ESIEE AMIENS

COMSOL CONFERENCE 2015 GRENOBLE

# Investigating Magnetic and Electric Fields Couplings for 3D models in Harmonic and Transient States

O. MALOBERTI
O. MANSOURI
(ESIEE Amiens)

# Context and Goal

- **Context:**

Pulsed magnetic technologies

Medium frequencies – pulsed magnetic fields

Eddy currents, skin effect and Lorentz force

- **Goal:**

**Investigate a 3D harmonic and transient formulation coupling magnetic and electric fields formulations in COMSOL**

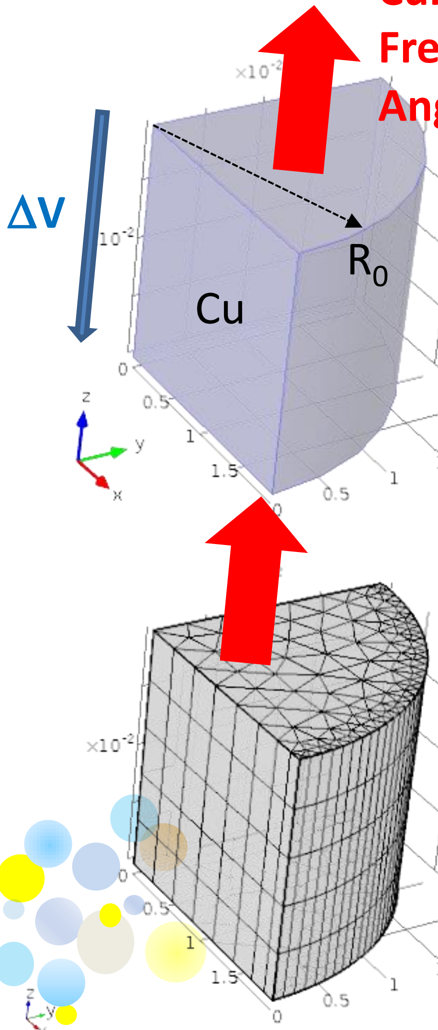
# Summary

- 1. Introduction: 2 or 3 test cases
- 2. Governing and resulting equations
- 3. Modelling of test cases and results
  - 3.1. Presentation of Test Case 1 – the wire
  - 3.2. Presentation of Test Case 2 – the wire in air
  - 3.3. Presentation of Test Case 3 – the coil
- 4. Conclusion and Forthcomings

# 1. Introduction

**Conducting material:** Copper (Conductivity $\sigma \cong 6e^{7}$ S.m$^{-1}$, permeability $\mu=\nu^{-1}=\mu_0=\nu_0^{-1}=4\pi.e^{-7}$ H.m$^{-1}$, permittivity $\varepsilon=\varepsilon_0=8.85.e^{-12}$ F.m$^{-1}$)

**wire**



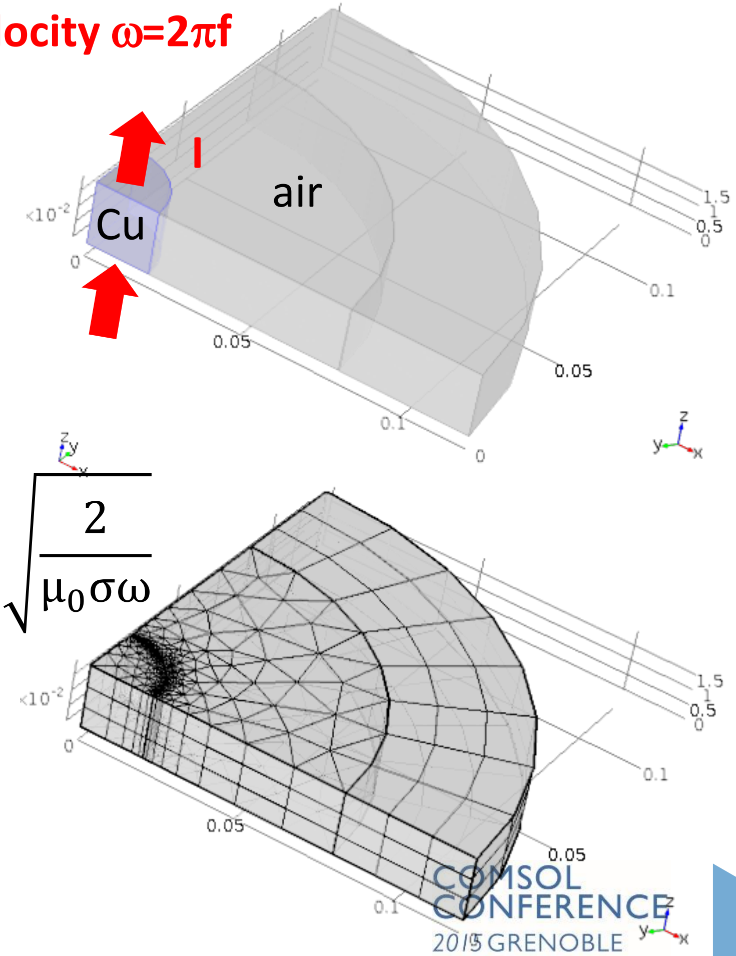
Current I

Frequency f

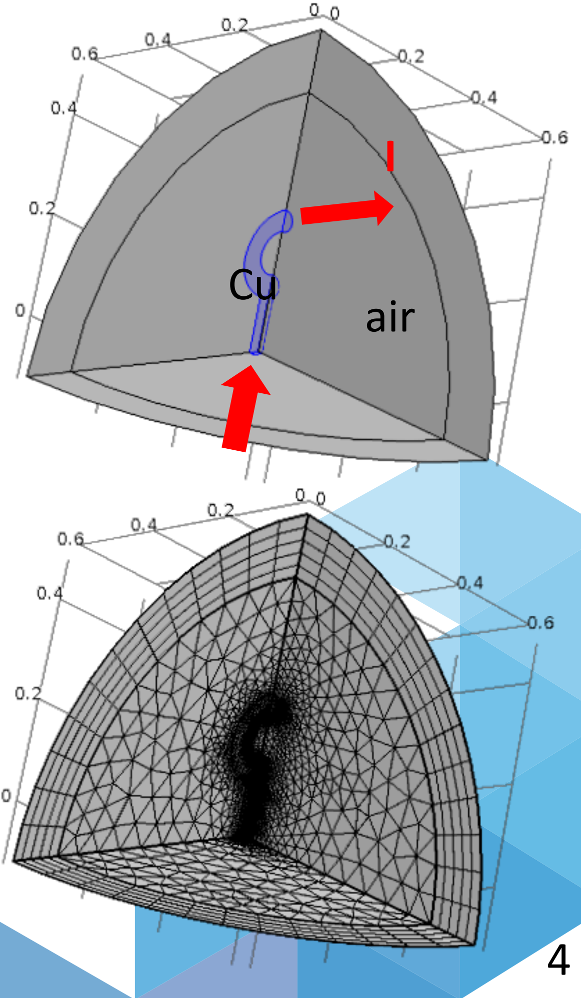
Angle velocity $\omega=2\pi f$

$$d \leq \frac{1}{3}\sqrt{\frac{2}{\mu_0\sigma\omega}}$$

**wire in air**



**coil**



# 2. Governing Equations

- Magnetic Field – Magnetic Vector Potential **A**:

$$\left\{\begin{matrix}\mathbf{\nabla}\times\mathbf{H}=\mathbf{j}\text{ and }\mathbf{H}=\nu\mathbf{B}=\nu_0\mathbf{B}\\ \text{with }\mathbf{B}=\mathbf{\nabla}\times\mathbf{A}\end{matrix}\right\}$$

- Electric Field – Electric Scalar Potential V:

$$\left\{\begin{matrix}\mathbf{\nabla}\cdot\mathbf{j}=\mathbf{0}\text{ and }\mathbf{j}=\sigma\mathbf{E}\\ \text{with }\mathbf{E}=-\mathbf{\nabla}\mathrm{V}\end{matrix}\right\}$$

- Coupling potentials **A** and V:

$$\left\{\begin{matrix}\mathbf{\nabla}\times(\nu\mathbf{\nabla}\times\mathbf{A})+\sigma\partial_t\mathbf{A}=-\sigma\mathbf{\nabla}\mathrm{V}\\ \mathbf{\nabla}\cdot\mathbf{A}=0\end{matrix}\right\}$$

$$\text{with }\mathbf{B}=\mathbf{\nabla}\times\mathbf{A}\text{ and }\mathbf{E}=-\mathbf{\nabla}\mathrm{V}-\partial_t\mathbf{A}$$

# 2. Resulting Equations

- Power losses and equivalent resistance

$$P_j = \iiint_{\text{space}} \frac{j^2}{2\sigma} d^3x \text{ and } \boxed{R = \frac{2P_j}{I^2}} \text{ with } I = \iint_{\pi R_0^2} \mathbf{j} \cdot \mathbf{d^2x}$$

- Magnetic energy and equivalent inductance

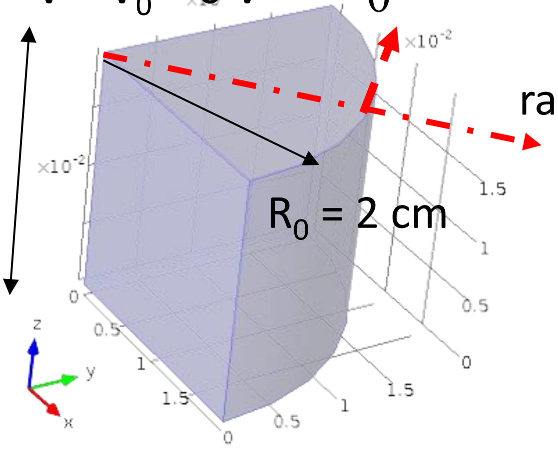
$$W_m = \iiint_{\text{space}} \frac{B^2}{2\mu} d^3x \text{ and } \boxed{L = \frac{2W_m}{I^2}}$$

- Electric energy and equivalent capacity

$$W_e = \iiint_{\text{space}} \frac{D^2}{2\varepsilon} d^3x \text{ and } \boxed{C = \frac{2W_e}{(\Delta V)^2}}$$

# 3. Modelling of test cases
## 3.1. Test case 1 – *the wire*

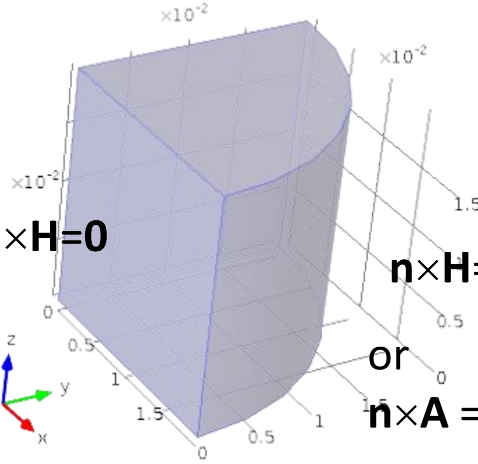
V = $V_0$ = 0 V



CL = 2 cm

$R_0$ = 2 cm

V = $V_1$ = 1 μV = 1 e$^{-6}$ V



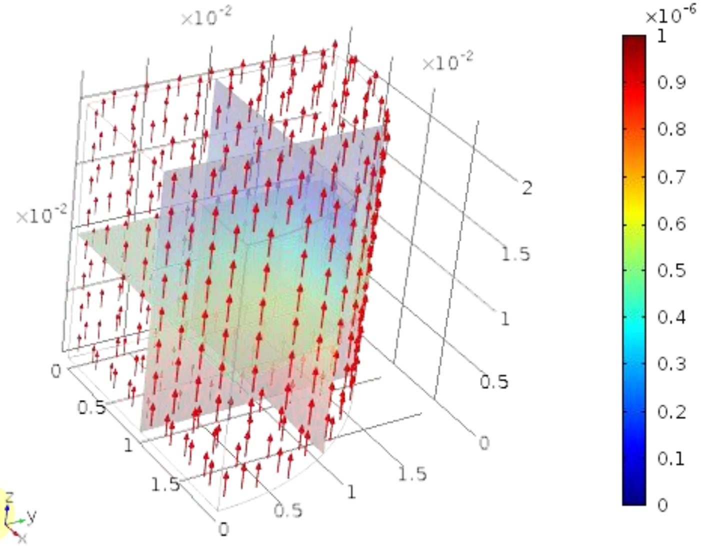
**n×H=0**

$$\mathbf{n}\times\mathbf{H}=\mathbf{H}_0 = \frac{\sigma\frac{V_1-V_0}{CL}\pi R_0^2}{2\pi R_0}\mathbf{u}_\theta$$

or

**n×A = 0**

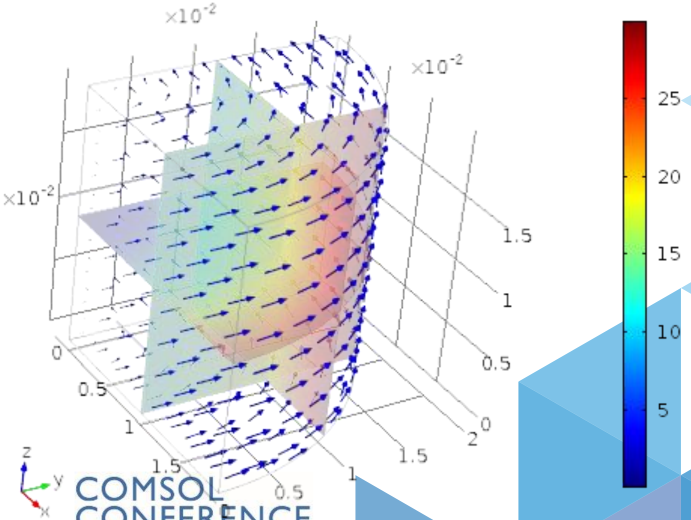
freq0(5)=50 freq(1)=50 Multicoupes: Potentiel électrique (V)
Flèches sur surface: Densité de courant



V

**j**

freq0(5)=50 freq(1)=50 Multicoupes: Champ magnétique, norme (A/m)
Flèches sur surface: Champ magnétique



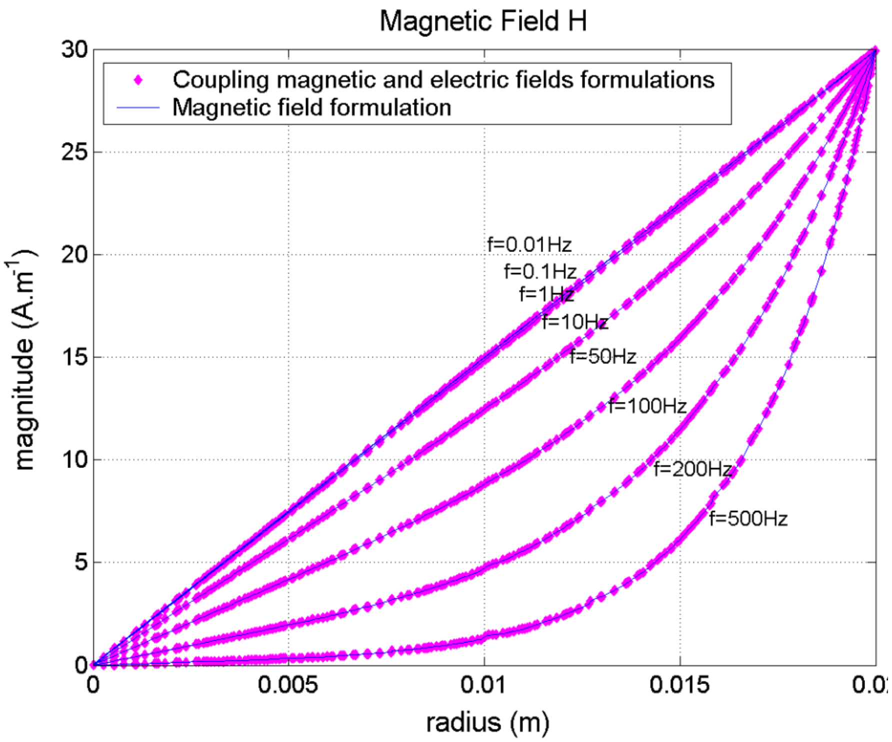
H

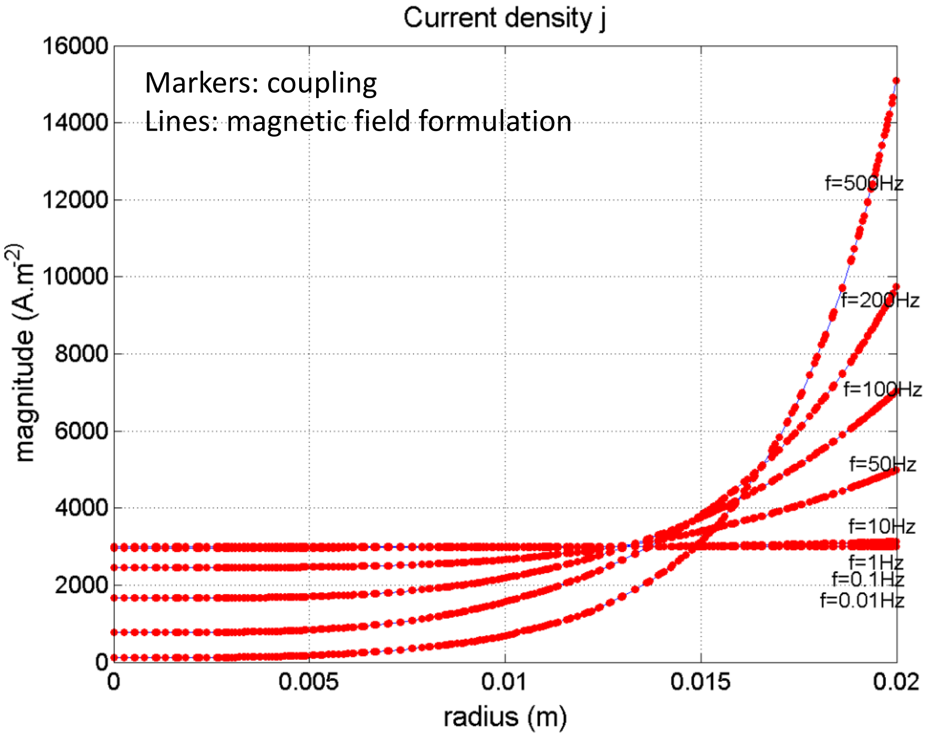
**H**

# 3. Modelling of test cases

## 3.1. Test case 1 – *the wire*

- H and j distribution with field constraint



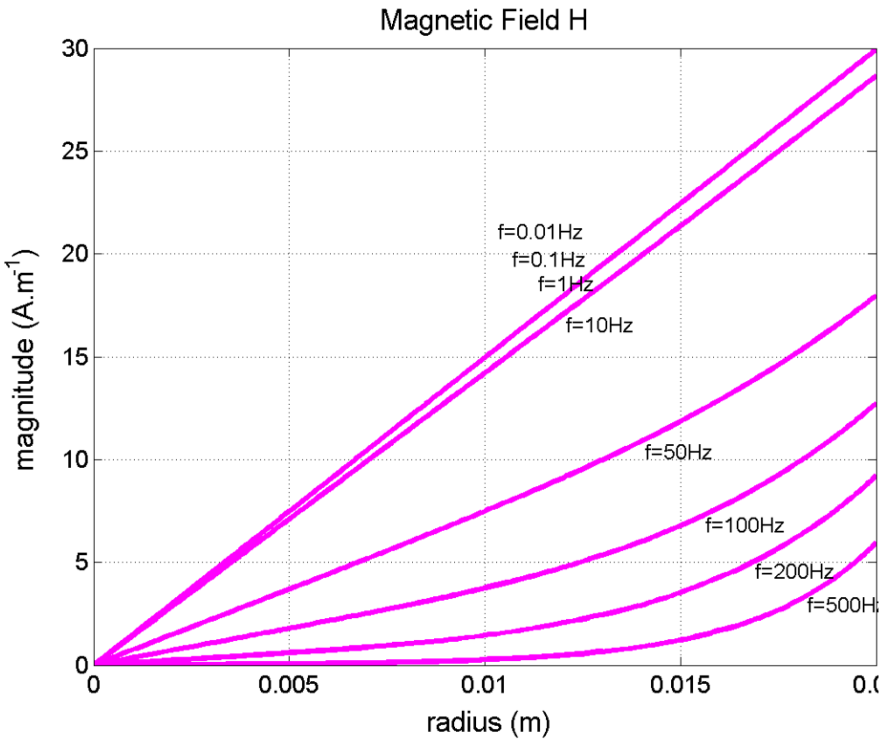


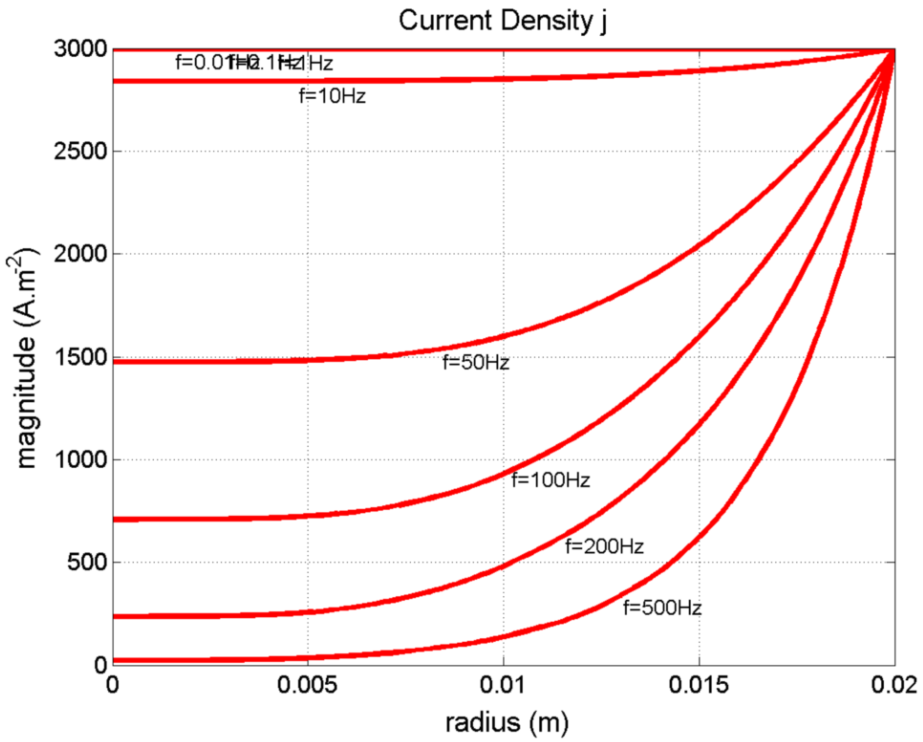
- Eddy currents and skin effect correctly described
- Agreement between the two methods and formulations

# 3. Modelling of test cases

## 3.1. Test case 1 – *the wire*

- H and j distribution without field constraint

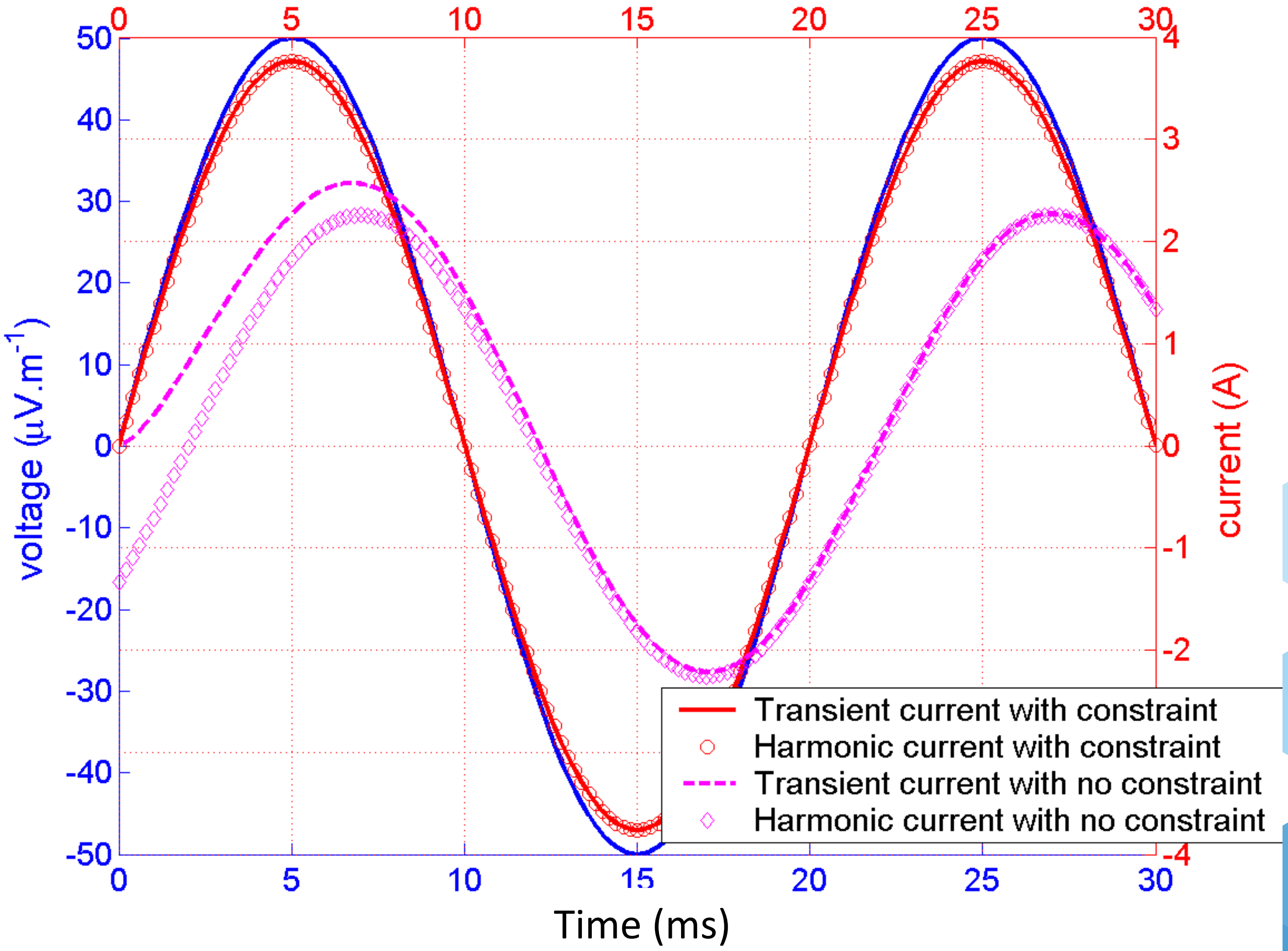




- Eddy currents and skin effect correctly described
- Electric field and total current density damping

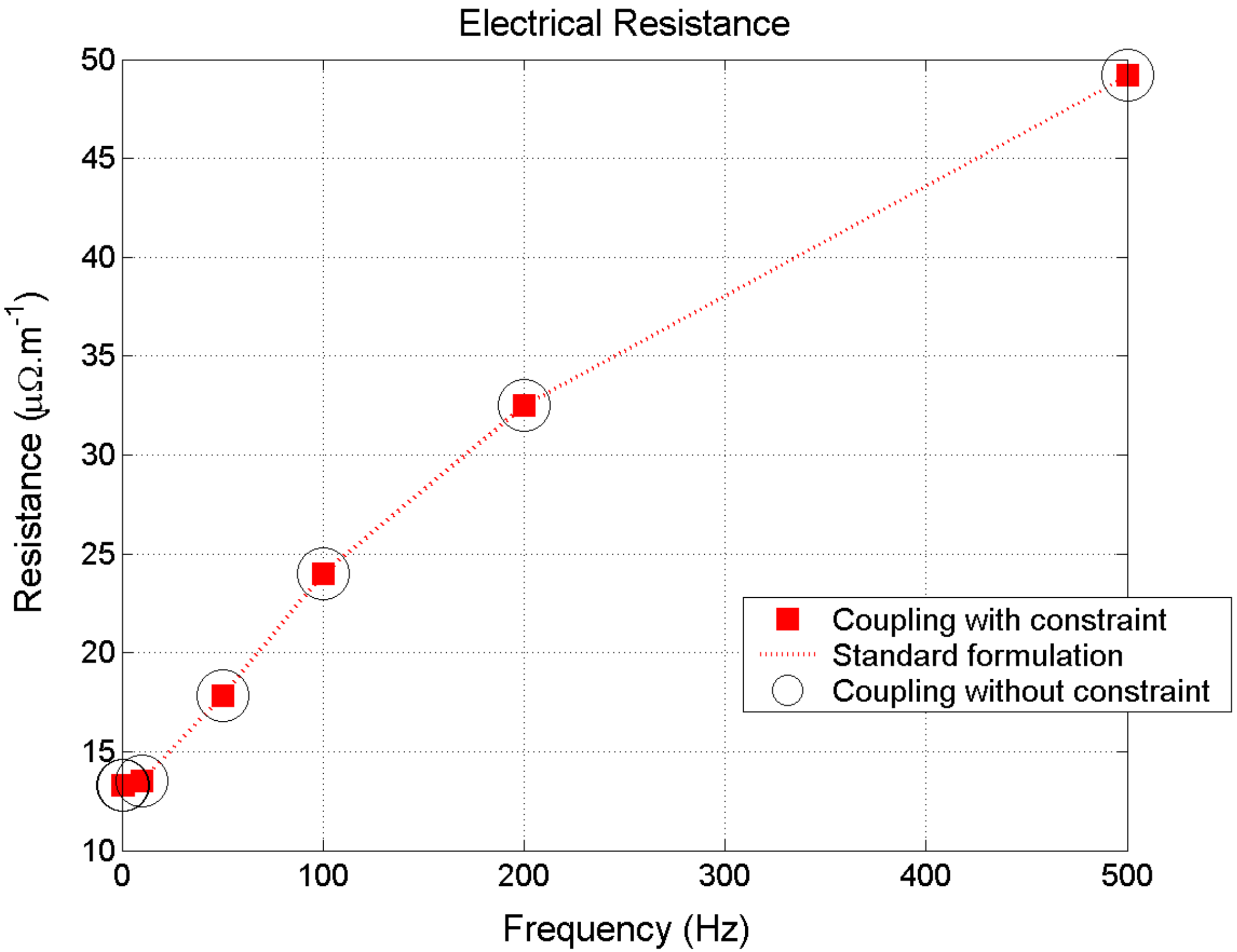
# 3. Modelling of test cases
## 3.1. Test case 1 – *the wire*

- Current-Voltage behaviour @ 50 Hz



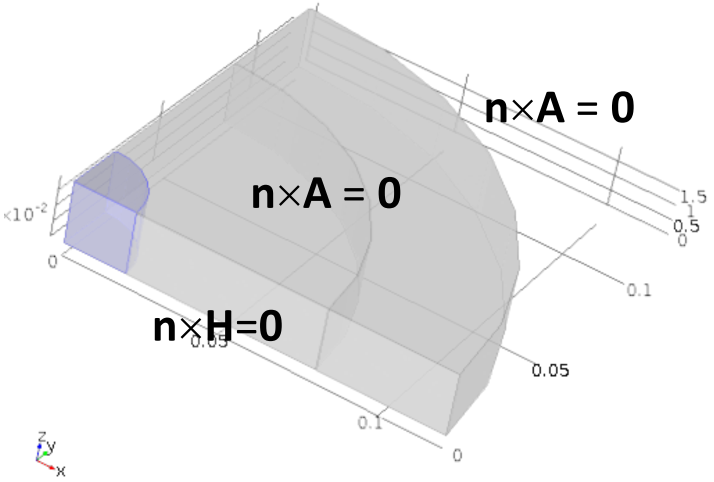
# 3. Modelling of test cases
## 3.1. Test case 1 – *the wire*

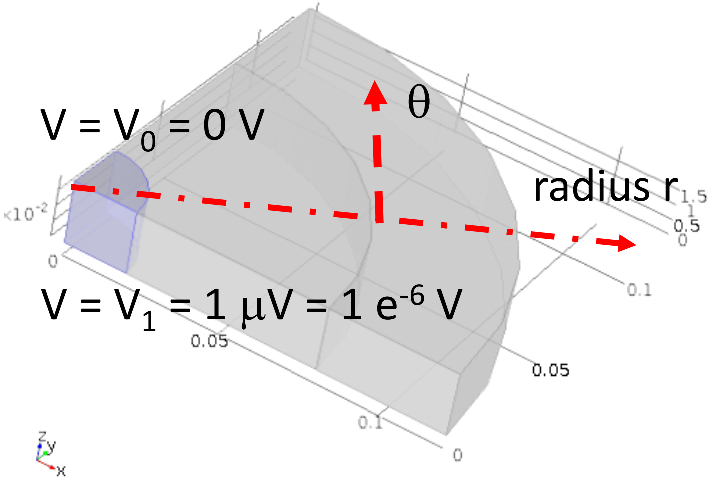
- Equivalent resistance per unit length



# 3. Modelling of test cases

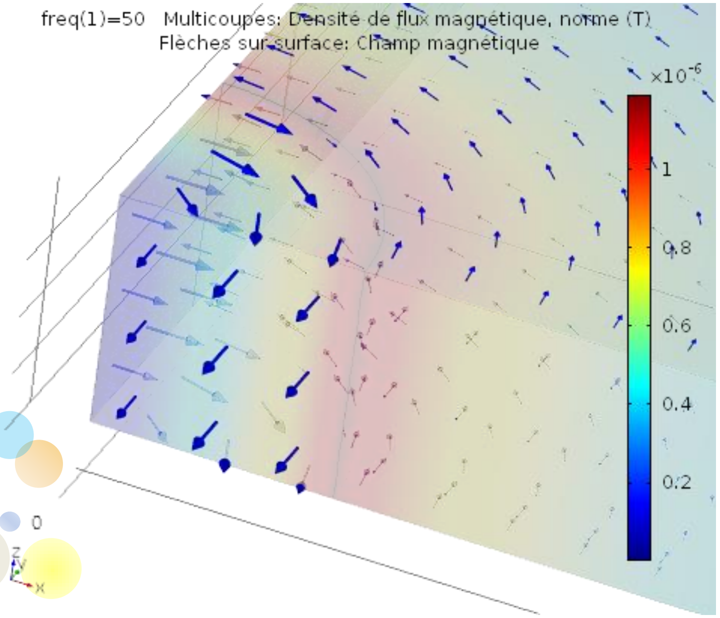
## 3.2. Test case 2 – *the wire in air*



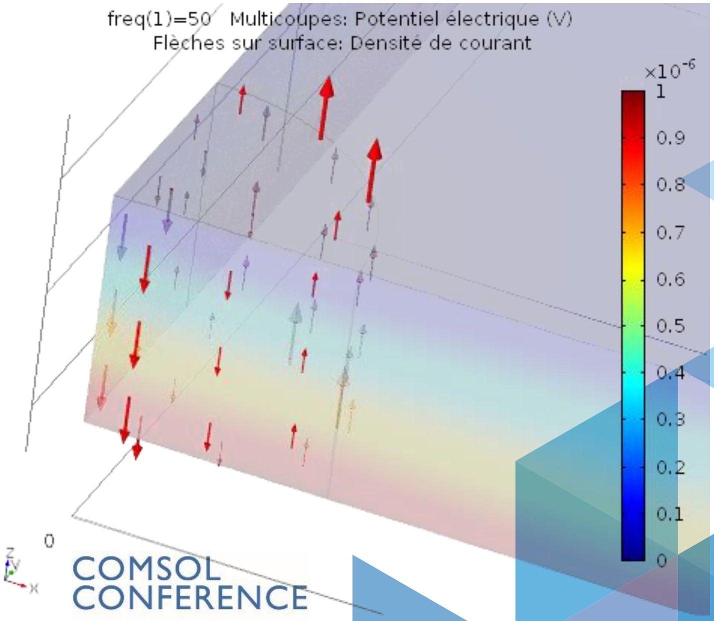


H

**H**



freq(1)=50 Multicoupes: Densité de flux magnétique, norme (T)
Flèches sur surface: Champ magnétique



freq(1)=50 Multicoupes: Potentiel électrique (V)
Flèches sur surface: Densité de courant

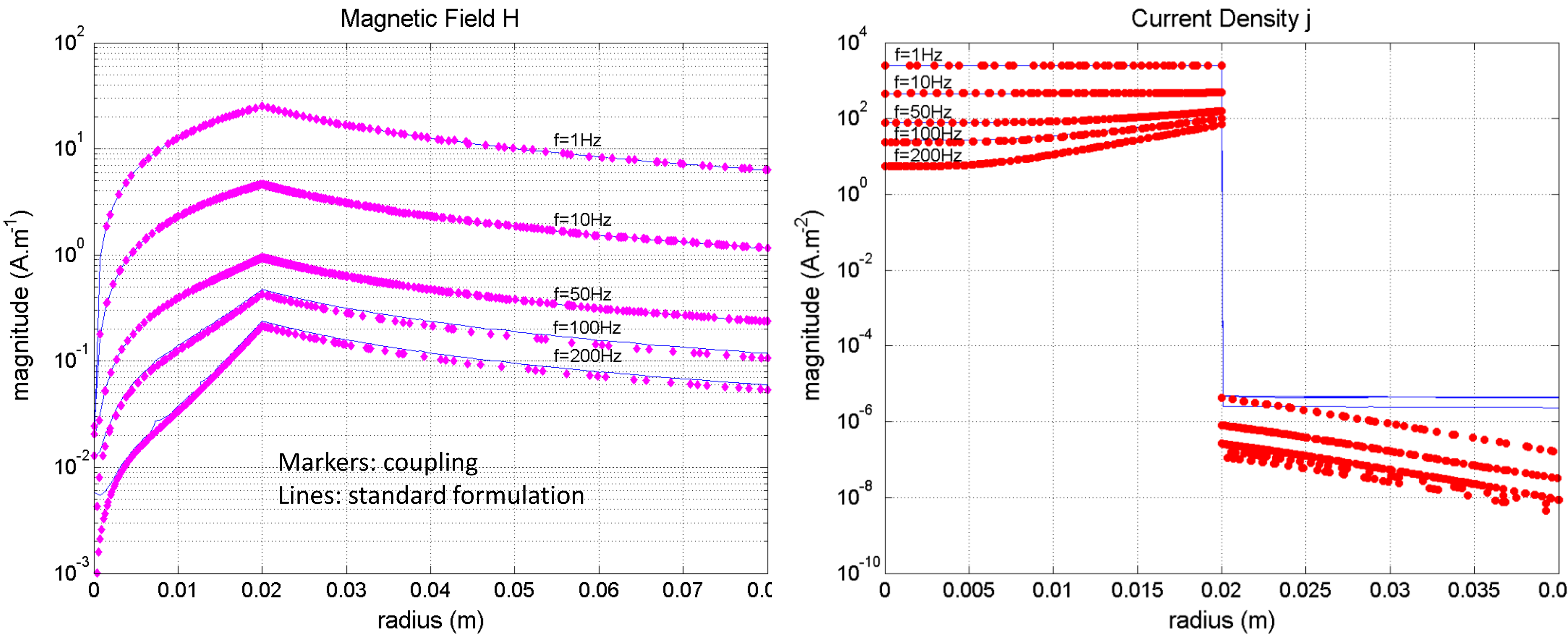
V

**j**

# 3. Modelling of test cases

## 3.2. Test case 2 – *the wire in air*

- Field H and Current density j distribution

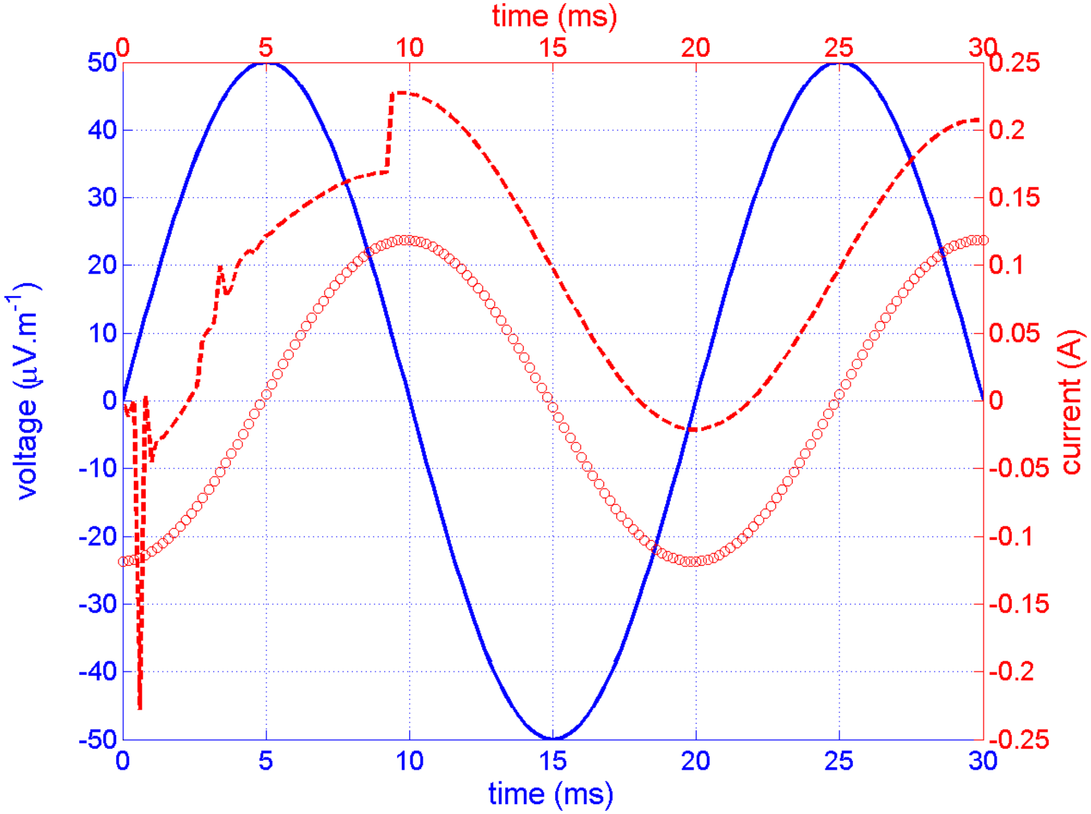


- Field damping due to both the inductance in air and eddy currents
- Eddy currents and skin effect correctly described
- Agreement between the two methods and formulations

# 3. Modelling of test cases

## 3.2. Test case 2 – *the wire in air*

- Harmonic vs transient behaviour @50 Hz



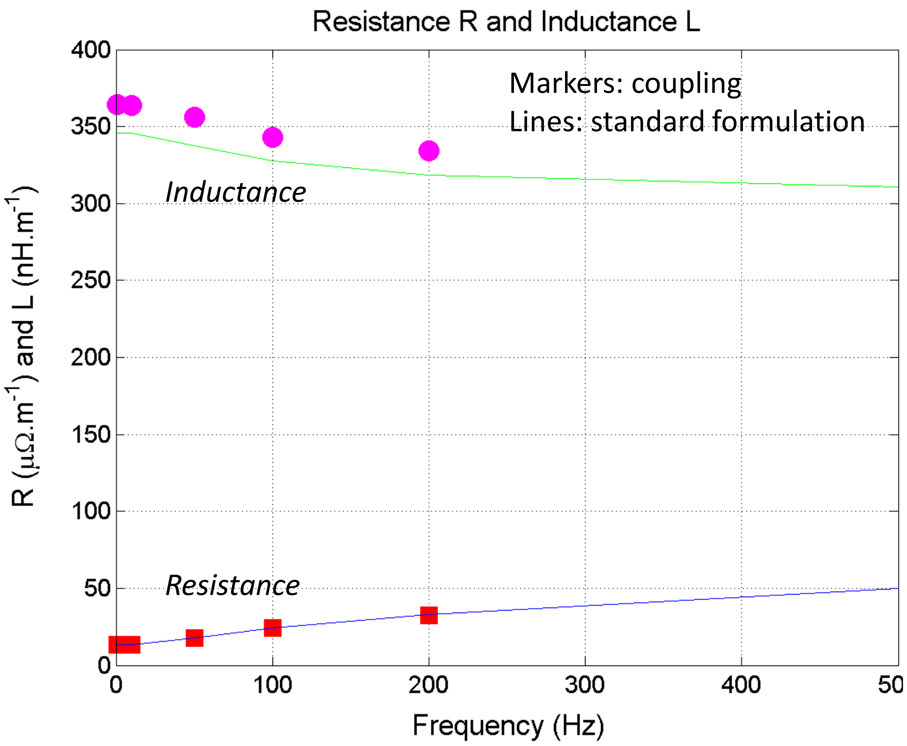
Markers: Harmonic
Lines: Transient

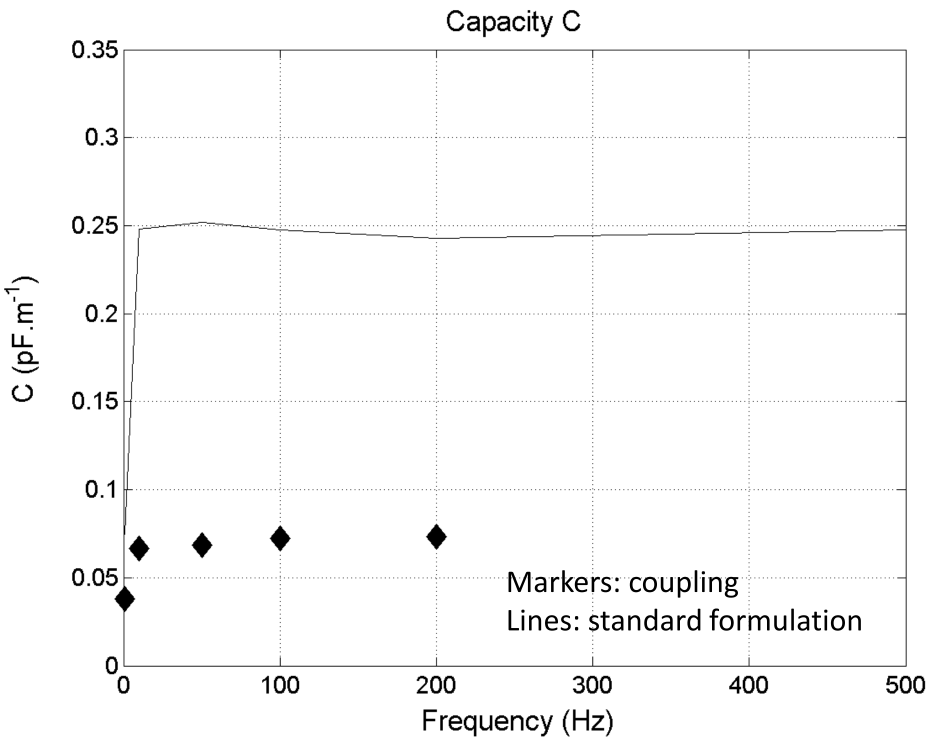
- Correct Magnitude and phase angle
- Wrong DC current (cause: Gauge? Convergence method?)

# 3. Modelling of test cases

## 3.2. Test case 2 – *the wire in air*

- Equivalent R, L and C per unit length





- Correct estimation of Resistance R and Inductance L
- Different Capacities C (cause: $\mathbf{E} = -\nabla V \neq \mathbf{E} = -\nabla V - \partial_t \mathbf{A}$)

# 4. Conclusion and Forthcomings

- **Conclusion:**

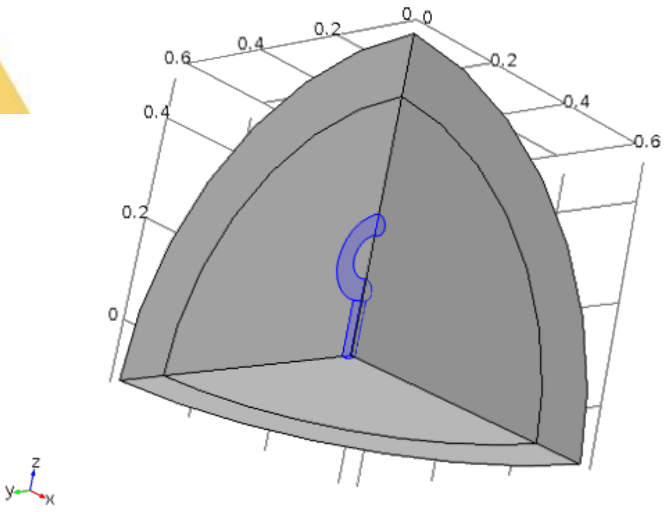
(+) no geometry constraint on the coil
(+) resistive, inductive and capacitive effects
(+) transient working condition
(-) big memory space needed
(-) hard convergence
(-) time consuming method

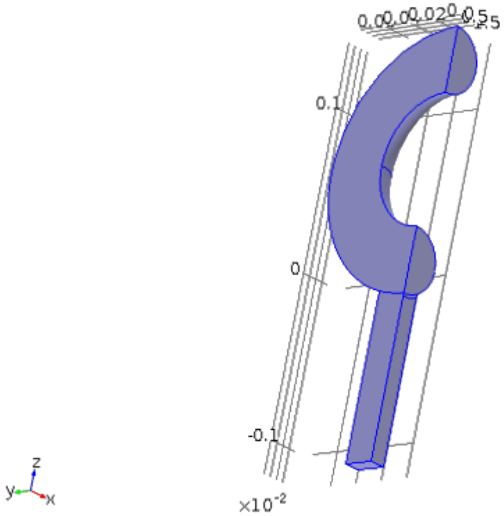
- **Forthcomings:**

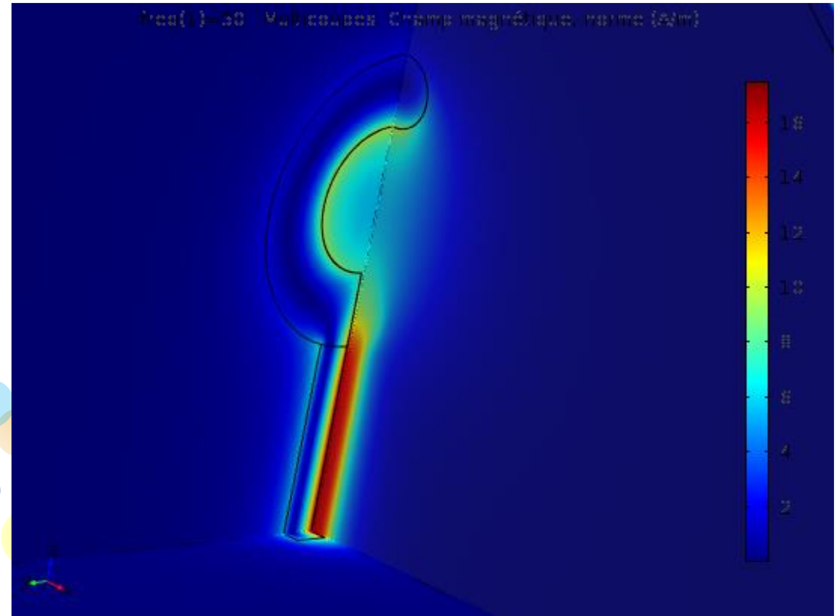
Apply the transient coupling formulation on the coil
Compare segregated and fully-coupled methods
Investigate other potentials formulations (**T**,$\Phi$)

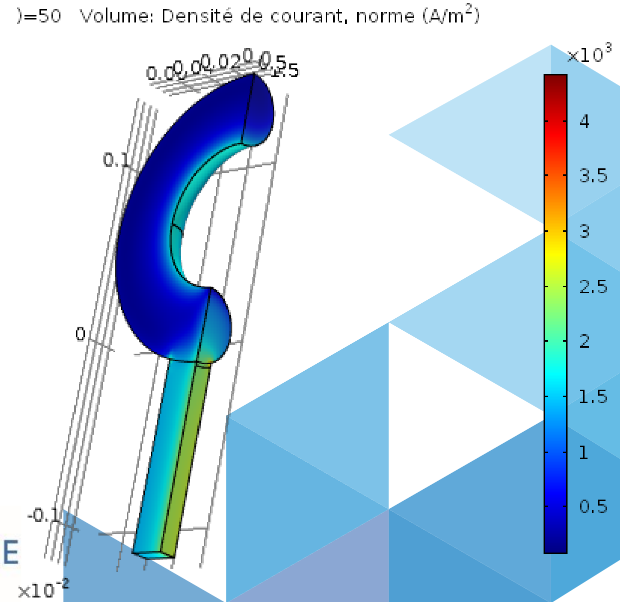
# 3. Modelling of test cases

## 3.3. Test case 3 – *the coil*



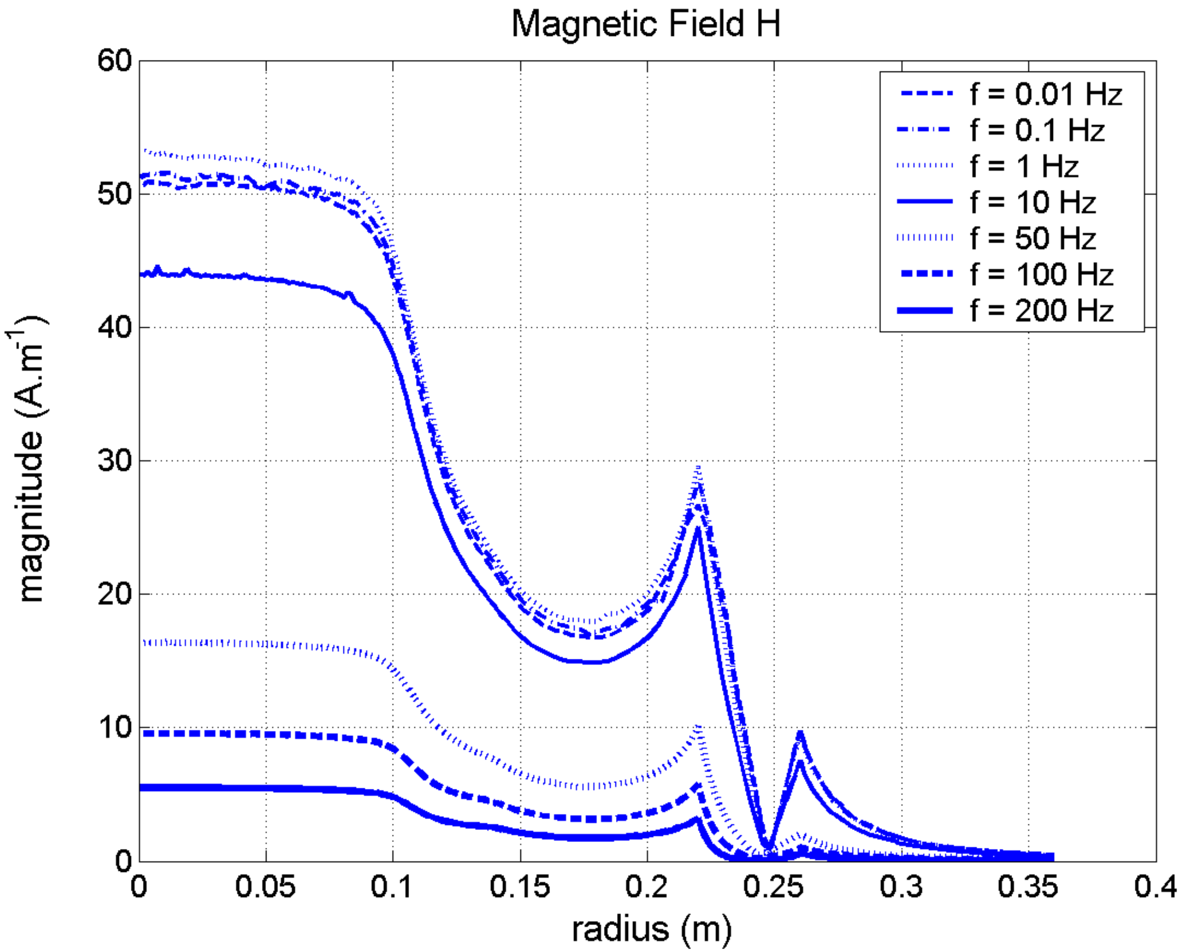






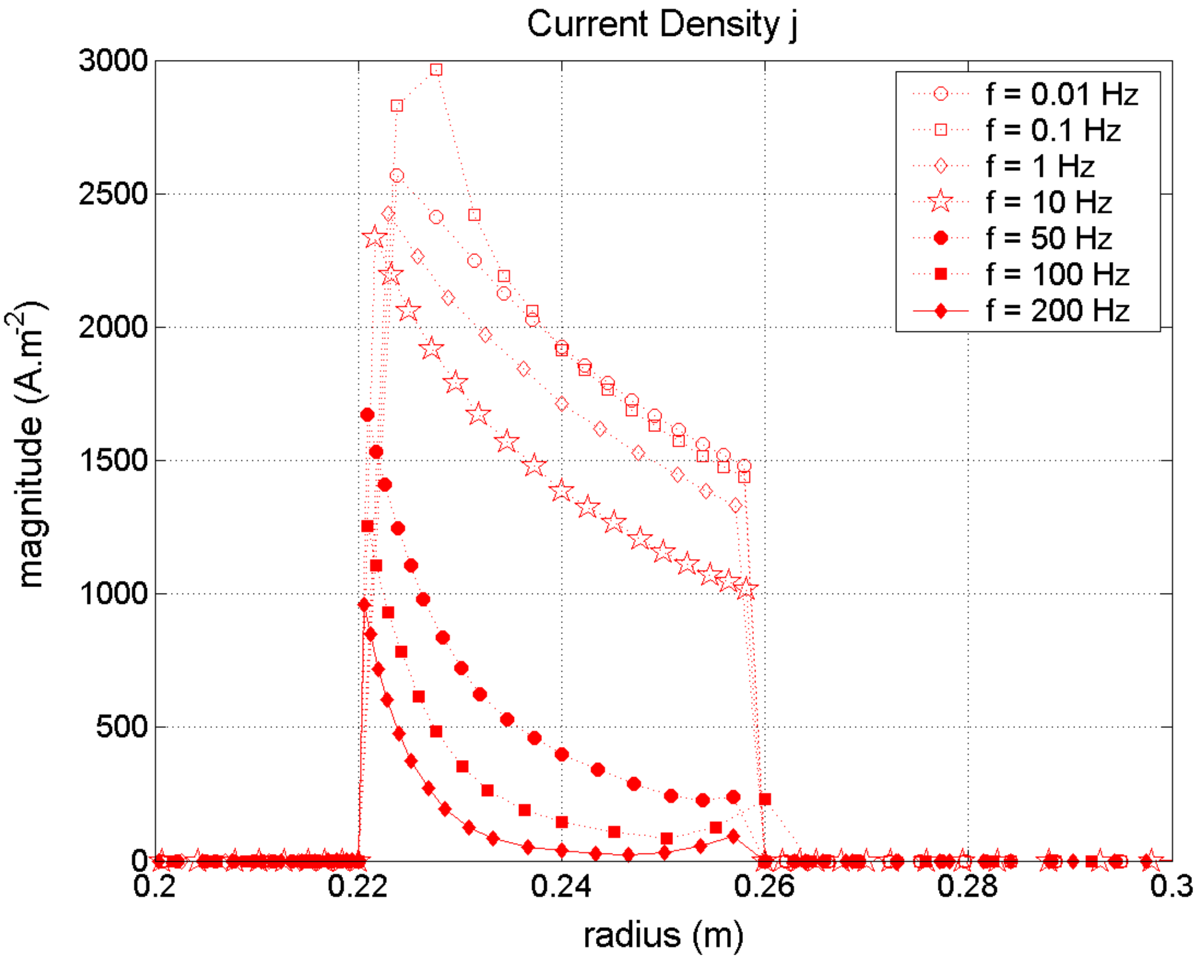
# 3. Modelling of test cases
## 3.3. Test case 3 – *the coil*

- Field distribution



# 3. Modelling of test cases
## 3.3. Test case 3 – *the coil*

- Current distribution



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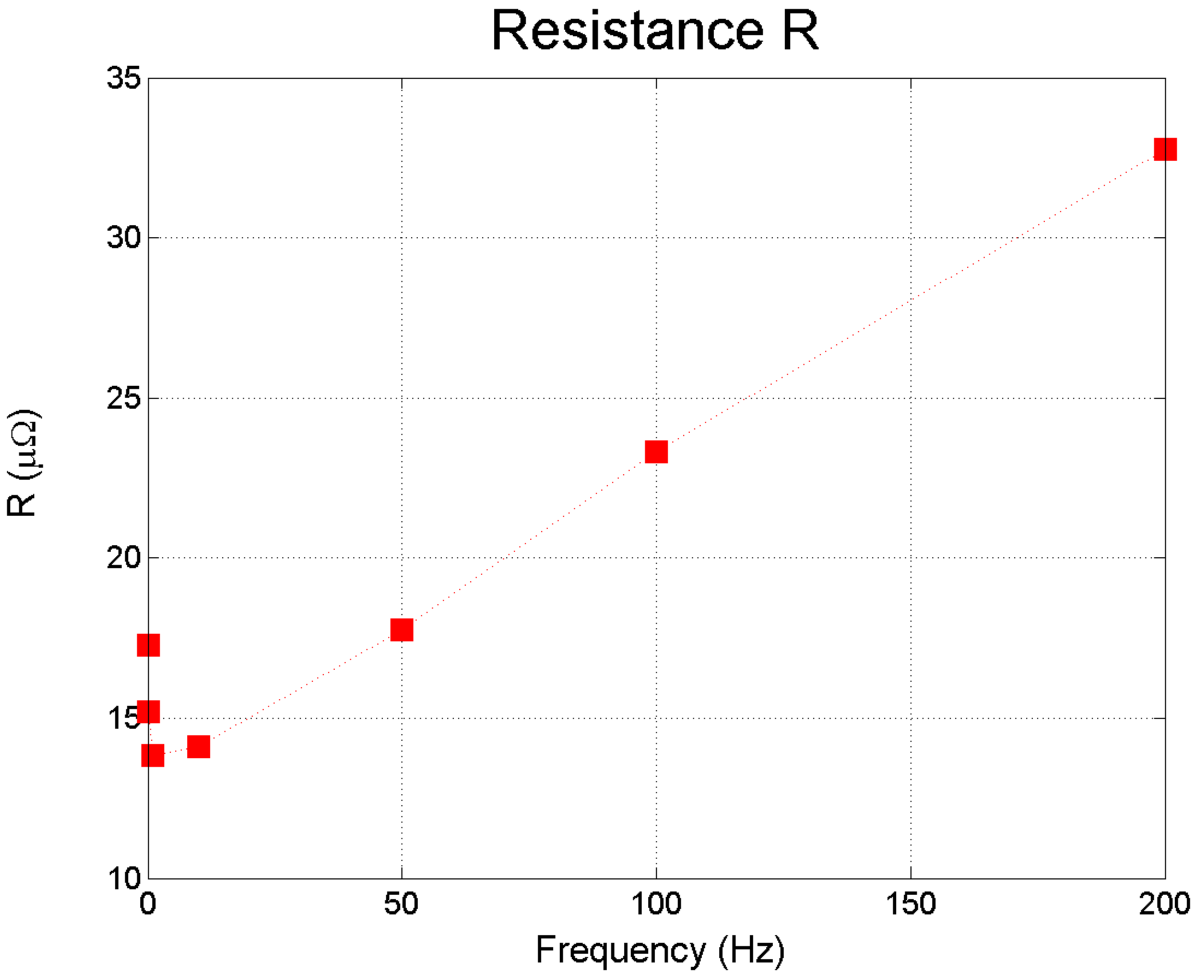
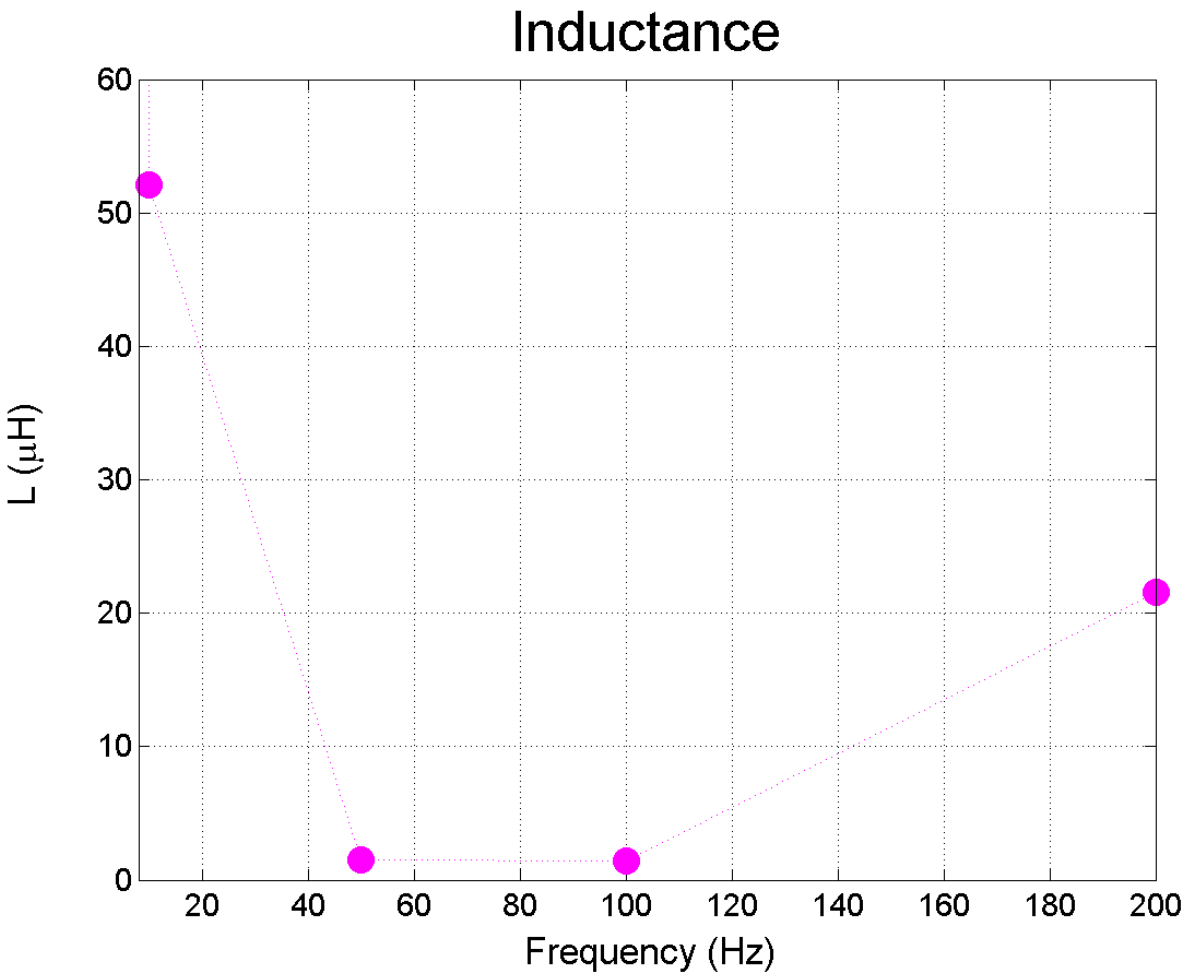
# 3. Modelling of test cases
## 3.3. Test case 3 – *the coil*

- Equivalent R, L and C per unit length



# 3. Modelling of test cases
## 3.3. Test case 3 – *the coil*

- Equivalent R, L and C per unit length



# 3. Modelling of test cases
## 3.3. Test case 3 – *the coil*

- Equivalent R, L and C per unit length