Acoustic Fluid-Structure Interaction Modelling of Gravity Dams in the Frequency Domain

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Abstract The assessment of the seismic safety of gravity dams is a topic of great importance in civil engineering.
In this paper, fluid structure interaction modeling of gravity dams during earthquakes is investigated. In particular, this work aims to provide physical significance of a plan numerical model simulating the dam and the infinite length reservoir when a horizontal ground motion acts at the dam foundation.
After a preliminary calibration of the model with analytical solutions, the dynamic properties of the numerical model are investigated via modal and frequency response analyses.
The fully coupled mechanical-acoustic model is also compared to the widespread "added mass" model [1] adopted in most national codes.

Keywords: Fluid Structure Interaction, Earthquake Engineering, Gravity Dams.

1. Introduction

The first attempt at solving the problem of a fluid domain under such conditions was performed by H. M. Westergaard in 1933 [1]. He adopted, for pressure *p* on the upstream face of the dam, the wave equation as a simplification of the full Navier-Stokes equation applied to the fluid domain:

$$— \quad —— \qquad (1)$$

where *c* is the sound speed in the water and *t* is the time. The following boundary conditions are imposed:

- rigid reservoir bottom: —
- rigid dam upstream face: —
- zero pressure at the free water surface:
- Sommerfeld radiation condition upstream of the reservoir:

$$\sqrt{\ } \left(— \quad ik\right) p \quad 0, \qquad (2)$$

where *k* is the bulk modulus of water.

The reference system (0, *x*, *y*) has the origin at the top of the upstream face of the dam with the *x* and *y* axes pointing upstream and the reservoir bottom, respectively. The Author finds the following solutions for the displacements ξ and η of a fluid particle in the *x* and *y* direction:

$$—— \cos\left(——\right) \sum - \quad \text{in}\left(——\right),$$

$$—— \cos\left(——\right) \sum —— \quad \cos\left(——\right),$$

with:

$$\sqrt{1 \quad ——} \qquad ——,$$

where *w* is the specific weight of water, α is the ratio of the horizontal acceleration to the gravity acceleration g, T is the period of the forcing acceleration and *h* is the depth of the reservoir.
The values of T that set c_n to zero, which lead η to infinity, correspond to the eigenfrequencies of the reservoir only:

$$—— \qquad (5)$$

The Author also proposes an approximation of the infinite Fourier series, by a parabolic expression in *y*. In this case, the pressure can be represented by horizontal inertial forces exerted by masses rigidly attached to the dam. The mass distribution is obtained by equating the expression of inertia forces to the above-defined pressure:

$$-\sqrt{h} \qquad (6)$$

This formulation is adopted in various national codes, under the name of "added mass" method.
The Westergaard's solution is valid only if the period of the exciting acceleration is greater than the first fundamental period of the reservoir.
For a perfectly rigid dam, as in this case, an analytic solution was developed by Chopra [2], [3], for any excitation frequency, and the pressure at the upstream face of the dam is given by equation (6), where ω is the pulsation of the exciting acceleration and $\lambda_n = (2n-1)\pi/h$.

However, there are no analytical solutions developed with deformable dam, so it is necessary to make use of finite element method.

$$p(y,t,\omega) = sin(\omega t) \cdot \left\{ \sum_{n=1}^{n_1-1} \left[\frac{(-1)^{n-1}}{(2n-1)\sqrt{\left(\frac{\omega}{c}\right)^2 - \lambda_n^2}} \cdot cos(\lambda_n y) \right] + \sum_{n=1}^{\infty} \left[\frac{(-1)^{n-1}}{(2n-1)\sqrt{\lambda_n^2 - \left(\frac{\omega}{c}\right)^2}} \cdot cos(\lambda_n y) \right] \right\} \quad (7)$$

2. The models

The models presented in this work simulate the behavior of an Italian dam located in Northern Italy, with height of about 50 m and base of 48 m. The fluid - structure interaction phenomenon is modelled by considering the forces and accelerations exchanged at the interface between the solid and the fluid domains.

The assumptions of small displacements and non-viscous fluid allow a simplification of the full Navier-Stokes equations into the D'Alembert wave equation, which becomes the Helmholtz equation under the hypothesis of sinusoidal excitation.

The 2D models are represented in figure 1 with the appropriate boundary conditions. The first model simulates the impounded reservoir only, the other the fluid with the dam, which is considered as rigid or deformable.

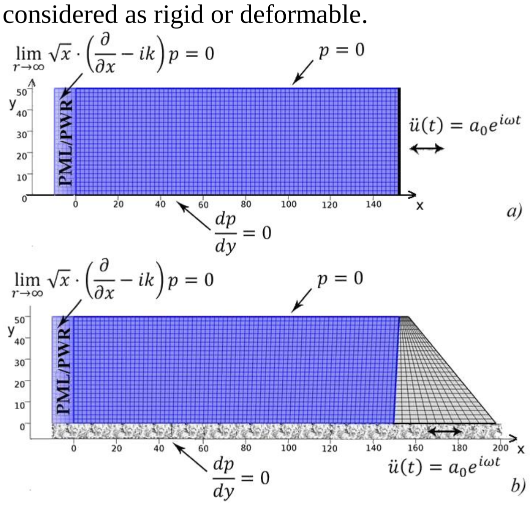

Figure 1. The model of the basin only (*a*) and the model of the interacting system (*b*).

The infinite length reservoir is truncated in both cases and the Sommerfeld radiation condition is applied in order to model a boundary that does not reflect incident plane waves. The method to simulate correctly the infinite length basin is deeply discussed. Successively, once the model has been calibrated through the analytical solution for the case of rigid dam, the deformability of the structure is introduced. In the first phase of the study, in order to discuss the more suitable approach to be adopted, the first model only is considered. In the second phase, in order to model also the reservoir sediment absorption effect, the rigid wall condition at the bottom is changed to an impedance condition and the domain of the dam structure is introduced, with the aim to evaluate the fluid-structure interaction effect to be compared to the traditional model with added masses.

3. Use of COMSOL Multiphysics®

Two COMSOL® modules are used in the simulation: the Solid Mechanics module and the Pressure Acoustics-Frequency Domain module, which solves the Helmholtz equation for the complex acoustic pressure *p* [4].

The materials applied are the standard Water material in the COMSOL® library and a concrete material with a density $\rho = 2400\ kg/m^3$, a Young's modulus $E = 35\ N/mm^2$ and a Poisson coefficient $\nu = 0.33$.

The mapped mesh (figure 1) has 1975 fluid quadrilateral elements and 300 solid elements in a plane strain formulation, with quadratic shape functions. The maximum element size has been chosen in order to satisfy the inequality: $h_{max} < \lambda/5$, where λ is the minimum wavelength of sound in water, corresponding to about 150 Hz.

As for the model with added masses, the above-defined mass is modelled as "*linear added mass*" in COMSOL®, by defining an analytic function for it. A mesh refinement is required in this case, in order to improve the discretization of the masses.

A "*sound-soft boundary condition*" and a "*sound-hard boundary condition*" are applied to the water free surface and to the reservoir bottom, respectively. A prescribed acceleration boundary condition of $0.1\ m/s^2$ is imposed at the dam upstream face in model (a), whereas an acoustic-solid boundary interface is imposed to full model (b), in which the prescribed horizontal acceleration is applied at the base.

Some considerations have to be made regarding the infinite length of the reservoir, simulated by

Sommerfeld radiation condition. The domain is extending off to infinity and the wave equation has no attenuation over distance, so in order to approximate an infinite space, two methods are available in COMSOL®: the Perfectly Matched Layer (PML) and the Plane Wave Radiation condition (PWR) are compared in this work.

The PWR is a Robin boundary condition, a linear combination of p and its derivative, and, in this case, since no dipole or monopole sources are present, it is equal to

$$-\frac{1}{\rho}(\nabla p) + \frac{ik}{\rho}p = 0, \qquad (8)$$

where ρ is the density of water. Its effectiveness is optimal only for incident waves normal to the boundary.

The PML is a complex coordinate stretching of the domain that introduces a decay of the oscillation without any reflection in the source domain, simulating a perfectly absorbing material. As suggested by [4], the PML mesh has to be assigned so that the mesh sides are directed along the radiation.

The PML settings were a "*rational stretching*" coordinate transformation with "*wavelength*" left at the default setting "*by physical interface*", and unitary scale factor and curvature parameter.

In order to use the PML technique, an additional domain is created on the upstream side of the reservoir. A stationary direct solver is used, and modal and frequency sweep between *0.1 Hz* and *100 Hz*, with an increment step of *0.1 Hz* are run.

A modal analysis with both PWR and PML approaches is performed using model (a). The first resulting eigenfrequencies are presented in table 1 and compared to the corresponding values from analytical solution.

Eigenmode	*Analytic* ($\frac{nc}{4h}$) *[Hz]*	*PWR [Hz]*	*PML [Hz]*
1	7.407	7.817	7.407
2	22.221	22.386	22.222
3	37.035	37.164	37.036
4	51.849	51.976	51.851
5	66.663	66.720	66.667

Table 1. First eigenfrequency values from analytical solution and from the model (a) with PWR and PML approaches.

One can observe that the model with PWR is affected by an appreciable error in the estimation of the natural frequencies of the system, while the PML provides an almost exact solution.

In figure 2 the first two eigenmodes of both the PML and PWR approaches are presented. It can be seen that with the PWR the pressure distribution is not constant in the upstream direction and the symmetry condition that follows from the hypothesis of infinite reservoir is not respected.

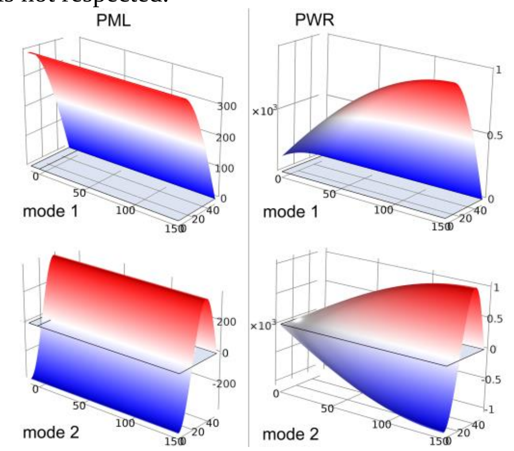

Figure 2. The first two eigenmodes of both the PML and PWR model.

The surfaces' height represents the pressure: it can be also shown that the pressure distribution on the upstream face of the dam is similar to the mode shape corresponding to the eigenfrequency which is closest to the excitation frequency (figure 3).

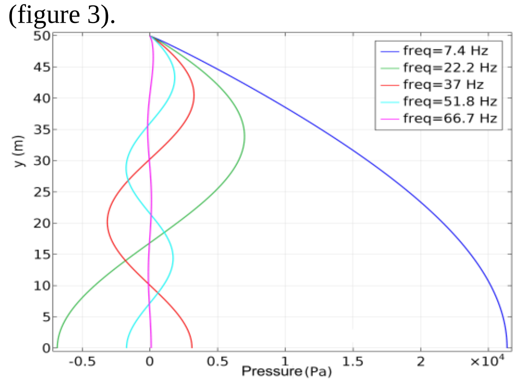

Figure 3. Pressure distributions on the dam upstream face for different values of the exciting frequency.

In figure 4 the response function of the model with basin only in terms of base pressure is presented in both cases PWR and PML. It can be observed that the PML option reproduces exactly the analytical solution [2], whereas the PWR option introduces noise and spurious peaks. The asymptotes, where the magnitude goes to infinite, occur at frequencies corresponding to the resonance of the reservoir (eq. (5)) and are

well matched by the COMSOL® solution only in the PML approach.
It can be observed that the PWR option produces "noise" in the frequency response and gives rise to secondary spurious peaks when some of the longitudinal modes of the truncated domain are excited. The application of PML provides cleaner results, which are almost exactly coinciding with the analytical solution.

When the absorption due to the sediments at the bottom of the reservoir is introduced, the "*sound hard*" boundary condition in COMSOL® is substituted by an "*impedance*" condition, with a value of impedance Z:

$$Z = \rho c \cdot \frac{1+\alpha_r}{1-\alpha_r}, \tag{9}$$

where α_r is the reflection coefficient, varying from 0 (perfect absorption) to 100% (perfect reflection).

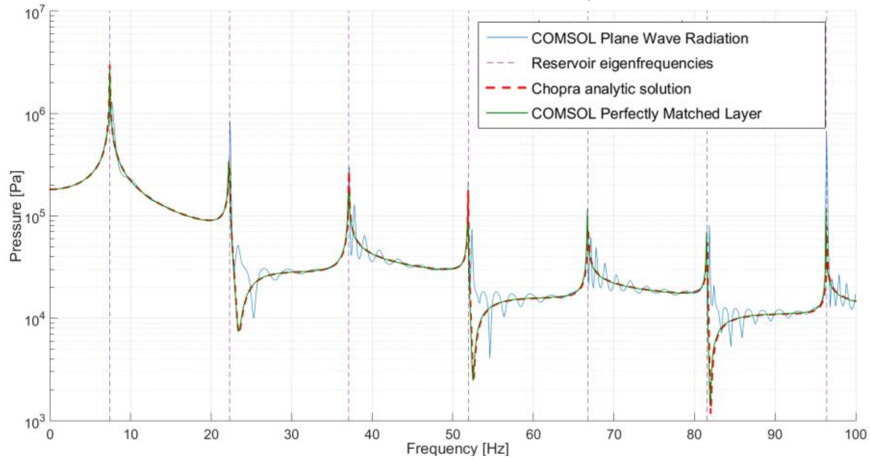

Figure 4. Pressure at the base of the dam upstream face. Comparison of the analytic solution with Plane Wave Radiation and Perfectly Matched Layer models; the purple dashed lines mark the eigenfrequencies of the reservoir.

Figure 5 shows that the absorption drastically reduces the amplification at the resonant frequencies.

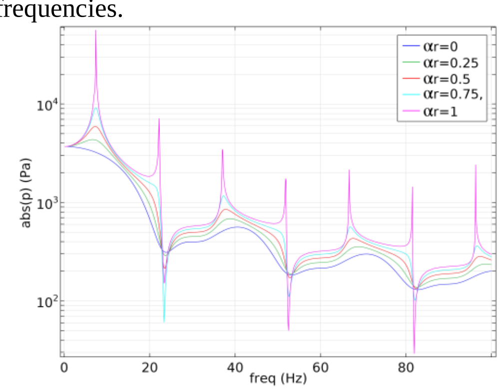

Figure 5 Hydrodynamic pressure at the base of the dam in model (a) for different values of the reflection coefficient.

In figure 6 the pressure distribution along the dam face is plotted for the different values of α_r. It can be seen that, for values of $\alpha_r > 0$, the maximum pressure decreases and no longer occurs at the bottom of the reservoir. Therefore, it appears more convenient from now on to represent the resultant force at the dam base instead of the pressure point value.

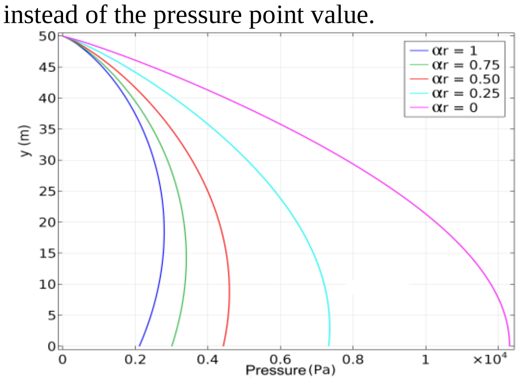

Figure 6. Pressure distribution in model (a) for an exciting frequency of 7 Hz at various values of the reflection coefficient.

4. The fluid-structure interaction

After the validation and comprehension of the model of the reservoir only, an analysis with deformable dam is performed.
A modal analysis is conducted on the dam body only and a frequency response curve with the same prescribed acceleration of *0.1 m/s²* is

obtained. The first eigenmodes of the dam body are presented in table 2, along with those resulting from the model with added masses. In figure 7 the first two modes are represented. It is noticeable the mode shapes similarity. The response function of the base horizontal reaction is represented in figure 8, in comparison with the case of the model with added masses. It can be seen that the latter model produces a shift to lower frequencies of the whole response spectrum, without modifying its shape by much.

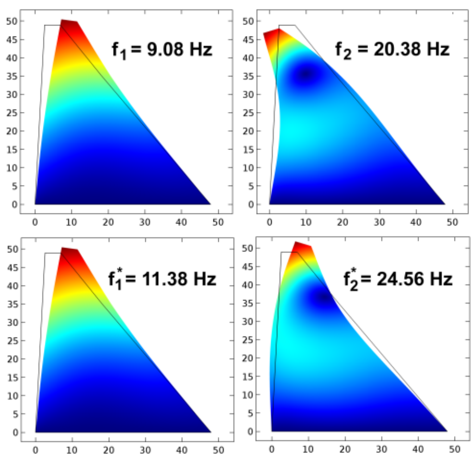

Figure 7. The two first mode shapes for the model with dam only (f) and with added masses (f*).

Eigenmode	*Dam body [Hz]*	*Dam with added masses [Hz]*
1	11.385	9.083
2	24.562	20.379
3	27.951	27.020
4	43.560	35.077

Table 2. First four eigenfrequency values for the dam only and for the model with added masses.

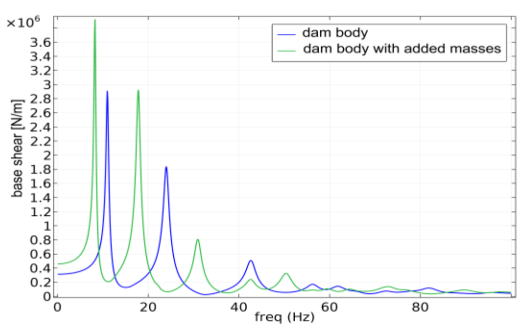

Figure 8. Base shear frequency response for the dam only (blue) and the model with added masses (green).

In figure 9 the system frequency response in terms of total horizontal base reaction (base shear) is represented in the following cases: fluid domain only (dashed blue curve), rigid dam (solid light blue curve), full fluid structure interaction (solid blue). It can be observed that the reservoir only and rigid dam cases differ by a constant value given by dam inertial forces, (in a liner-scale plot the two curves would be parallel); in both curves the resonance peaks of the reservoir only (eq. (5)) are evident. The interaction curve shows also a notable amount of peaks, in addition to the other cases.

The same curves reported in figure 9 are plotted in figure 10 for different structural damping ξ and reflection coefficient α_r values (black and red curves). It can be seen that the presence of structural damping determines a noticeable height reduction of the peaks that are generated by the resonance of the deformable dam: red curves correspond to a greater structural damping (ξ=25%) than the black ones (ξ=5%).

On the other hand, the introduction of the bottom sediment absorption effect produces the attenuation of the peaks corresponding to the basin resonance frequencies, while it does not alter the ones of the dam body only.

This phenomenon allows a sort of identification of the peaks, relating each one mainly to the reservoir or the dam body. However, a general rule cannot be easily defined. In fact, the first resonant peak in the surroundings of 10 Hz shows a strong coupling between the two physics when the structural damping or the reflection coefficient is varied, while the peaks at higher frequencies display an almost uncoupled behavior.

In figure 11 the full interaction model is compared to the model with added masses: it can be observed that, for low frequency values, the base shear value almost coincides in both cases.

However the first resonant peak frequency is quite similar and the behavior of the system differs greatly for higher frequencies. The height of the peaks is comparable, but the shift of the resonant frequency due to the interaction is not accurately represented by the model with added masses. In fact, while the shift of the first mode could be considered approximately identified, the same cannot be said for the other peaks. It can be stated that the model with added masses cannot reproduce the interaction phenomenon because it does not take into account the full behavior of the fluid domain.

However, such model is able to roughly identify the first eigenfrequency of the system and also its dynamic behavior al low frequencies, up to 10 Hz, in the typical range covered by earthquake accelerograms.

A final step to investigate the problem and increase the comprehension of the phenomenon is taken by analyzing the same model with a material sweep over the dam concrete, varying the Young modulus E, from 10 to 35 MPa. In figure 12 the frequency response of the fluid structure interaction for various values of the dam's Young modulus is represented. The analysis in this case is performed for a reservoir level of 40 m, with a theoretical reservoir eigenfrequency of $9.256\,Hz$. The response curves of the dam body only are represented with dotted lines. It can be seen that a linear increase in the elastic modulus value produces a corresponding shift of the first frequency of the system, which asymptotically approaches to the reservoir resonance frequency in a nonlinear manner. This behavior is not shown by the model with added masses, where the frequency shift of the peak remains constant.

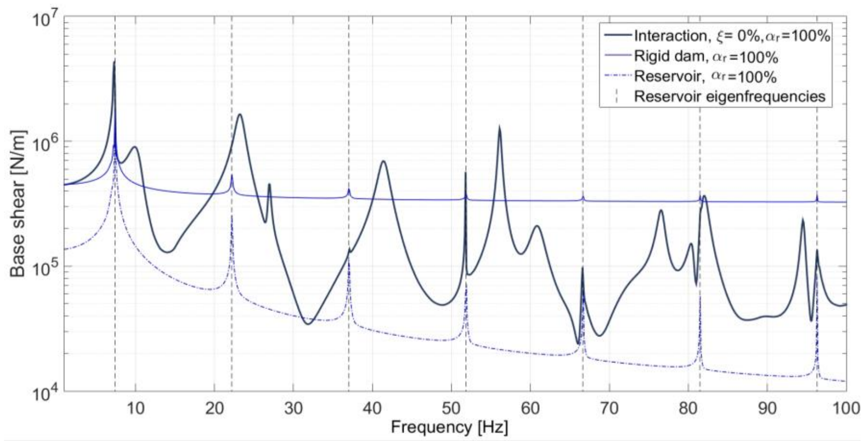

Figure 9. Frequency response function of the base shear in the cases of reservoir only, rigid dam, and elastically deformable dam.

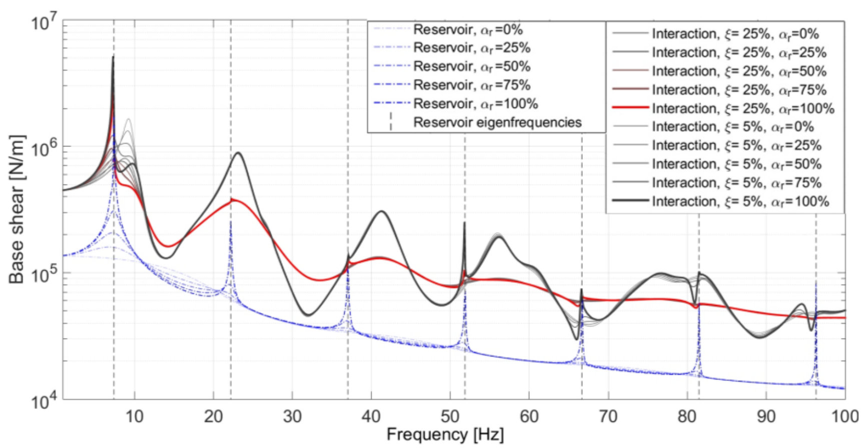

Figure 10. Response function for different values of the structural damping and bottom sediment reflection coefficient.

This behavior of the system could not be captured with the traditional approach with added masses but only with a full multiphysics coupling.

5. Conclusions

COMSOL® Multiphysics® has proved to be a suitable software to perform frequency response analyses of basin-dam interaction.

In this work the basin has been modelled with the acoustic approach and the problem of modelling an infinite reservoir has been examined and efficiently resolved by the application of the Perfectly Matched Layer method, simulating a perfectly absorbing domain. The COMSOL® model of the fluid domain only perfectly reproduces the analytic solution given by Chopra [2].

The subsequent modelling of the fluid-structure interaction has been then performed providing a deep knowledge of the dynamic behaviour of the interaction system in sight of the seismic safety assessment of gravity dams. Furthermore, this study demonstrates that the widespread model with added masses is not able to accurately represent the dynamic behaviour of the system, even though it is suitable to roughly reproduce its behaviour in the range of usual earthquake frequencies.

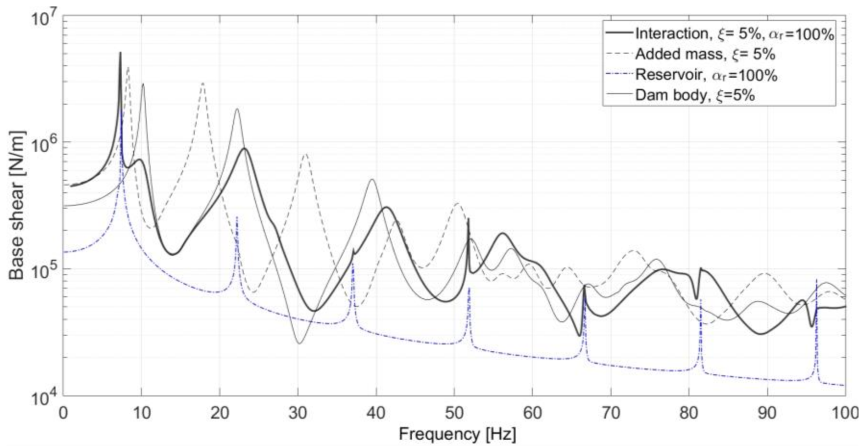

Figure 11. Comparison of the response function between the interaction, the model with added masses and the dam only model.

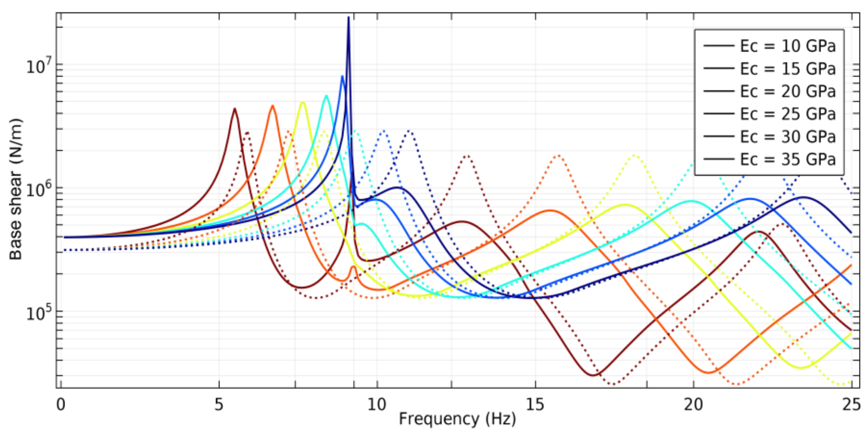

Figure 12. Frequency response of the fluid structure interaction for various values of the dam's Young modulus *E* (solid lines), compared to the dam-only responses (dotted lines).

6. Bibliography

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