Hydrodynamic Modeling of a Rotating Cone Pump Using COMSOL Multiphysics®

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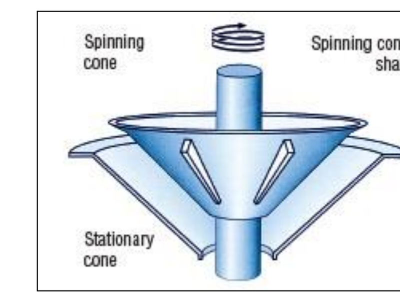
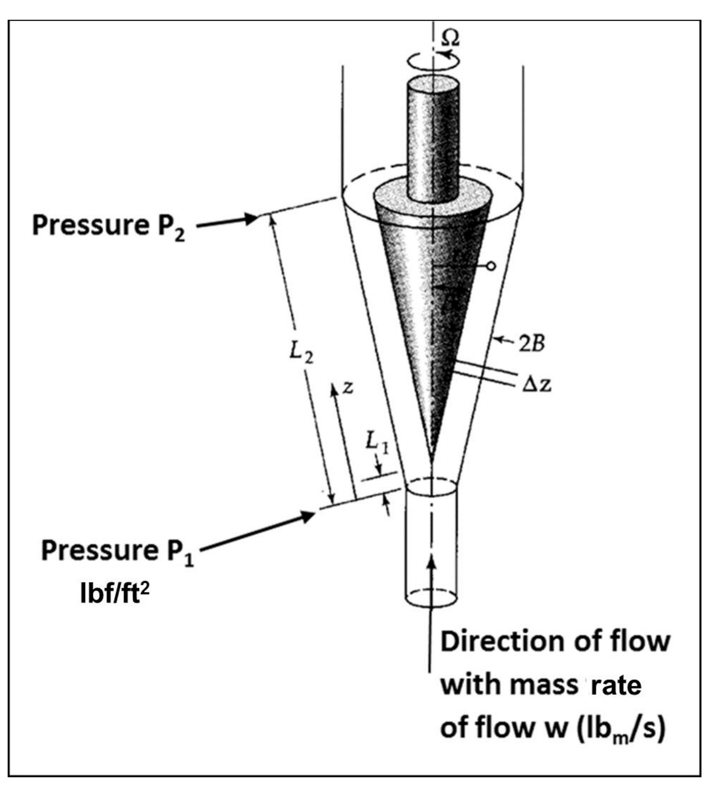
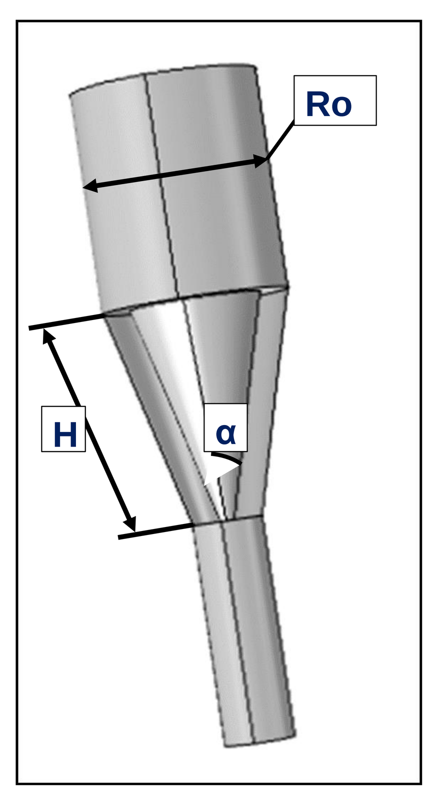


Introduction

The hydrodynamics of thin liquid films flowing over rotating conical surfaces are of considerable importance in industry. The efficiency of important process equipment, such as spinning cone columns, fluid degassers, centrifugal disc atomizers, centrifugal film evaporators, and rotating packed-bed reactors, is greatly influenced by the nature of the fluid velocity and pressure distributions. COMSOL Multiphysics™ is a powerful numerical engine for modeling complex fluid dynamic systems, such as rotating cone pumps. Conical pumps offer simple alternatives to conventional pumps yet the analysis of pump performance has received minimal attention in the literature. Robust models that allow prediction of pump performance would be useful for guiding new designs for various applications versus using less efficient empirical approaches.



Problem Description



Assumed Geometry

Model Equations

Key Assumptions

- Incompressible, isothermal Newtonian fluid.
- Constant fluid density (ρ) and fluid viscosity (μ).
- 3-D flow so that $\mathbf{u} = [u_x \;\; u_y \;\; u_z]$
- No slip condition at all solid walls.
- Both laminar flow and turbulent flow (k-ε model).
- Steady-state and transient behavior.

Laminar Flow

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot \left[-p\mathbf{I} + \mu\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) - \frac{2}{3}\mu(\nabla \cdot \mathbf{u})\mathbf{I} \right] + \mathbf{F}$$

Turbulent Flow k-ε model

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla)\mathbf{u} = \nabla \cdot \left[-p2\mathbf{I} + (\mu + \mu_T)\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) - \frac{2}{3}(\mu + \mu_T)(\nabla \cdot \mathbf{u})\mathbf{I} - \frac{2}{3}\rho k2 \mathbf{I} \right] + \mathbf{F}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\rho \frac{\partial k2}{\partial t} + \rho (\mathbf{u} \cdot \nabla) k2 = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \nabla k2 \right] + P_k - \rho \epsilon$$

$$\rho \frac{\partial \epsilon}{\partial t} + \rho (\mathbf{u} \cdot \nabla)\epsilon = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_\epsilon} \right) \nabla \epsilon \right] + C_{\epsilon 1}\frac{\epsilon}{k2}P_k - C_{\epsilon 2}\rho\frac{\epsilon^2}{k2}, \quad \epsilon = ep2$$

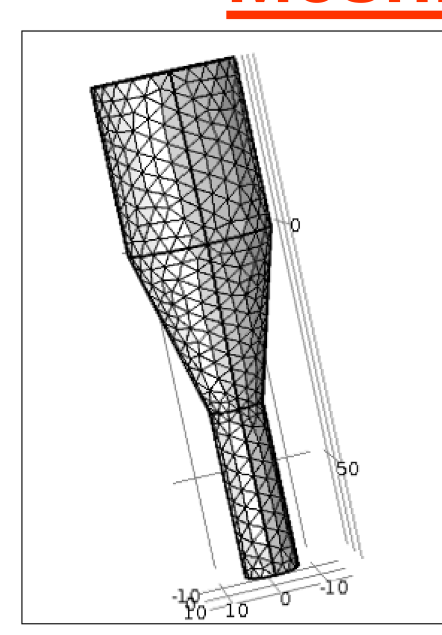
$$\mu_T = \rho C_\mu \frac{k2^2}{\epsilon}$$

$$P_k = \mu_T \left[\nabla \mathbf{u} : \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) - \frac{2}{3}(\nabla \cdot \mathbf{u})^2 \right] - \frac{2}{3}\rho k2 \nabla \cdot \mathbf{u}$$

Objectives

- Develop a realistic model for fluid hydrodynamics based on the 3-D transient Navier-Stokes Equations for an assumed geometry using COMSOL Multiphysics™ in the laminar and turbulent flow regimes.
- Examine the effect of rotational speed (Ω), cone semi-angle (**α**), and ratio of outer to inner radii (κ) on pump performance.

Meshing and COMSOL Parameters



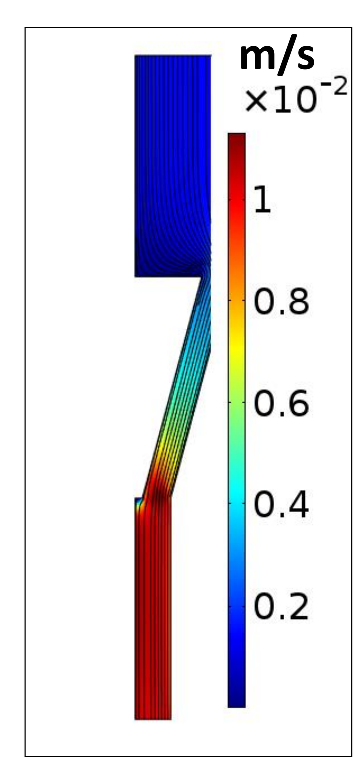
Tetrahedral mesh

Parameter	Value		Units
	Min	Max	
H	10	35	mm
$\kappa = R_i/R_0$	0.7	0.9	-
R_0	15	15	mm
P_{in}	1	1	atm
Ω	3	10	rpm·**10^{-3}**
α	10	45	deg

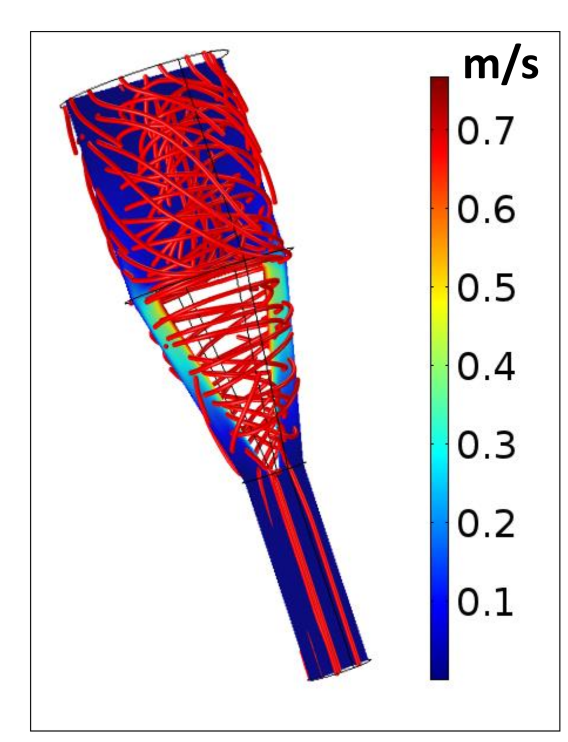
Model parameters

Results and Discussion

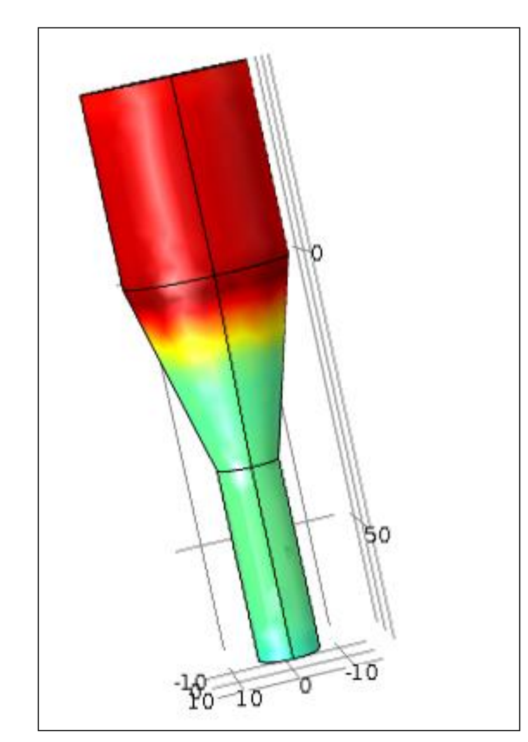
Example of simulation results (at Ω = 6000 rpm, α = 15°, w = 1 g/s)



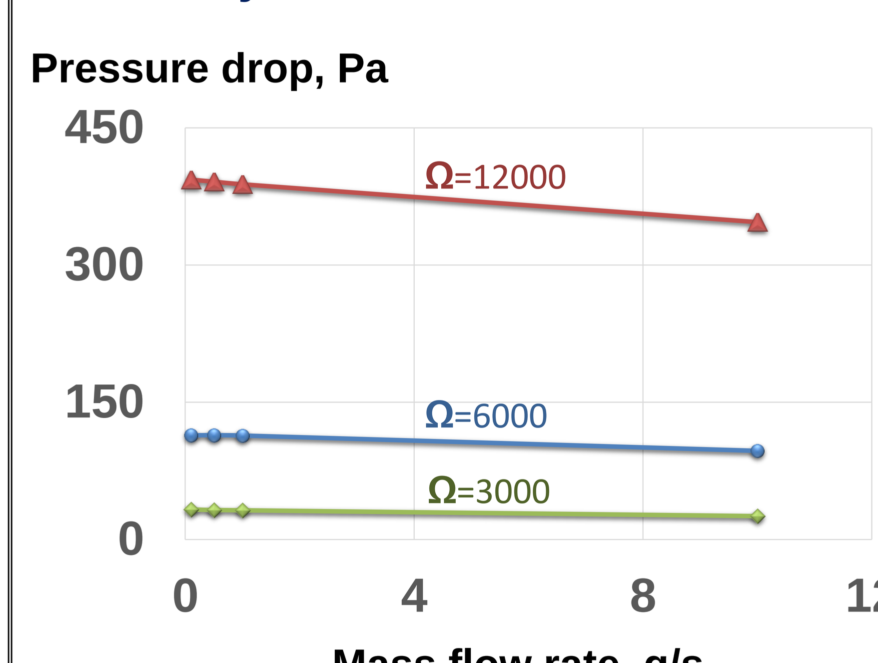
Initial Estimate for Velocity Profiles in 2-D



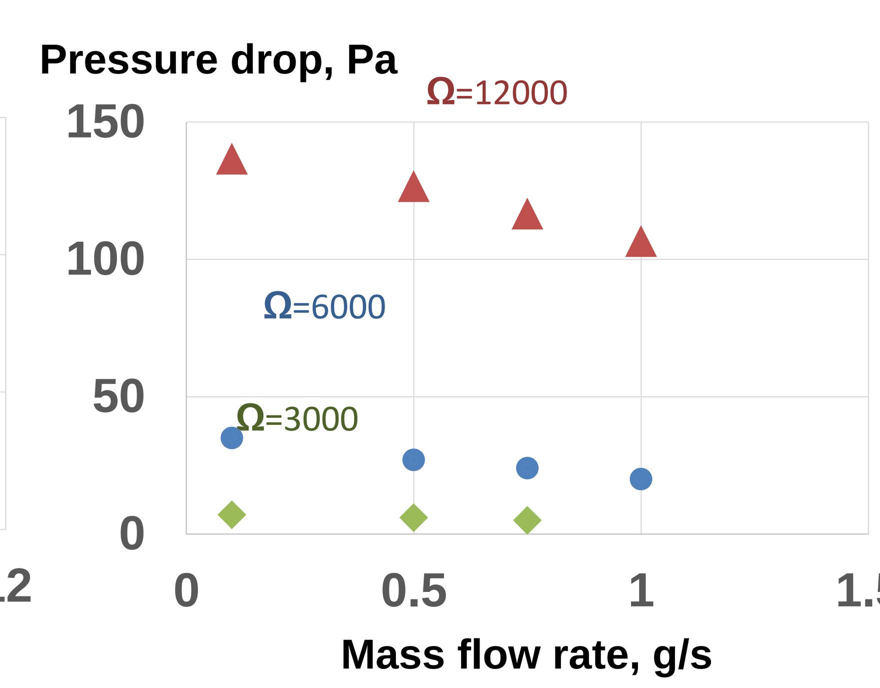
3-D Velocity Profiles



3-D Pressure Field



Head curve for semi angle = 12° and different Ω



Head curve for semi angle = 45° and different Ω

Conclusions

- COMSOL provides a convenient tool for investigating various design parameters for system optimization and captures important non-linear fluid mechanical effects
- The rotating cone pump is capable of creating comparatively large throughputs; However, it cannot create heads comparable to conventional centrifugal pumps but is less complex.

References

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