Modeling Metamaterials with a Time-Domain Perfectly Matched Layer Formulation

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Abstract

INTRODUCTION: Perfectly matched layers (PML) have been widely used for simulating wave propagation in unbounded media to effectively avoid spurious wave reflections from the computational domain boundaries. Time-domain PML formulations, especially for elastic waves, usually use a complex system of first-order equations.

Compact second-order time-domain formulations are particularly desired for finite element method (FEM) simulations in COMSOL Multiphysics® software. Such PML formulations were introduced by the authors [1, 2] for modeling wave propagation in unbounded fluid, solid, as well as for coupled fluid-solid media. In this work, we demonstrate the use of a time-domain PML formulation for the complex case of wave propagation in acoustic/elastic metamaterials. In particular, the time-domain superfocusing of acoustic waves in water by a slab of a solid metamaterials lens (see Zhou et al. [3]) is presented, enabling many interesting characteristics of metamaterials to be observed. The limitations of the PML in modeling metamaterials will be addressed, particularly the use of periodic structures within the PML, as considered in [4].

USE OF COMSOL MULTIPHYSICS®: Our PML formulations were reformulated in a weak form to facilitate the use of the mathematical models using the PDE interfaces in the COMSOL software. The fluid-solid coupling boundary conditions were presented in a form that enables these conditions to be easily integrated into the weak formulation. Second order Lagrange finite elements were used for space discretization, while the generalized alpha method was used for time discretization.

RESULTS: The solid metamaterial lens proposed by Zhou et al. [3], was used in our simulations. It consists of a periodic structure of brass cylinders and cavities embedded in Al-SiC foam, with a unit cell as shown in the top right panel of Figure 1. The left column of Figure 1 shows snapshots of the simulations for the case were the slab of solid consists only of the foam matrix with water on both sides, while the right column shows the corresponding snapshots when the metamaterial periodic structure is included, but well within the physical domain. The very slow group velocity, reversed phase velocity, and other theoretically expected properties of metamaterials are observed in the simulations. Figure 2 shows results corresponding to those of Figure 1, but with the periodic structure extending to the inner edge of the PML. The way the total energy evolves in time within the solid slab is presented in Figure 3 for the three simulations in the first two figures. Figure 4 shows the simulations for the case where the periodic structure

extends inside the PML. It is clear that the solution blow up marking a limitation of the PMLs in this case.

CONCLUSION: The use of compact second-order time-domain formulations in modeling the complex and interesting case metamaterials, which are usually modeled in frequency domain, is presented. The limitation of the PML in the case of periodic metamaterials is addressed. Some proposed future work to overcome these limitations for periodic and homogenized metamaterials will be discussed.

Reference

[1] H. Assi and R.S.C. Cobbold, Compact second-order time-domain perfectly matched layer formulation for elastic wave propagation in two dimensions, Mathematics and Mechanics of Solids, (2015) DOI: 10.1177/1081286515569266

[2] H. Assi and R.S.C. Cobbold, A perfectly matched layer formulation for modeling transient wave propagation in an unbounded fluid-solid medium, Submitted (2015)

[3] X. Zhou et al., Acoustic superfocusing by solid phononic crystals, Applied Physics Letters, 105(23), 233506 (2014)

[4] A.F. Oskooi et al., The failure of perfectly matched layers, and towards their redemption by adiabatic absorbers, Optics Express, 16(15), 11376–11392 (2008)

Figures used in the abstract

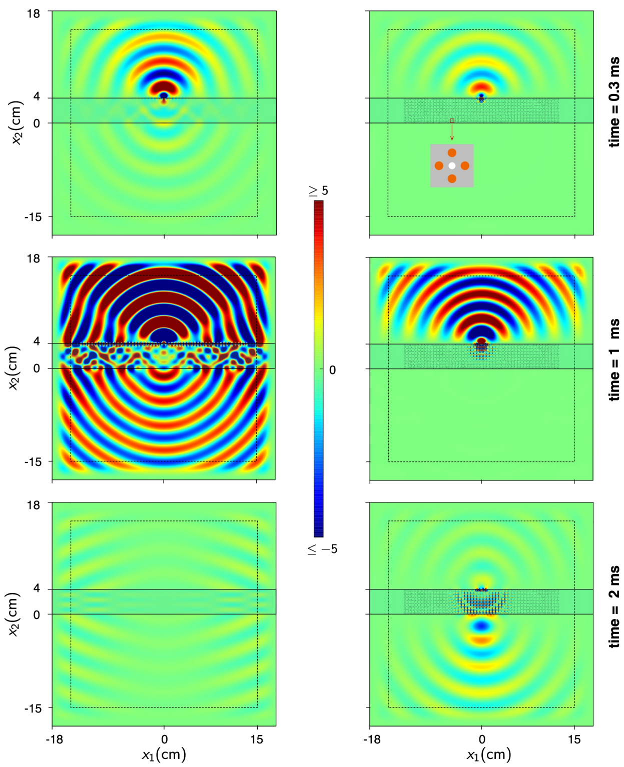

Figure 1: Snapshots of simulations of a radiating line source in water close to an infinite slab of solid Al-SiC foam are shown in the right column. Periodic structure of 8X50 unit cells, as the one shown enlarged in the left column's top panel, were inserted in the foam to generate left-handed metamaterial. The snapshots of the simulation presented in the right column show the slow group velocity in the metamaterials and the focusing at the 2 ms snapshot.

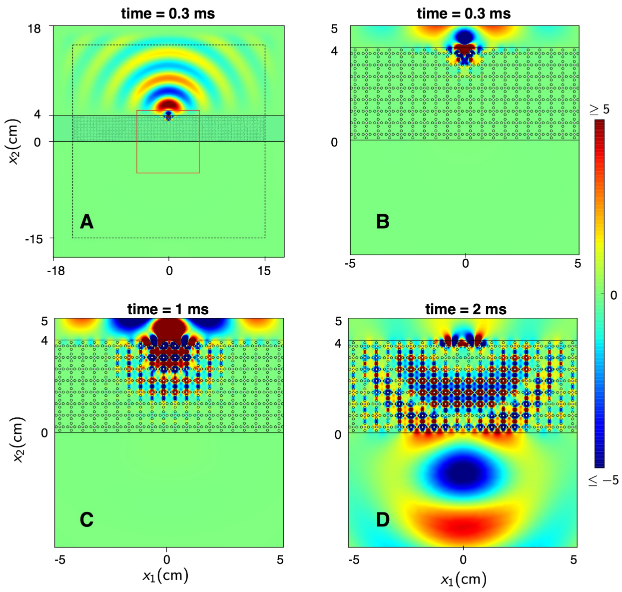

Figure 2: The periodic structure in Figure 1 is extended to the edge of the PML (8x60 unit cells) as shown in (A). In (B) (C) and (D), the snapshots of the simulation show a zoomed view of a region of the periodic structure corresponding to the red square in (A).

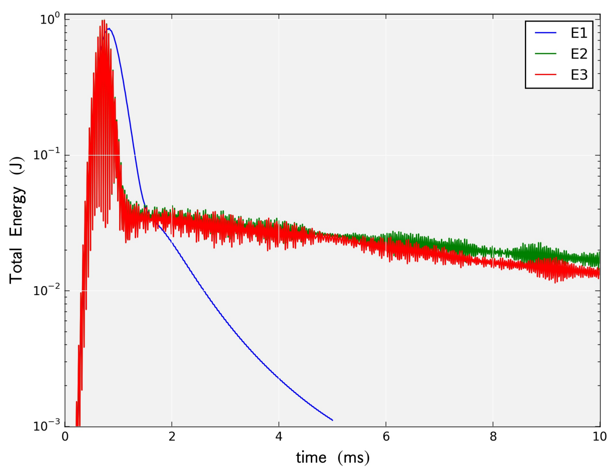

Figure 3: Total energies evolving in time in the solid slab region occupied by 8x50 cells were calculated for the three simulations in the right columns of Figures 1 &2. Total energies, E2 and E3, which belong to periodic structure simulations in Figure 1 and Figure 2 respectively, are very close. They show that the energy decays rapidly, but slower than for the case of homogeneous foam (E1) in the left column of Figure 1.

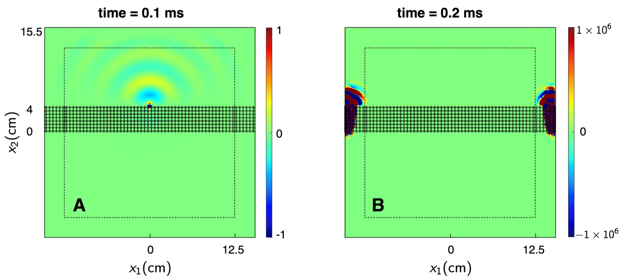

Figure 4: When the metamaterial is extended to the PML region, the solution blows up early in the simulation. Note that the color map scale in (B) is six orders of magnitude grater than in other snapshots. This marks a limitation of the PML.