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Modeling and Simulation of the Consolidation Behaviour of Cemented Paste Backfill

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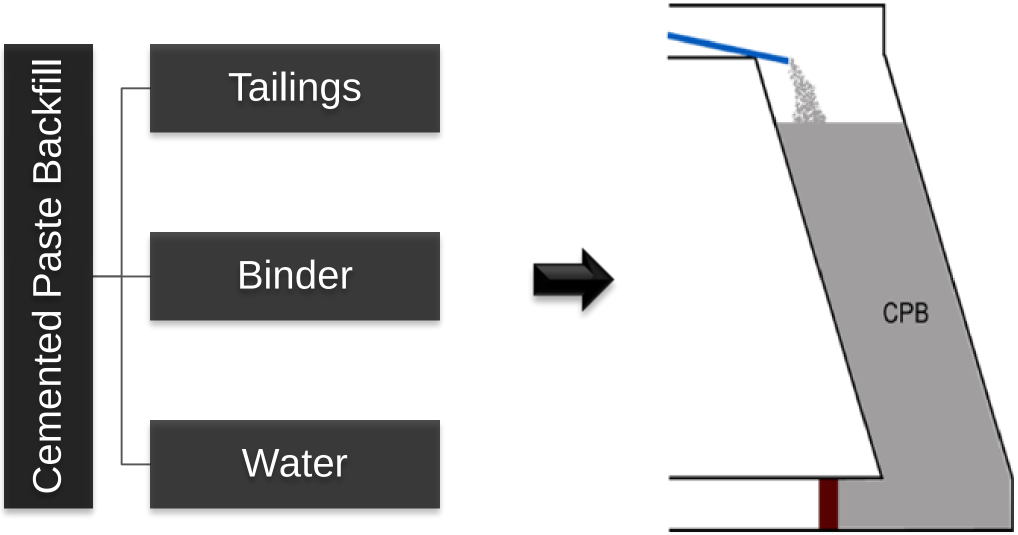
COMSOL
CONFERENCE
2015 BOSTON

OUTLINE

- **Introduction**
- **Coupled THMC formulation**
  - Relevant phenomena
  - Balance equations
  - Constitutive equations
- **Consolidation formulation**
- **Model Validation Examples**
- **Conclusions**

Introduction

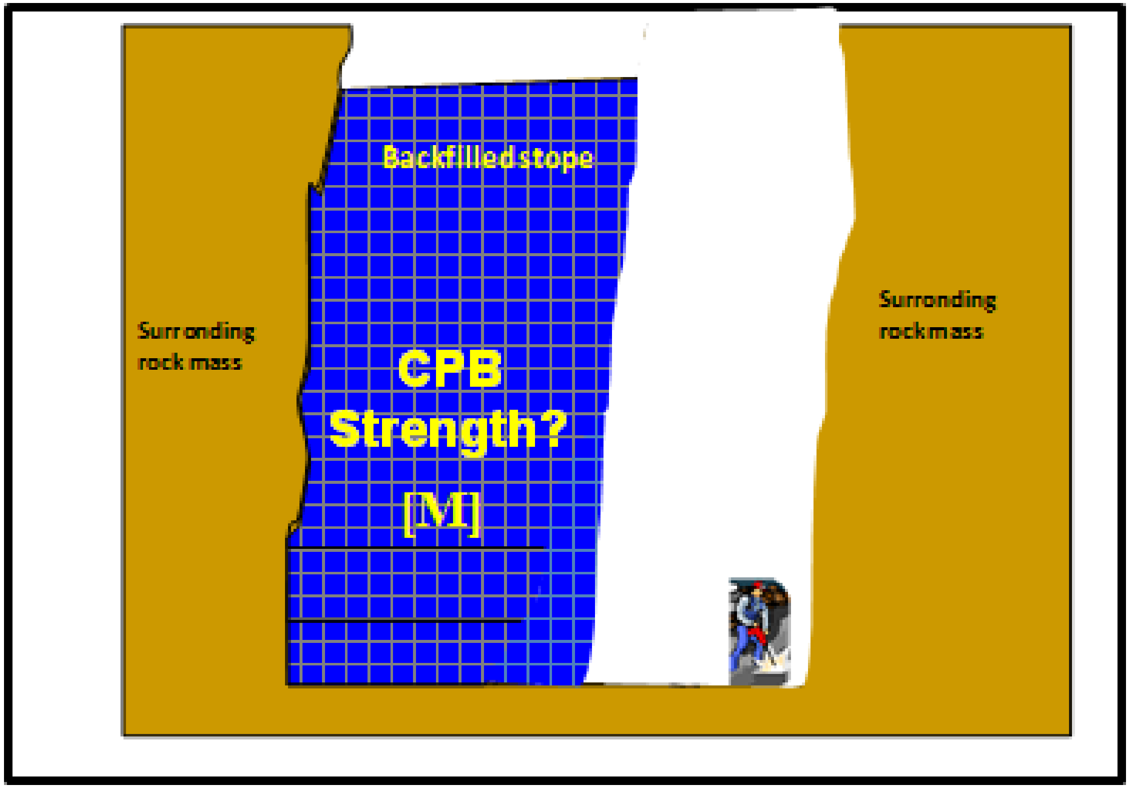
- Cemented Paste backfill (CPB)



Source: Cui and Fall a, 2015

Introduction

- **M**echanical Process

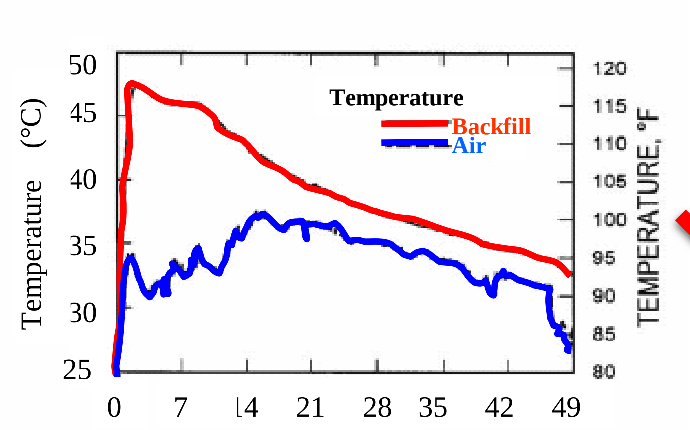


Source: Fall, 2013

Mechanical Stability:

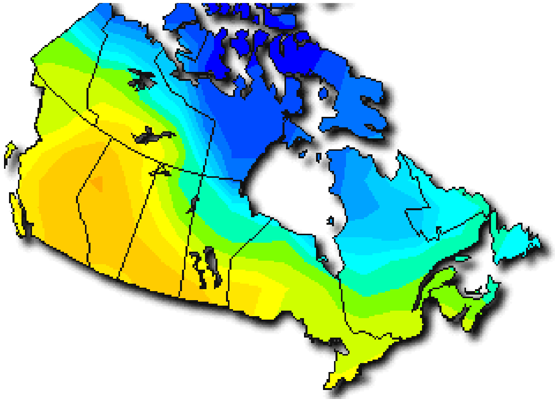
- **Strength**
- **Stress**
- **Deformation**

Heat developed by binder hydration (CPB in field)

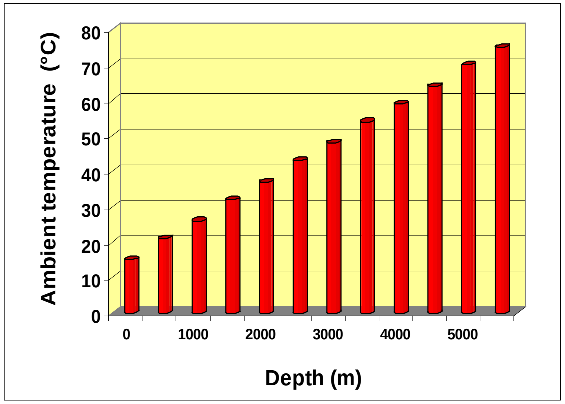


Williams et al. 2001

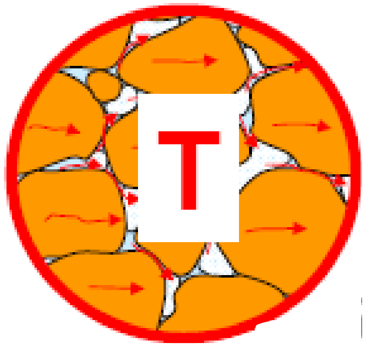
Permafrost regions



Deep mine temperatures (geothermal gradient)

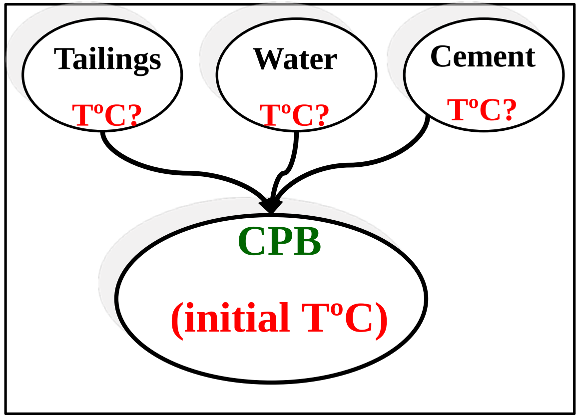


Source: Rawlins et Phillips, 2001



Heat transfer

Temperature of CPB components



Tailings

T°C?

Water

T°C?

Cement

T°C?

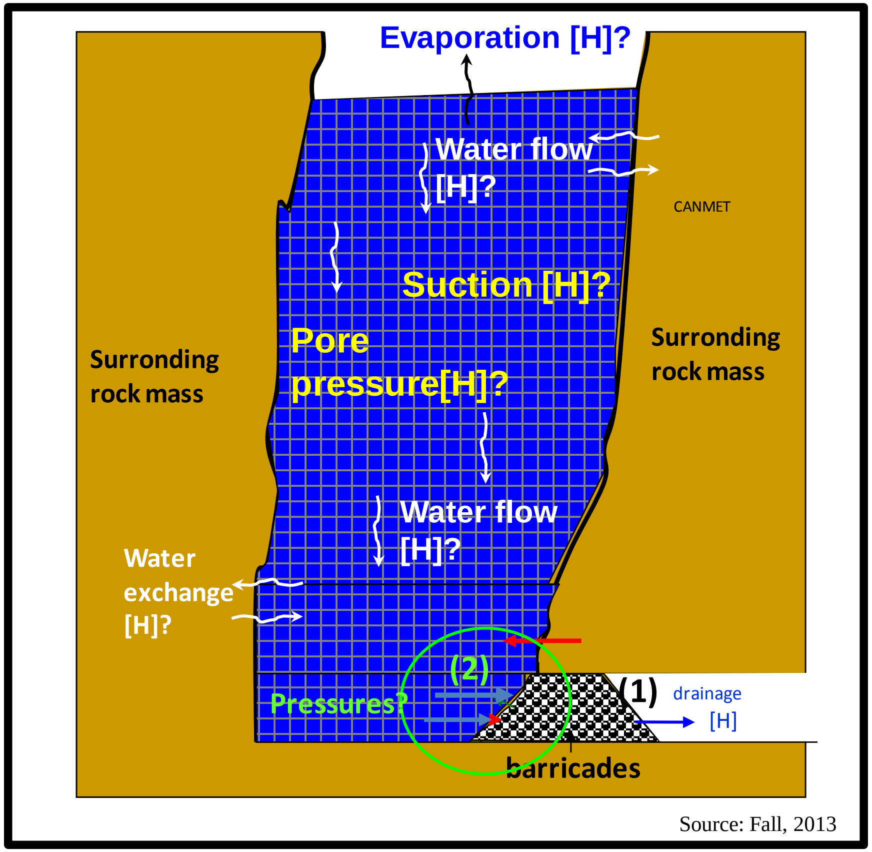
CPB

(initial T°C)

Source: Fall, 2013

Introduction

- **H**ydrualic Process



Source: Fall, 2013

Introduction

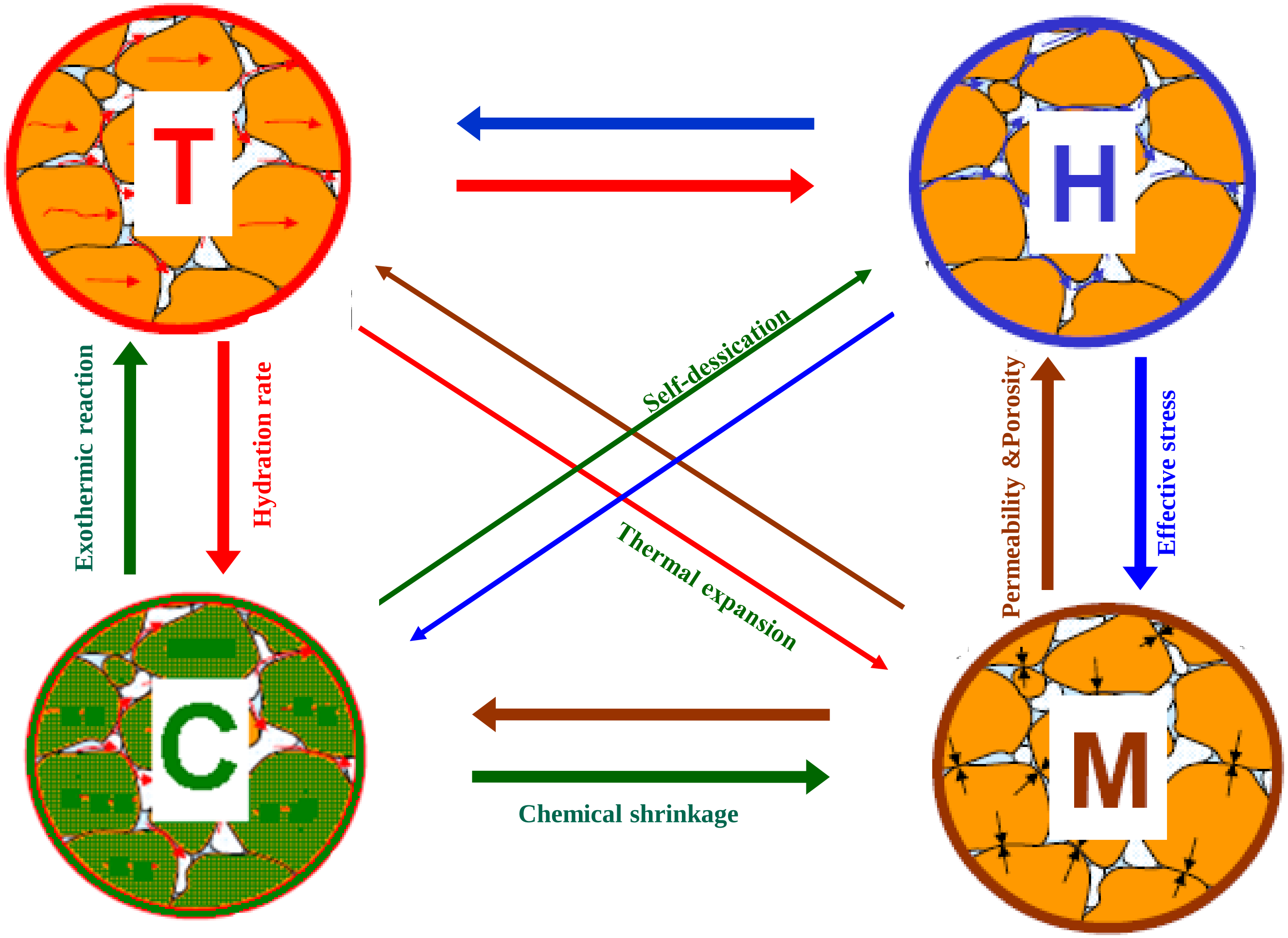
- **C**hemical Process

Chemical Processes

- Binder hydration
- Tailings types
- Mixing water chemistry

Source: Fall, 2013

All 4 Processes, T, H, M, C are coupled



Source: Fall, 2013; modified

THMC: relevant phenomena

- THERMAL PROCESS (**T**)
  - Heat conduction
  - Heat advection
  - Heat release from binder hydration(source term)
- HYDRAULIC PROCESS(**H**)
  - Fluid flow
  - Pore-water consumption (sink term-liquid phase; source term-solid phase)
- MECHANICAL PROCESS(**M**)
  - Strain components
    - Elastoplastic strain
    - Thermal expansion(contraction)
    - Chemical shrinkage
- CHEMICAL PROCESS (**C**)
  - Binder hydration degree
  - Water consumption and heat release
  - Evolution of material properties with binder hydration

Balance equations

- Mass balance equation:

$$\phi\frac{\partial(S\rho_w)}{\partial t}+S\rho_w\left[\frac{(1-\phi)}{\rho_s}\frac{\partial\rho_s}{\partial t}+(1-\phi)\frac{\partial\varepsilon_v}{\partial t}\right]+\phi S\rho_w\frac{\partial\varepsilon_v}{\partial t}-\phi S\dot{m}_{hydr}\left(\frac{S\rho_w}{\rho_s}-1\right)=-\nabla\cdot\left(\phi S\rho_w\mathbf{v}^{rw}\right)$$

$$\phi\frac{\partial\left[(1-S)\rho_a\right]}{\partial t}+(1-S)\rho_a\left[\frac{(1-\phi)}{\rho_s}\frac{\partial\rho_s}{\partial t}+(1-\phi)\frac{\partial\varepsilon_v}{\partial t}\right]+\phi(1-S)\rho_a\nabla\cdot\mathbf{v}_s-\frac{(1-S)\rho_a}{\rho_s}\phi S\dot{m}_{hydr}=-\nabla\cdot\phi(1-S)\rho_a\mathbf{v}^{ra}$$

- Momentum balance equation:

$$\nabla\cdot\left(\frac{\partial\boldsymbol{\sigma}}{\partial t}\right)+\frac{\partial\left[(1-\phi)\rho_s+\phi S\rho_w+\phi(1-S)\rho_a\right]}{\partial t}\mathbf{g}=0$$

- Energy balance equation:

$$\left[(1-\phi)\rho_s C_s+\phi S\rho_w C_w+\phi(1-S)\rho_a C_a\right]\frac{\partial T}{\partial t}+Q_{ad}+Q_{cd}=Q_{hydr}$$

Coupled processes	Constitutive relation
Thermal	$Q_{ad} = \left(\rho_v C_{w-p} \mathbf{v}^{rw} + \rho_a C_{a-p} \mathbf{v}^{ra} \right) \cdot \nabla T$ $\mathbf{q} = -k_{eff} \nabla T$
Hydraulic	$\mathbf{v}^{ri} = -K \dfrac{k_{ri}}{\rho_i g} \nabla \left(P_i - \rho_i g \right)$ $S_{eff} = \dfrac{1}{\left[1 + \left(\alpha P_c \right)^{\frac{1}{1-m}} \right]^m}$
Mechanical	Elastoplastic strain Thermal strain(expansion or contraction) Chemical shrinkage
Chemical	$Q_{hydr} = \left(H_{cem} \cdot X_{cem} + 461 \cdot X_{slag} + 1800 \cdot x_{CaO/FA} \cdot X_{FA} \right) C_b \left(\dfrac{\tau}{t_e} \right)^{\beta} \left(\dfrac{\beta}{t_e} \right) \xi \left(t_e \right) \cdot \exp \left[\dfrac{E}{R} \left(\dfrac{1}{273 + T_r} - \dfrac{1}{273 + T} \right) \right]$ $\dot{m}_{hydr} = 2 m_{hc-initial} \left(0.187 x_{C_3S} + 0.158 x_{C_2S} + 0.665 x_{C_3A} + 0.2130 x_{C_4AF} \right) \left\{ \left(\dfrac{\tau}{t_e} \right)^{\beta} \left(\dfrac{\beta}{t_e} \right) \xi \exp \left[\dfrac{E}{R} \left(\dfrac{1}{273 + T_r} - \dfrac{1}{273 + T} \right) \right] \right\}$

Constitutive equations

- Mechanical constitutive relation
  - Strain components

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_p + \boldsymbol{\varepsilon}_T + \boldsymbol{\varepsilon}_c$$

- Elastic strain $\boldsymbol{\varepsilon}_e$
- Plastic strain $\boldsymbol{\varepsilon}_p$
- Thermal expansion(contraction) $\boldsymbol{\varepsilon}_T$
- Chemical shrinkage $\boldsymbol{\varepsilon}_c$

Constitutive equations

- Thermal expansion (contraction):

$$\boldsymbol{\varepsilon}_{\boldsymbol{T}} = \alpha_T T$$

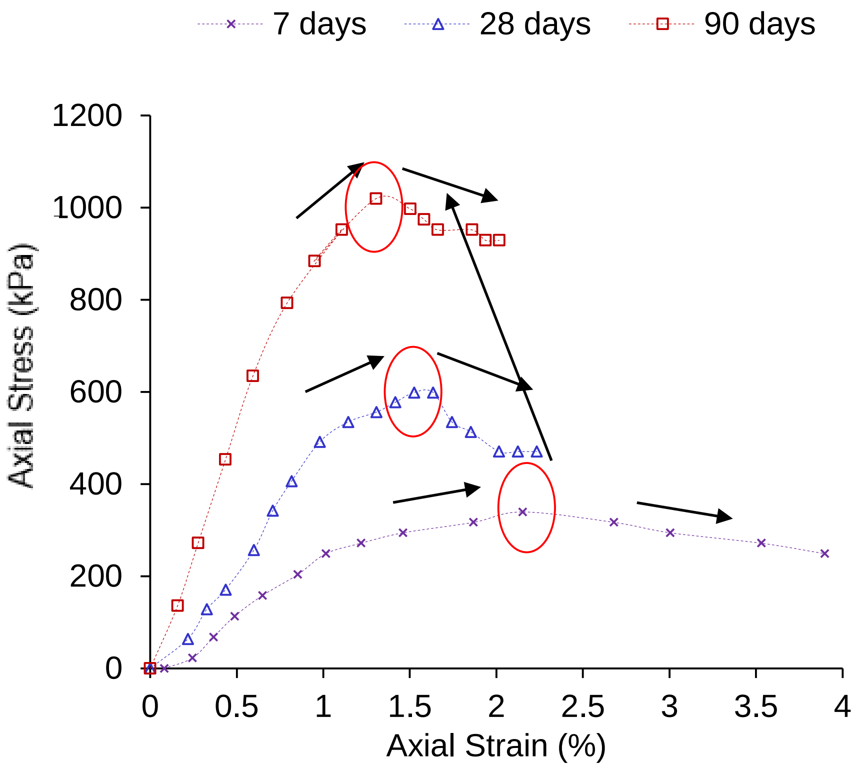
$$a_T = a_p - \frac{2v_t\left(1/C_m - 1\right)\left(a_p - a_t\right)}{2v_t\left(1/C_m - 1\right) + \left[\left(w/c\right)v_w + v_c\right]\left\{1 + \left[L_1 \cdot \left(1 - \phi_h\right)^{L_2}\right]/E_t\right\}}$$

- Chemical shrinkage:

$$\boldsymbol{\varepsilon}_{ch} = \alpha_{ch}\xi$$

$$\alpha_{ch} = \frac{\left(2v_w - v_n - v_{ab-w}\right)R_{n-w/hc}}{\left(w/c\right)v_w + v_c + \left(1/C_m - 1\right)v_{tailings}}$$

Constitutive equations



Evolution of UCS with curing time (Cui and Fall, 2014)

Yield function and potential function

- Strain hardening/softening behavior

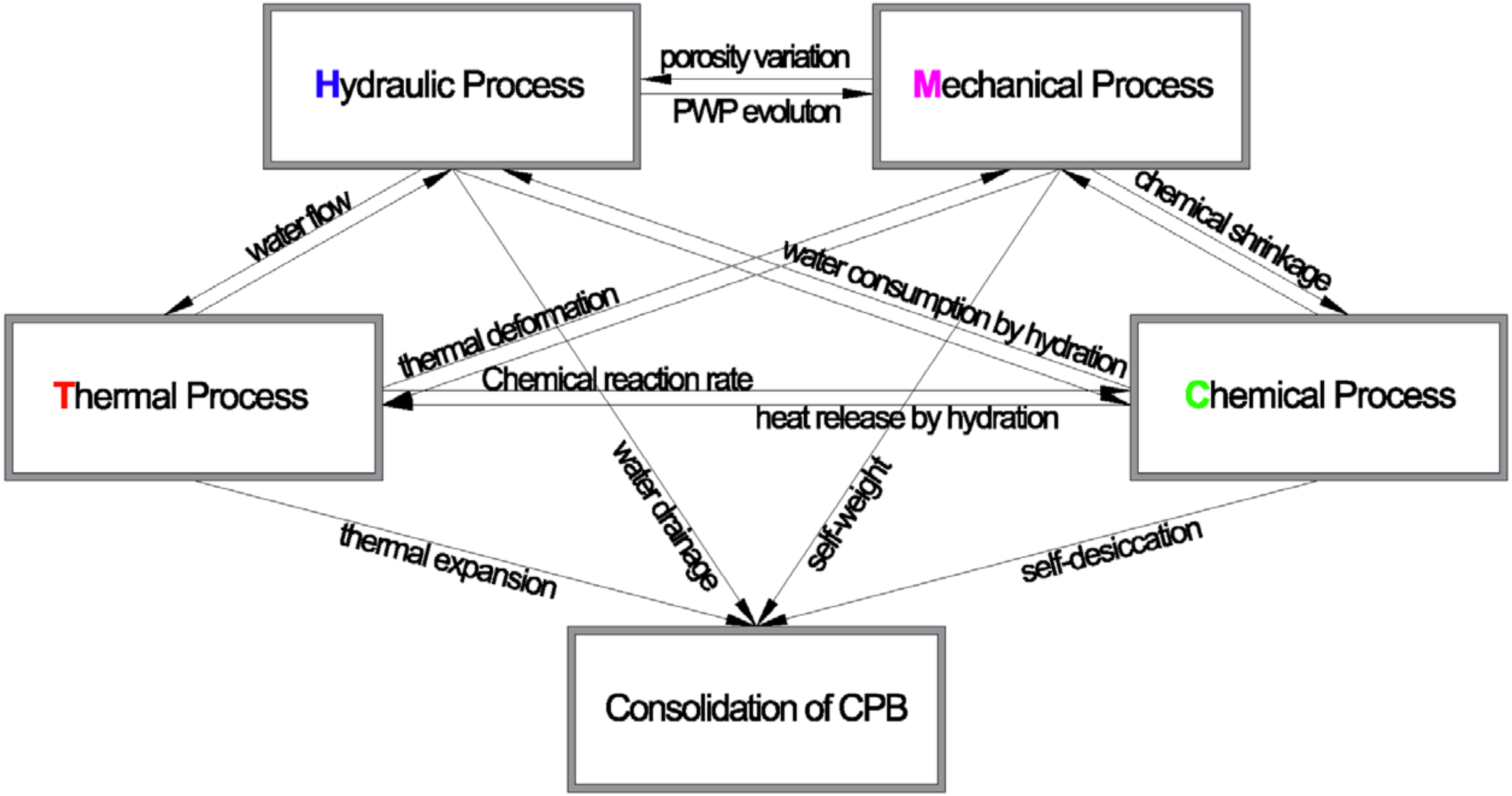
$$F(I_1,\sqrt{J_2},\xi,\kappa)=\alpha(\xi,\kappa)\left[I_1+C(\xi)\right]+\sqrt{J_2}=0$$

- Chemical hardening behavior

$$G=\frac{2\sin\psi(\xi)}{\sqrt{3}\left[3+\sin\psi(\xi)\right]}I_1+\sqrt{J_2}$$

Reference: Cui and Fall b, 2015

Consolidation Equation



Consolidation Equation

- Continuity of pore space

$$\underbrace{\left(-dV_{ch\_s}+dV_{m}+dV_{Ts}\right)}_{A_1}=\underbrace{\left(-dV_{ch\_w}+dV_{Tw}+dV_{sp}+dV_{Pw}\right)}_{A_2}+\underbrace{dV_{a}}_{A_3}$$

skeleton and solid phase

pore water

pore air

Consolidation Equation

- Continuity of pore space

$$\left\{\frac{\left(2v_w - v_n - v_{ab-w}\right)R_{n-w/hc}}{\left(w/c\right)v_w + v_c + \left(1/C_m - 1\right)v_{tailings}} - \left\{\frac{\sigma + \alpha_{Biot}\left[SP_w + \left(1-S\right)P_a\right]}{E}\right\}\left\{\frac{9\left[1-2\upsilon\right]}{E}\frac{\partial E}{\partial \xi} + 18\frac{\partial \upsilon}{\partial \xi}\right\}\right.$$

$$\left. + \left[SP_w + \left(1-S\right)P_a\right]\frac{1-2\upsilon}{E}\frac{\partial \alpha_{Biot}}{\partial \xi} - \alpha_{Biot}\left(P_a - P_w\right)\frac{1-2\upsilon}{E}\left\{\left[1 - S_e\left(P_w, P_a, \xi\right)\right]\frac{\partial \theta_r}{\partial \xi} + \left(\frac{e}{1+e} - \theta_r\right)\frac{\partial S_e}{\partial \xi}\right\}\right\}\frac{\partial \xi}{\partial t}$$

$$+\left\{\alpha_{Biot}S\frac{1-2\upsilon}{E} + \frac{e}{1+e}S\beta_{Pw} - \alpha_{Biot}\left(P_a - P_w\right)\frac{1-2\upsilon}{E}\left(\frac{e}{1+e} - \theta_r\right)\frac{\partial S_e\left(P_w, P_a, \xi\right)}{\partial P_w}\right\}\frac{\partial P_w}{\partial t}$$

$$+\frac{\partial \lambda}{\partial t}\frac{\partial g}{\partial I_1} - \frac{eSRT}{\left(1+e\right)\left(M_a P_a - \rho_a RT\right)}\frac{\partial \rho_a}{\partial t} + \frac{1-2\upsilon}{E}\frac{\partial \sigma}{\partial t} + k\frac{k_{rw}}{\mu_w}\nabla^2 P_w$$

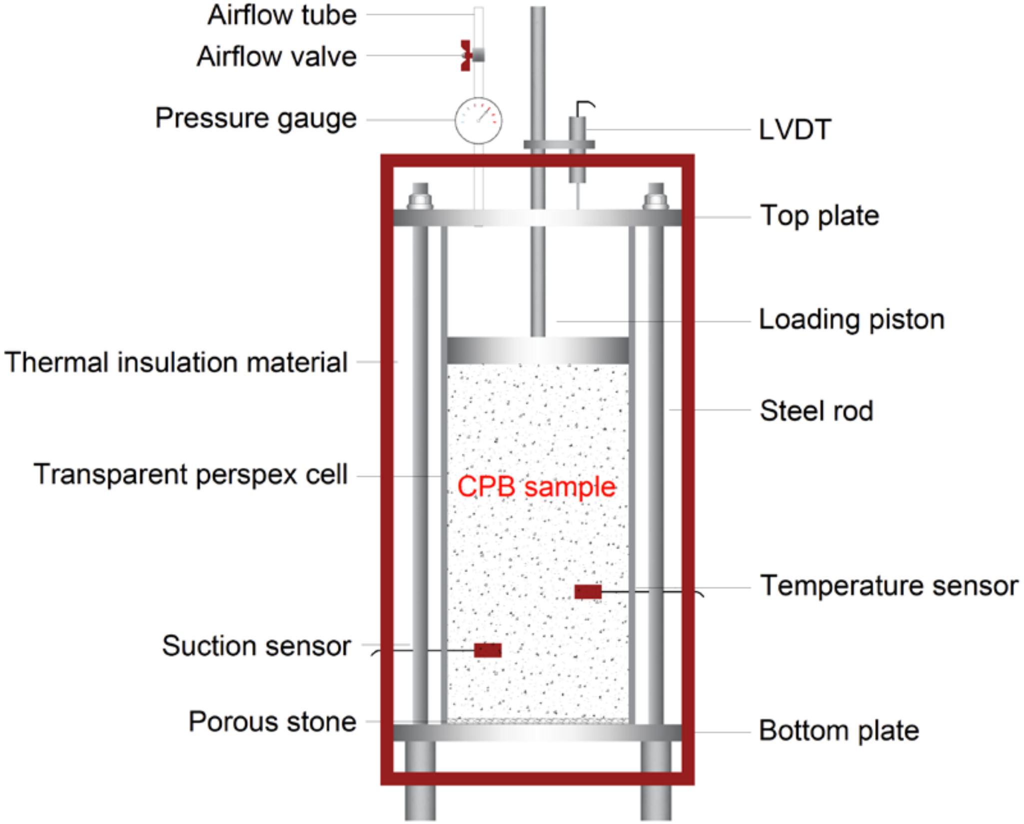
$$+\left\{\alpha_{Biot}\left(1-S\right)\frac{1-2\upsilon}{E} + \frac{eSM_a}{9\left(1+e\right)\left(M_a P_a - \rho_a RT\right)} - \alpha_{Biot}\left(P_a - P_w\right)\frac{1-2\upsilon}{E}\left(\frac{e}{1+e} - \theta_r\right)\frac{\partial S_e}{\partial P_a}\right\}\frac{\partial P_a}{\partial t}$$

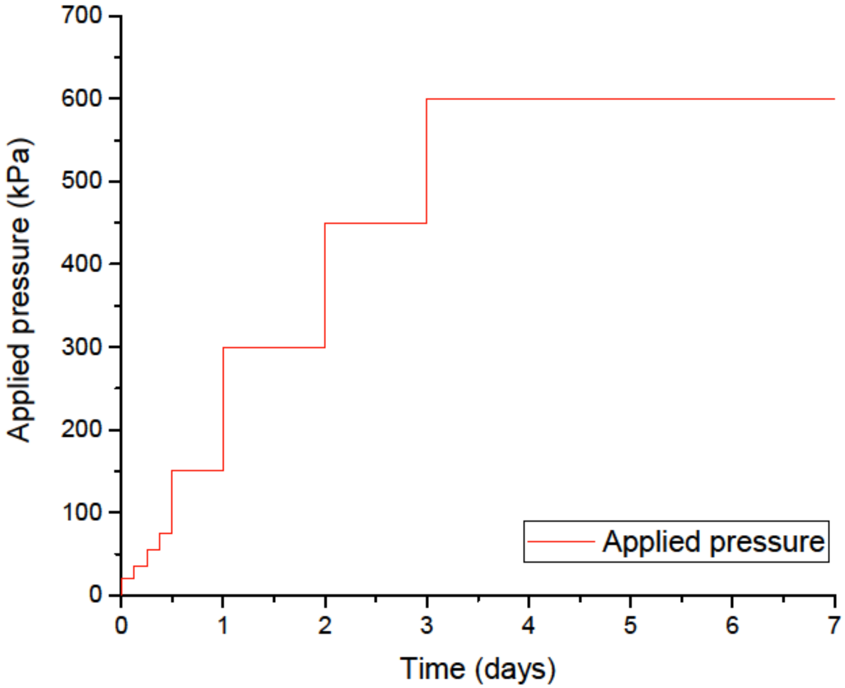
$$+\left[\alpha_{Ts} - \frac{e}{1+e}S\alpha_{Tw} - \frac{eSR\rho_a}{\left(1+e\right)\left(M_a P_a - \rho_a RT\right)}\right]\frac{\partial T}{\partial t}$$

$$= -\alpha_{Biot}\left(P_a - P_w\right)\frac{1-2\upsilon}{e^2\left(1+e\right)^2 E}\left[S_e e + \left(1+e\right)^2\left(1 - S_e\right)\theta_r\right]\frac{\partial e}{\partial t}$$

Multiphysics modelling of CPB

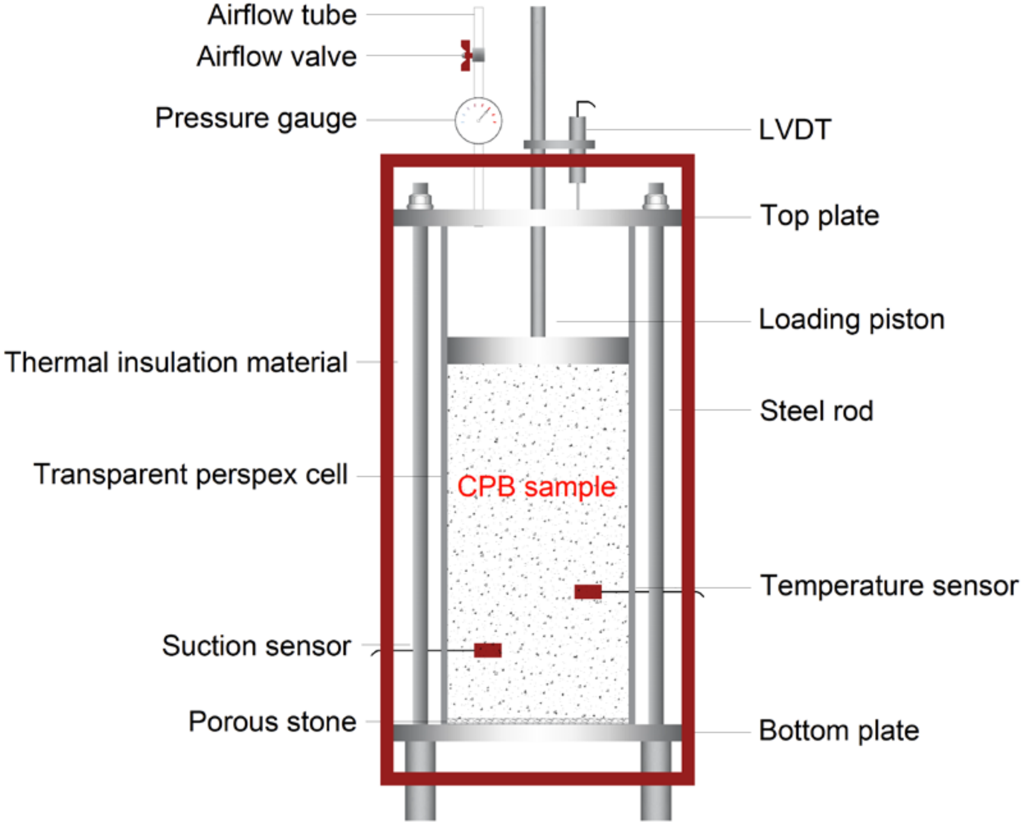
- Pressure cell test





Multiphysics modelling of CPB

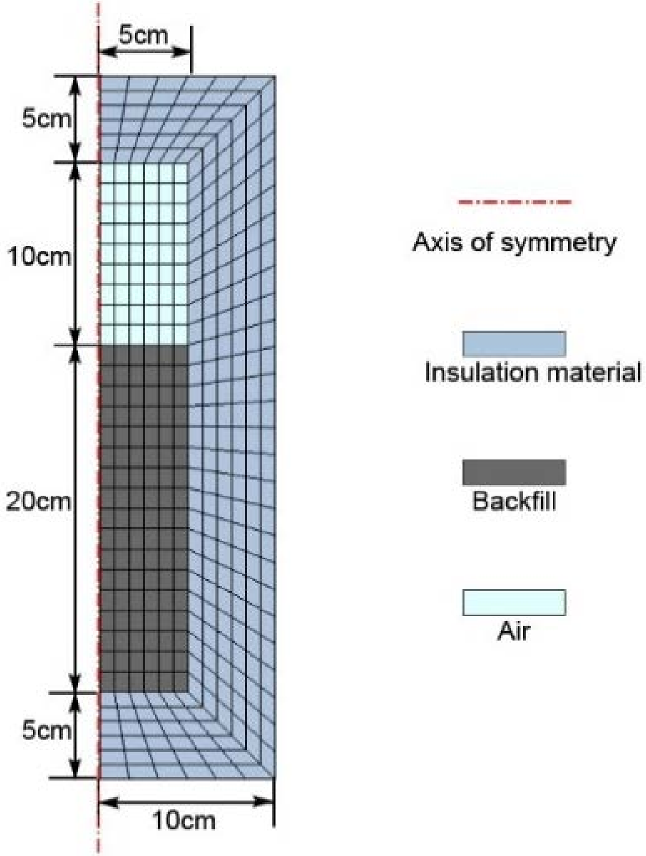
- Pressure cell test



- ➢ Measured data:
  - ➢ Temperature
  - ➢ Pore water pressure
  - ➢ Vertical displacement

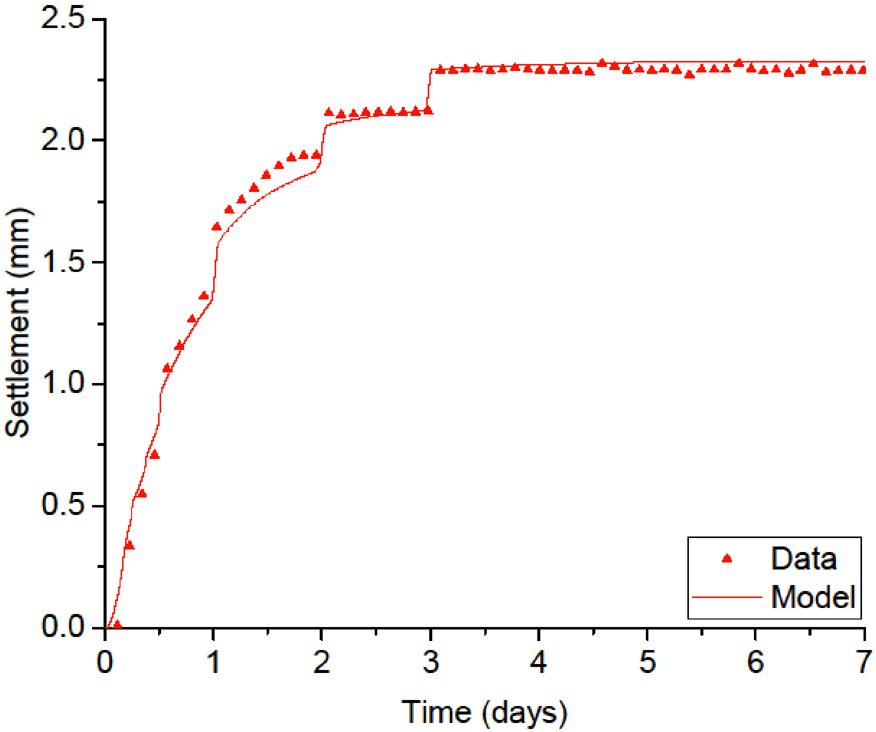
Multiphysics modelling of CPB

- Pressure cell test



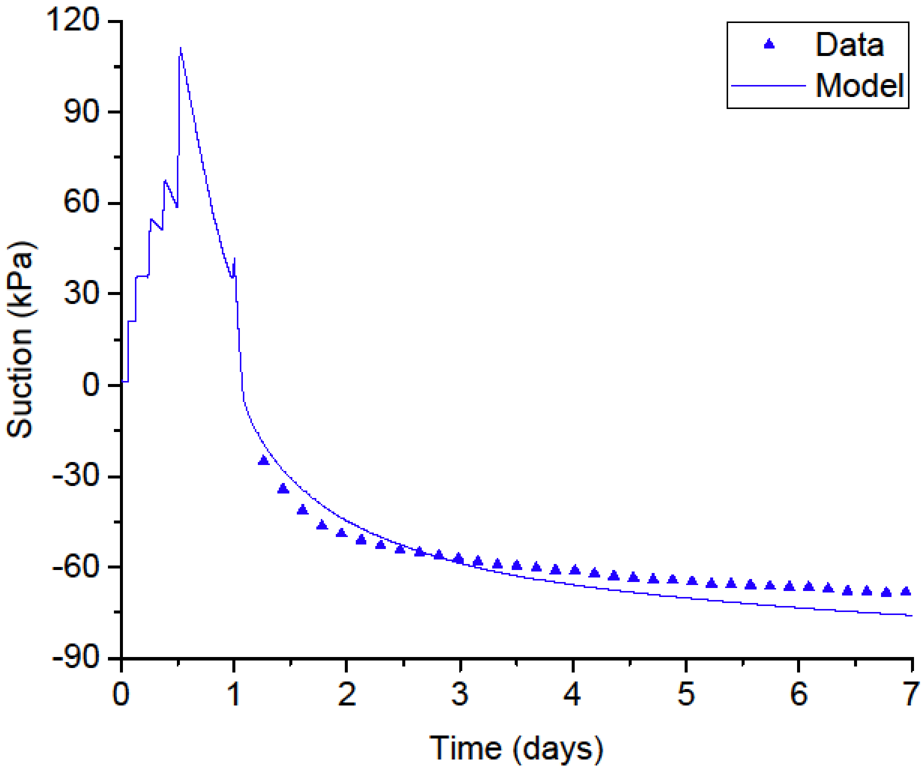
Multiphysics modelling of CPB

- Evolution of Settlement



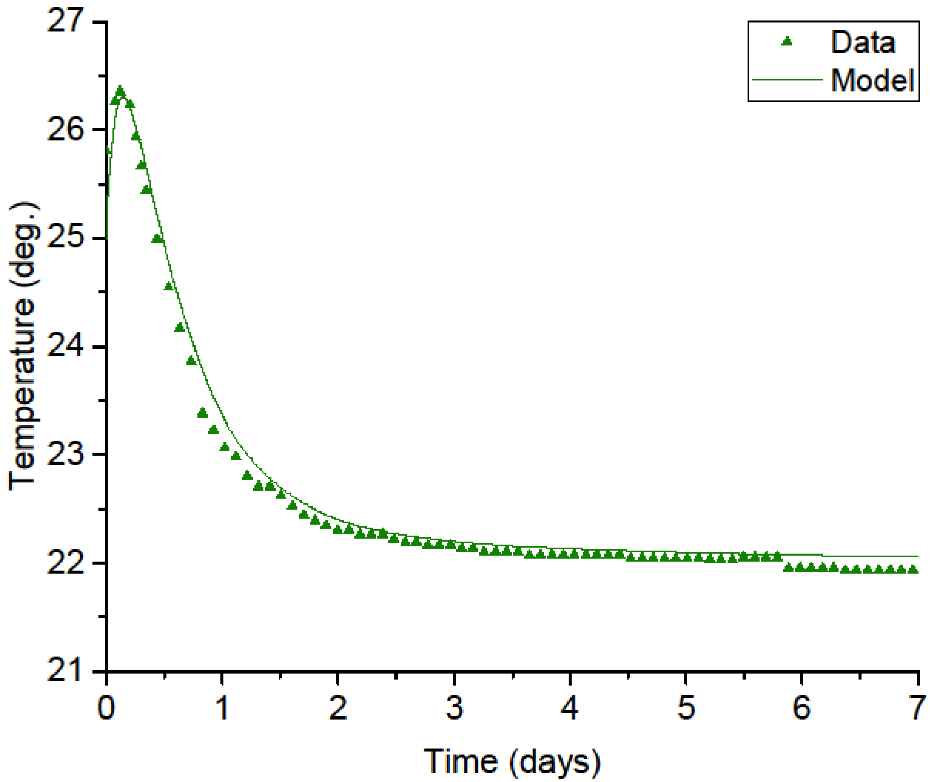
Multiphysics modelling of CPB

- Evolution of Suction



Multiphysics modelling of CPB

- Evolution of Temperature



Conclusions

- The strongly interacting multiphysics within CPB dominate the evolution of CPB behaviour and properties.

- Consolidation behavior of CPB is closely related to the strongly coupled THMC process. Therefore, the conventional consolidation analysis technique (i.e., analysis of coupled H-M process) is unsuitable to predict the deformation of CPB.

- Good agreement between simulation results and experimental data is obtained, which confirm the predictive capability of the developed model.

Thank you!

any questions?