{\partial z^2} + A(z,t,T)$$

- ρ -density
- C_p -specific heat
- u -vertical groundwater velocity
- K -thermal conductivity of soil
- Suffixes '*s*' and '*f*' refers to solid matrix and fluid
- A(z,t) - Source/sink

- Initial condition:

A super-imposed linear and exponential function is used and is given by

T(x,0)=T0 + a*x + delt*exp(d*x)

- Boundary condition:

Robin type boundary condition that relates surface heat flux due to both SAT and SST is used to access the atmospheric contribution to subsurface thermal profile and is given by

$$k\frac{\partial T}{\partial t} = H*(T-(T_A + b*\exp(c*t)))$$

Parameters:

Name	Expression	Description
T0	15[degC]	Temperature of soil surface
T_A	16.5[degC]	Air temperature on surface
k	2.5[W/(m*K)]	thermal conductivity
K	6.1[m^2/yr]	thermal diffusivity
u	0.5[m/yr]	Mean annual vertical groundwater velocity
H	0.2[W/(m^2*degC)]	Heat transfer coefficient
a	0.022[degC/m]	Temperature gradient
b	1.60[degC]	
c	0.0116[/yr]	
delt	2.0[degC]	
d	-0.02[/m]	

Solution of the problem

- Analytical solution for simple initial and boundary condition is given by (Rajeev et al. 2012)
- *Initial condition:*

$$T = T0 + aZ \qquad \text{at } t = 0 \qquad \text{for all } Z > 0$$

- *Boundary condition:*

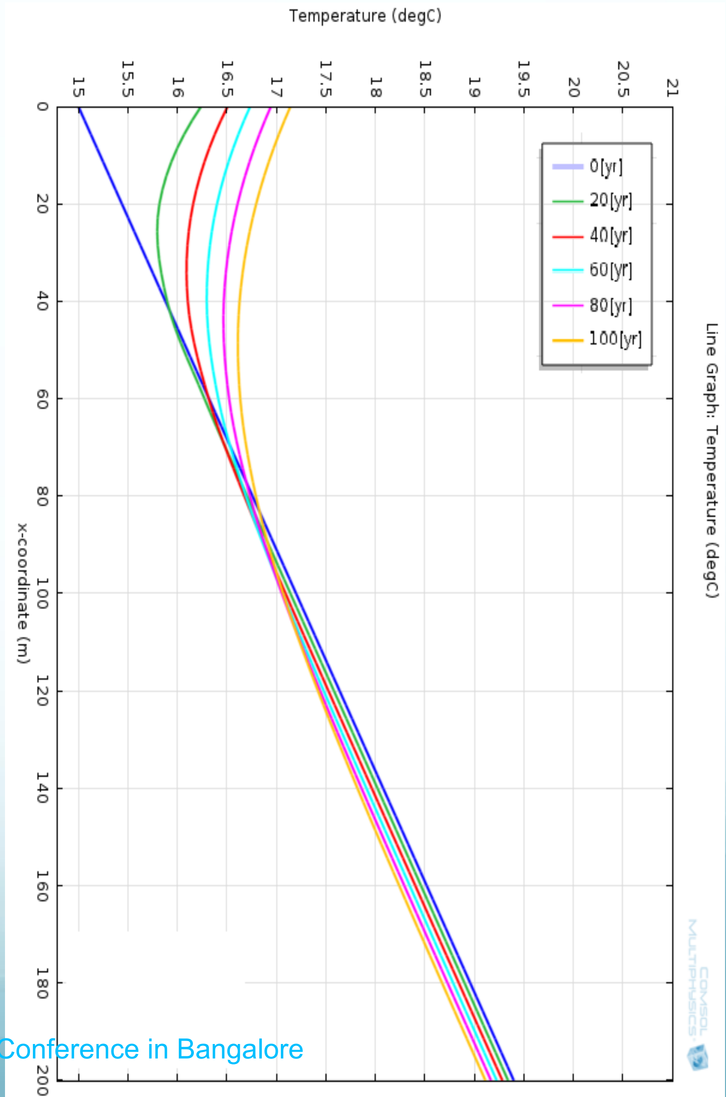
$$K*dT/dz = H(T - T_A - bt) \qquad \text{at} Z=0 \qquad \text{for all } t > 0$$

- *Solved Temperature distribution with space and time in laplace domain.*

$$T(z,t) = (T0 + aZ)/p - aU/p2 + [K1\,(T0 - T_A) - a]\exp(m2Z)/(p(m2 - K1)) - (K1\,(aU + b)/(p2(m2 - K1)))\exp(m2Z)$$

where $m2 = U/(2k) - \mathrm{sqrt}(U^2/(4*k^2 + p/k))$

➢ The above problem is Solved numerically by using COMSOL Multiphysics



Finite element method using COMSOL Multiphysics

- The problem is solved numerically by Finite Element Method using COMSOL Software.
- One dimensional heat conduction–advection equation in the subsurface is:

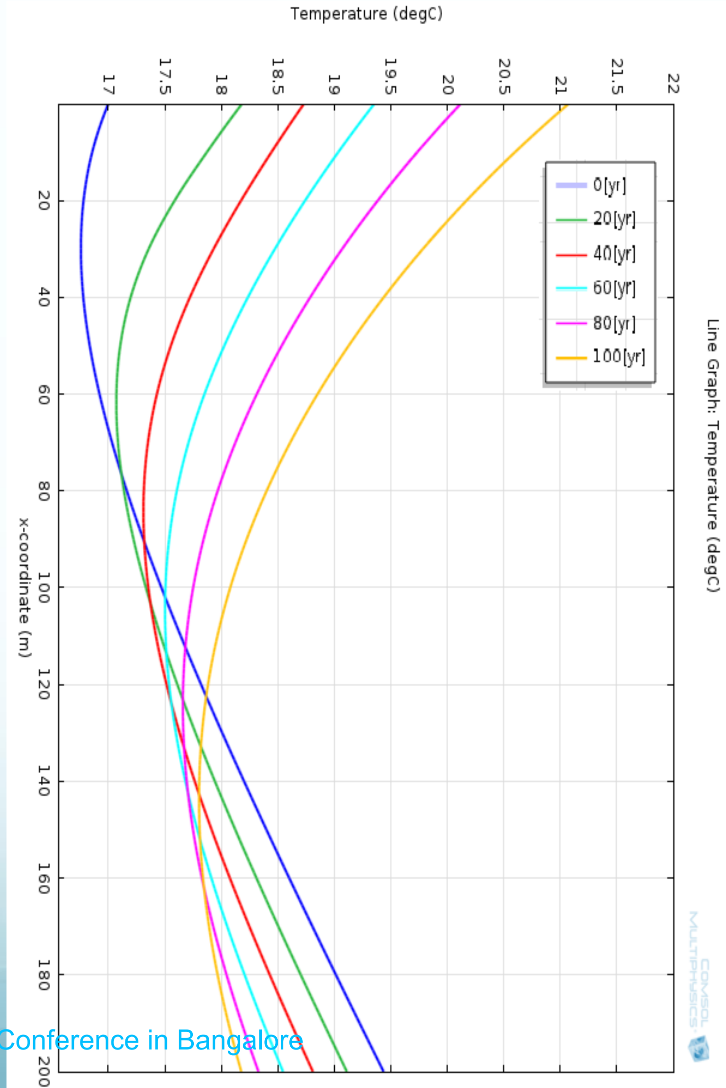
$$(\rho C_p)_{eq}\frac{\partial T}{\partial t}+\rho C_p\mathbf{u}\cdot\nabla T=\nabla\cdot(k_{eq}\nabla T)+Q$$

- Number intervals are 2
- Discretized for 200 elements
- The solution is obtained by using iterative method
- This iterative solver uses conjugate gradient method with error less than 0.001

Results and discussions

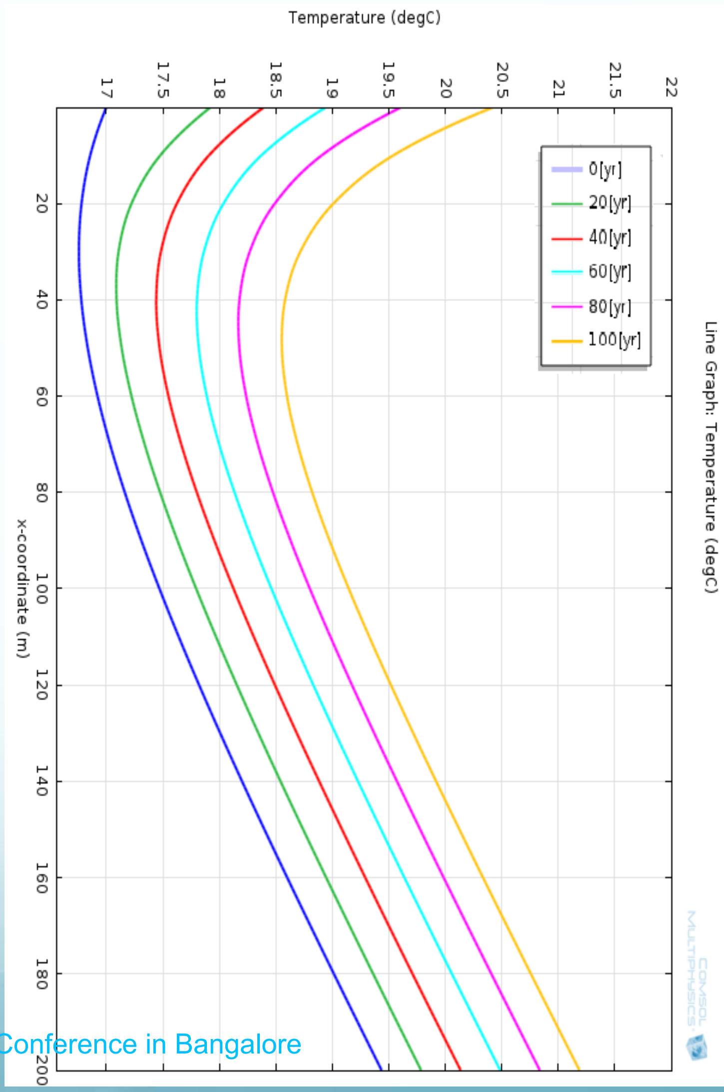
Temperature-depth profiles for annual groundwater recharge (0.5[m/yr])

- When the ground is recharging with annual mean vertical ground-water velocity, surface air(climate) is forcing the earth's surface to warm.
- Temperature profiles are observed to be reversed after certain depth where the effect of air and soil interaction ceases.



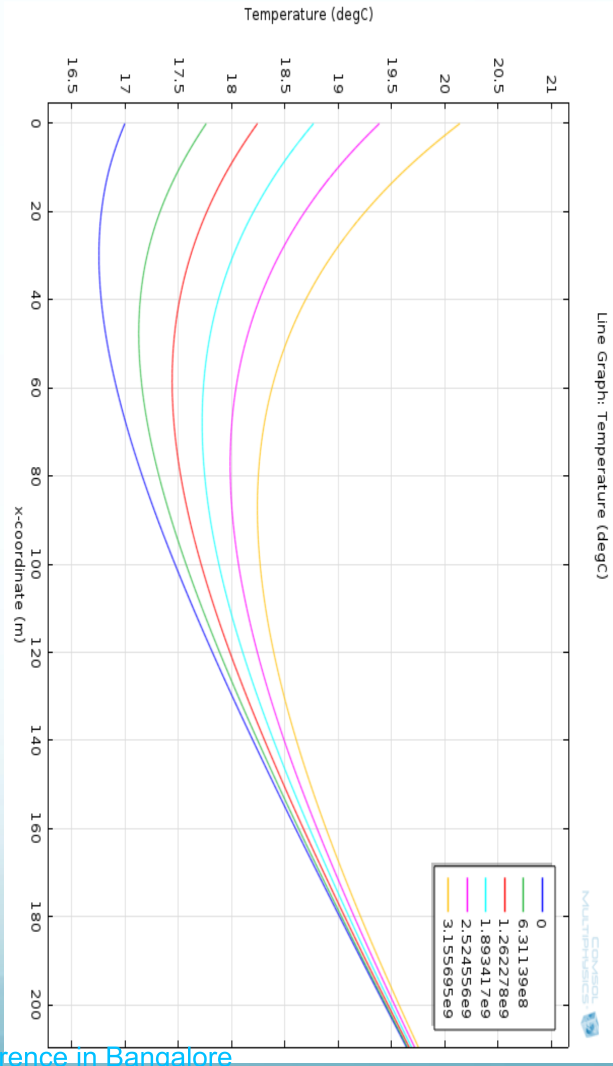
Temperature-depth profiles for annual discharge (-0.5[m/yr])

- Discharge will enhances the surface to warm from the deeper subsurface by advection.
- When the ground is discharging the temperature profiles will follows a linearly decreasing trend from the bottom of the subsurface due to temperature gradient.

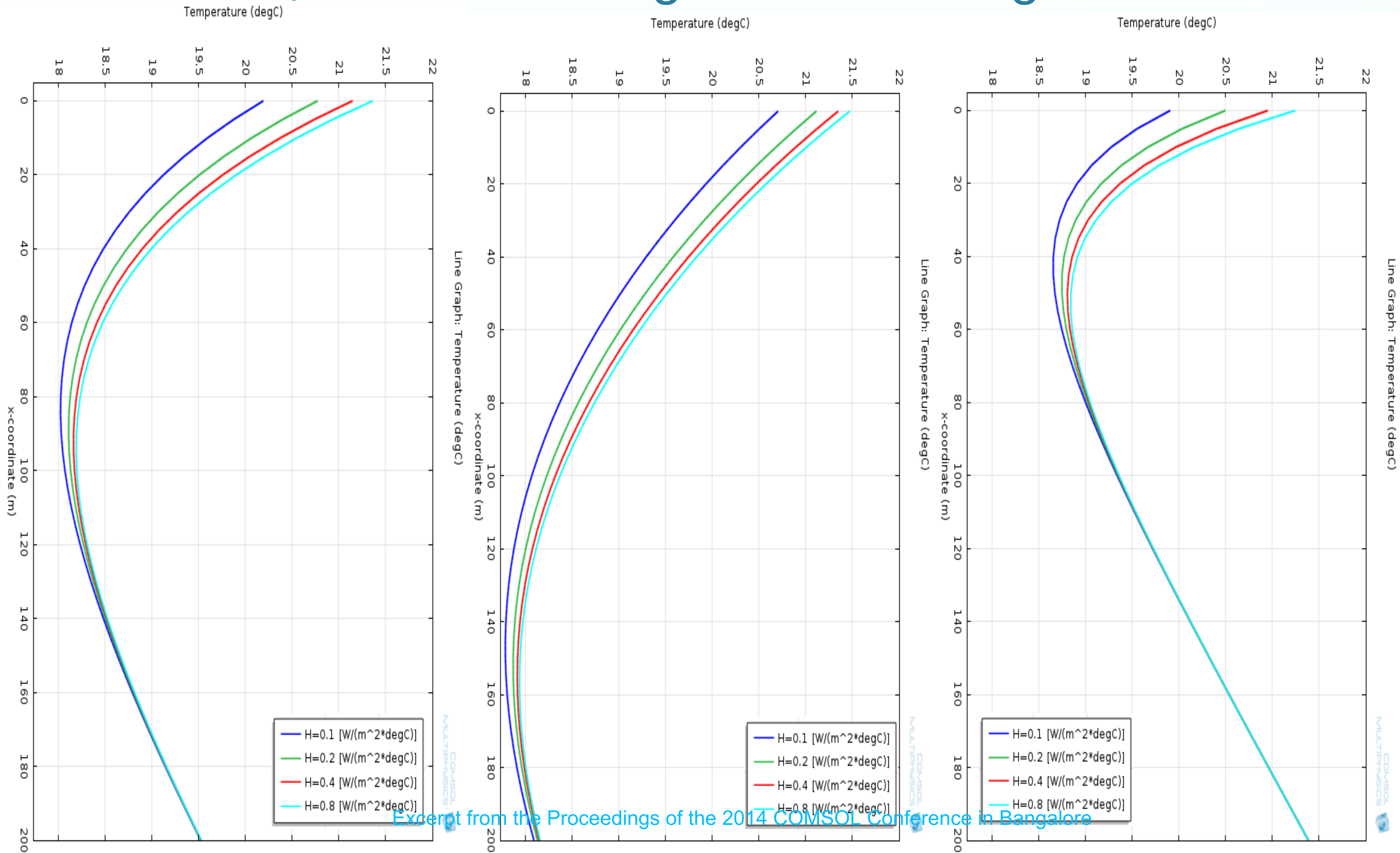


Temperature-depth profile for annual groundwater velocity (10^-5[m/yr])

- It is observed that when the ground is covered with ice and the surface air with snow, thermal profile follows an exponential trend due to initial condition and then it coincide with the geothermal gradient.

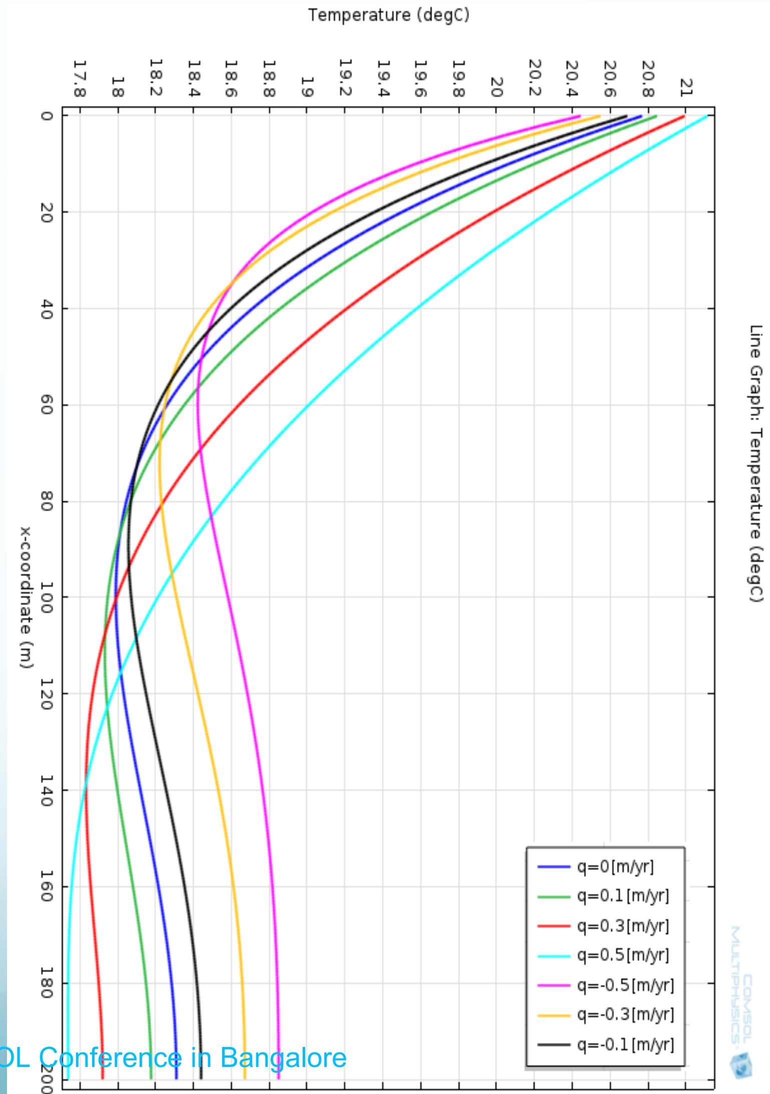


Temperature-depth profiles for different ‘H’ with no flow, annual recharge and discharge rates



Temperature-depth profile for various annual groundwater velocities

- Groundwater velocity is an intensive parameter in transporting heat in the subsurface.
- As annual groundwater velocity increases, subsurface is warming from the above with respect to depth.



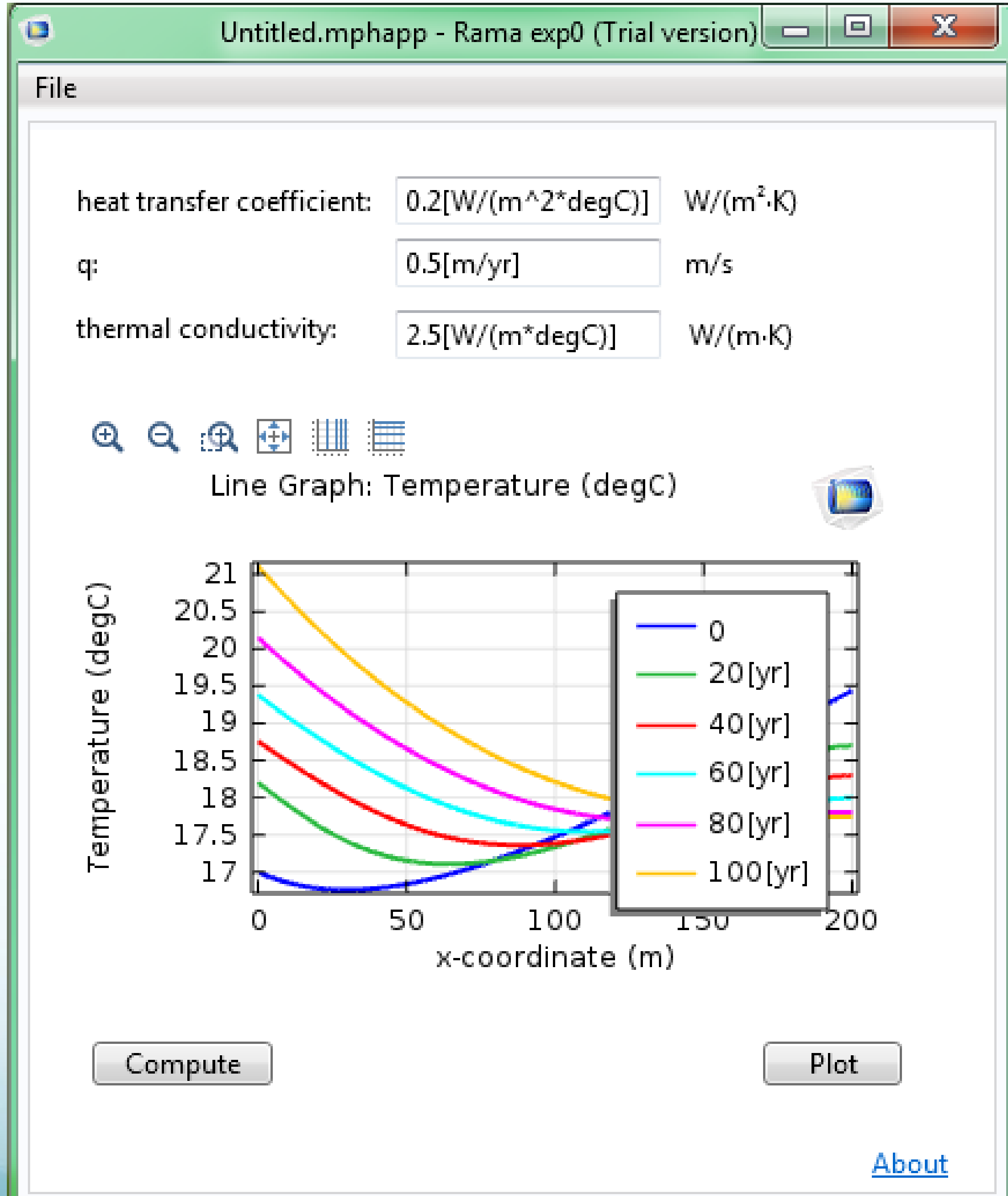
Discussions

- Temperature –depth profiles obtained by the exponential model show that the recharge can accelerate shallow subsurface warming, whereas upward groundwater discharge can enhance deeper subsurface warming.

- With higher values of H, faster propagation of heat is observed in the subsurface.

Conclusions

- A numerical solution to the transient one-dimensional conduction-advection equation was developed for air temperature projections.
- These studies are useful to set the numerical results for the complex problems where it is not possible to get analytical solutions.
- Sensitivity of ground recharge or discharge, interaction of surface air with surface soil on temperature profiles for subsurface are discussed.
- These numerical solutions have potential use in understanding the future climate change by observing the thermal profiles.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File
heat transfer coefficient:
0.2[W/(m^2*degC)]
W/(m²·K)
q:
0.5[m/yr]
m/s
thermal conductivity:
2.5[W/(m*degC)]
W/(m·K)
Line Graph: Temperature (degC)
Temperature (degC)
21
20.5
20
19.5
19
18.5
18
17.5
17
0
50
100
150
200
x-coordinate (m)
0
20[yr]
40[yr]
60[yr]
80[yr]
100[yr]
Compute
Plot
About

Thank You