COMSOL CONFERENCE
2016 BOSTON

PENNSTATE 1855

Fully Variational Implementation of Immersed Finite Element Method for Fluid-Structure Interaction applications

Nitesh Nama, Tony Jun Huang, and Francesco Costanzo

Department of Engineering Science and Mechanics

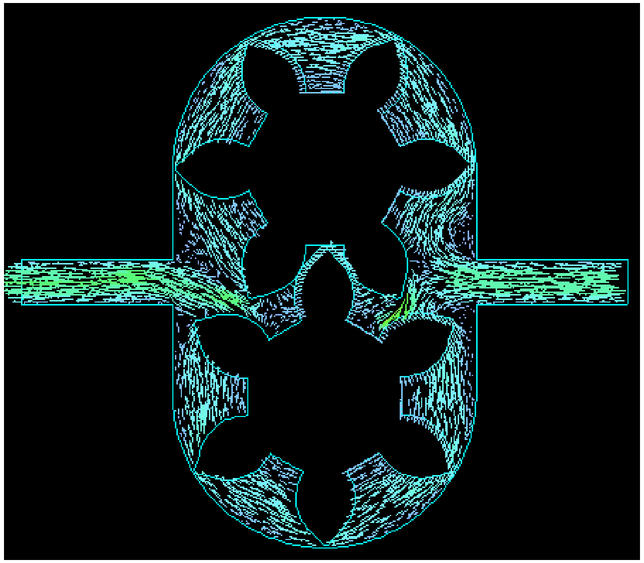
The Pennsylvania State University, State College, PA, USA

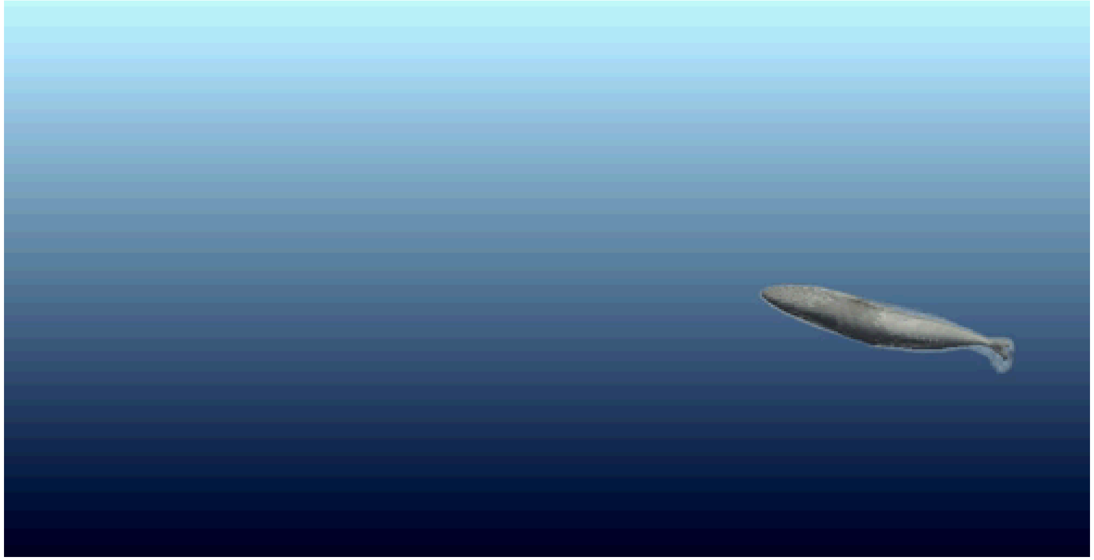
Outline

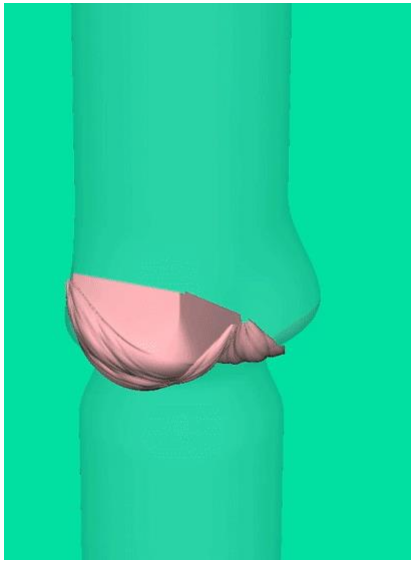
- Introduction
- Different approaches for Fluid-Structure Interaction (FSI)
- Immersed Finite Element Method
- Numerical Implementation and Workflow
- Test Cases
- Summary and Future Work

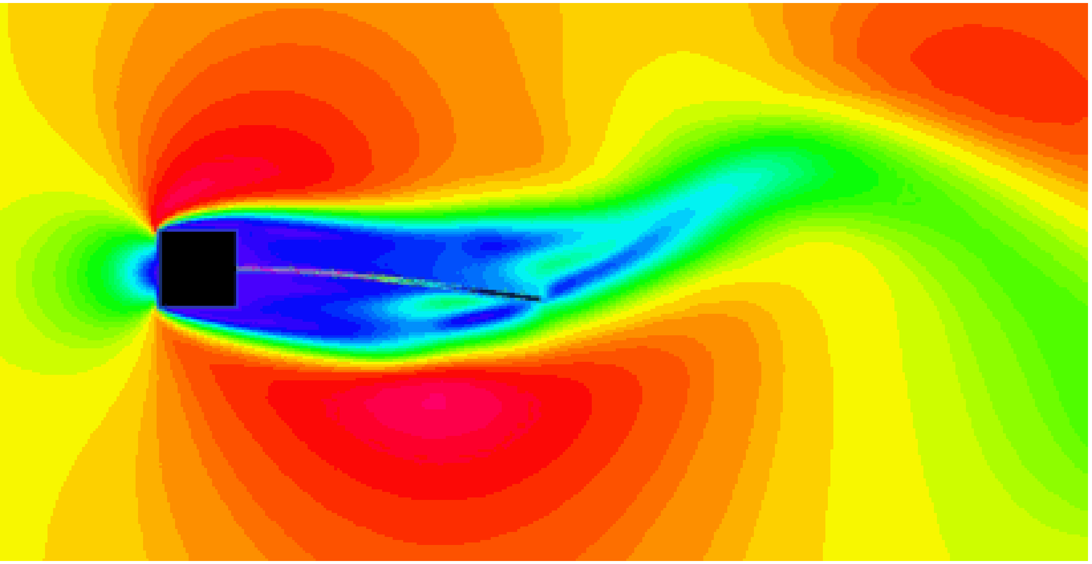
Fluid-Structure Interaction (FSI): Overview

- Interaction of a solid body with fluid and vice versa.





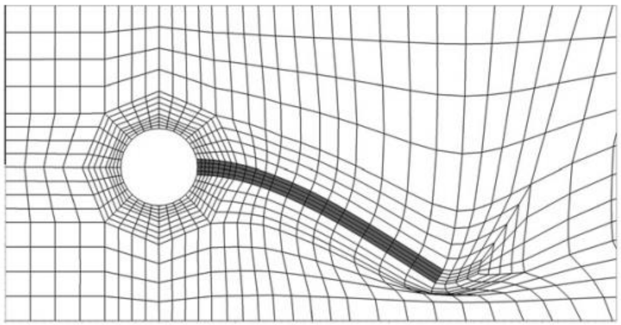




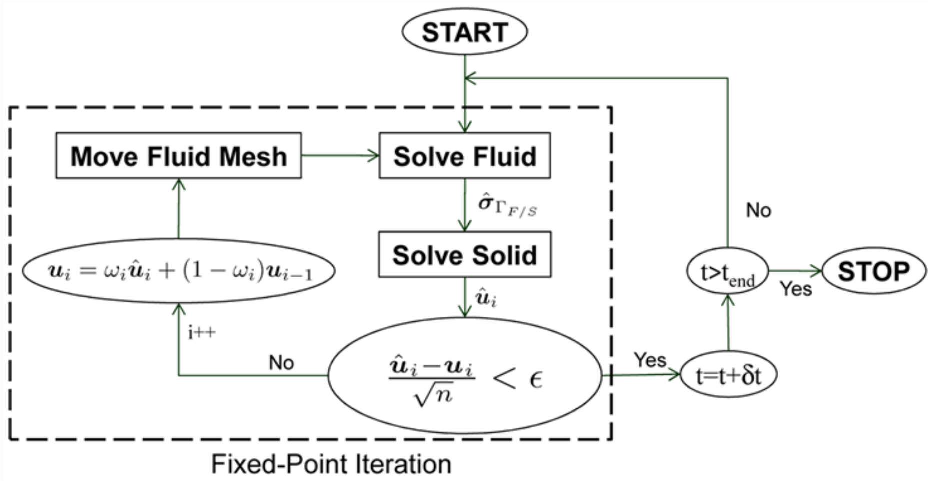
FSI Simulations: Overview

Primary Challenges:

- Tracking large solid deformation
  - Mesh becomes distorted very quickly.
- Strong nonlinearity.
- Very few analytical solutions.

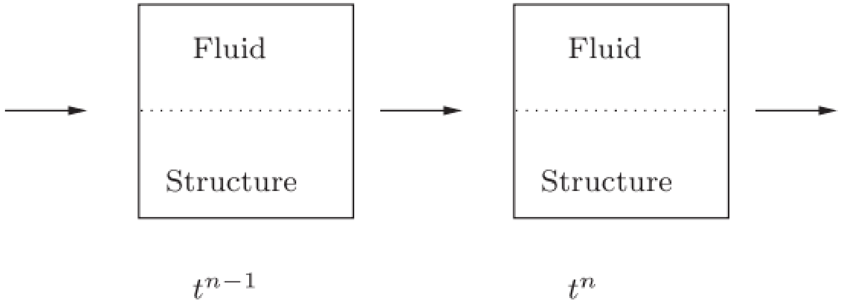


Partitioned Approach



- Specialized algorithms can be used.
- Stable and accurate coupling algorithm required.

Monolithic Approach

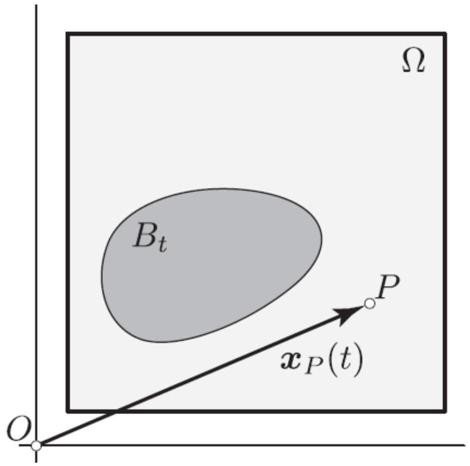


- Interface conditions are implicit in the solution procedure.
- Specialized codes (and significantly more computationally extensive)

Governing equations

Balance of mass $\dot{\rho} + \rho \operatorname{div} u = 0, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$

Balance of momentum $\operatorname{div} \mathsf{T} + \rho b = \rho \dot{u}, \quad x \in \Omega \setminus (\partial\Omega \cup \partial B_t),$



- Same governing laws, Different properties and constitutive equations
- Velocity and pressure can be defined everywhere!

What about solid displacement?

Constraint and Interface Conditions

Immersed body velocity $\dot{w}(s,t) = u(x,t)\big|_{x=\zeta(s,t)}$

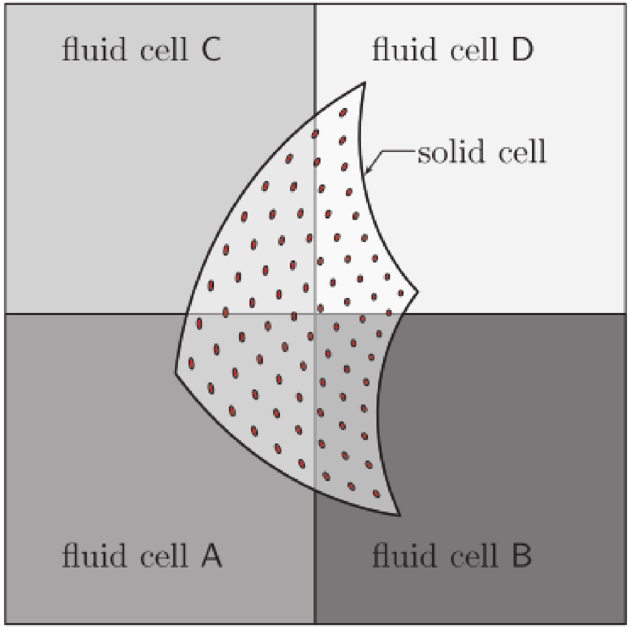
Boundary condition $u(\check{x}^+,t) = u(\check{x}^-,t) \quad \text{and} \quad \mathsf{T}(\check{x}^+,t)n = \mathsf{T}(\check{x}^-,t)n, \quad \check{x} \in \partial B_t,$

Overall system to implement

$$\int_\Omega \rho_f(\dot{\boldsymbol{u}} - \boldsymbol{b}) \cdot \boldsymbol{v}\,\mathrm{d}v + \int_{B_t} (\rho_s - \rho_f)(\dot{\boldsymbol{u}} - \boldsymbol{b}) \cdot \boldsymbol{v}\,\mathrm{d}v + \int_\Omega \hat{\mathsf{T}}_f \cdot \nabla_{\boldsymbol{x}} \boldsymbol{v}\,\mathrm{d}v + \int_{B_t} (\hat{\mathsf{T}}_s - \hat{\mathsf{T}}_f) \cdot \nabla_{\boldsymbol{x}} \boldsymbol{v}\,\mathrm{d}v - \int_{\partial\Omega_N} \boldsymbol{\tau}_g \cdot \boldsymbol{v}\,\mathrm{d}a = 0 \quad \forall \boldsymbol{v} \in \mathscr{V}_0$$

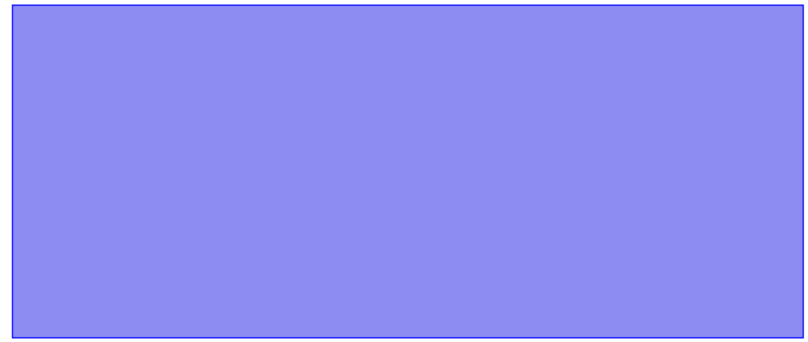
$$\int_\Omega q\,\mathrm{div}\,\boldsymbol{u}\,\mathrm{d}v = 0 \quad \forall q \in \mathscr{Q}.$$

$$\int_B \Big[\dot{\boldsymbol{w}}(s,t) - \boldsymbol{u}(\boldsymbol{x},t)\big|_{\boldsymbol{x}=\zeta(s,t)}\Big] \cdot \boldsymbol{y}(s)\,\mathrm{d}V = 0 \quad \forall \boldsymbol{y} \in \mathscr{H}_Y,$$

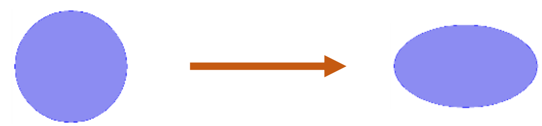


COMSOL Implementation

- 3D_LidDrivenCavity.mph *(root)*
  - Global Definitions
    - Parameters
    - Materials
  - Fluid Component *(comp1)*
    - Definitions
    - Geometry 1
    - Materials
    - Balance of Momentum on Fluid *(u)*
    - Balance of Mass on Fluid *(w2)*
    - Fluid Mesh
  - Solid Component *(comp2)*
    - Definitions
    - Geometry 2
    - Materials
    - Solid Contibution Terms *(w)*
    - Solid Mesh
  - Study 1
  - Results



- Definitions
  - Actuation
  - Real Imaginary Definitions
  - Formulation
  - Domain Average *(avgV)*
  - General Extrusion 1 *(genext1)*



Destination Map

x-expression: x+comp2.u0x

y-expression: y+comp2.u0y

- ➢ The position of solid is tracked via a mapping, which is used to query whether a particular point lies on the solid or not.

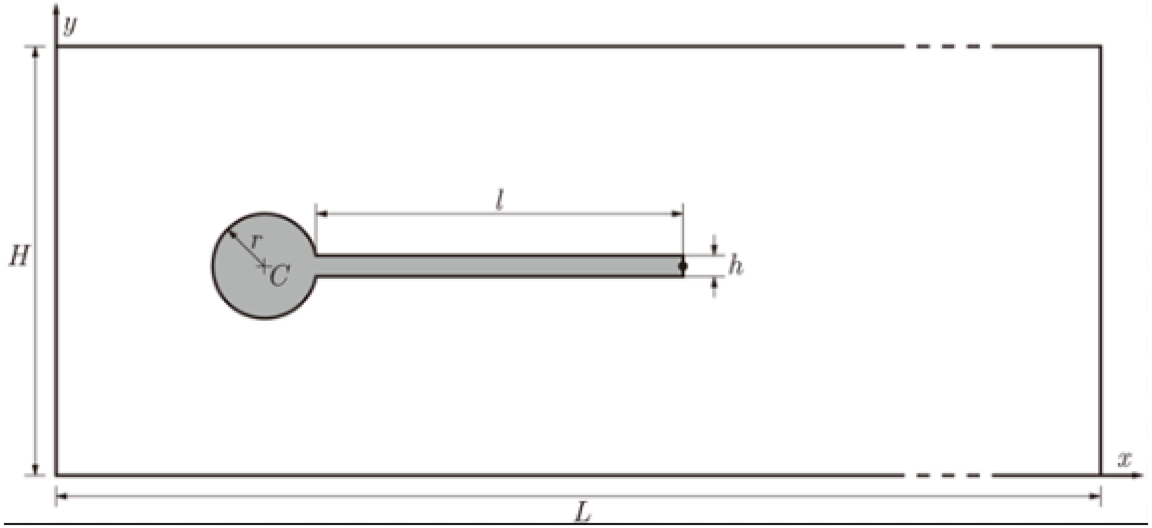
Mathematica based workflow

```
uEqIWCOmega = FullSimplify[IP[Tu, rF (ut + Du.u) - fOmega] + IP[TDu, SigmaFv] - IP[Tr[TDu], p] /.
    FormulationRules];

pEqIWCOmega = FullSimplify[-IP[Tp, Tr[Du]] /. FormulationRules];

pEqIWCB0 = FullSimplify[IP[Tpm, Tr[Dum] ] + IP[Tpm, J c1 pm] /. FormulationRules];

uEqIWCB0 = FullSimplify[IP[TDum, 1/J (ElasticPK1S.Transpose[F])] /. FormulationRules];

wEqIWCB0 = FullSimplify[IP[Tw, wt - um] /. FormulationRules];
```

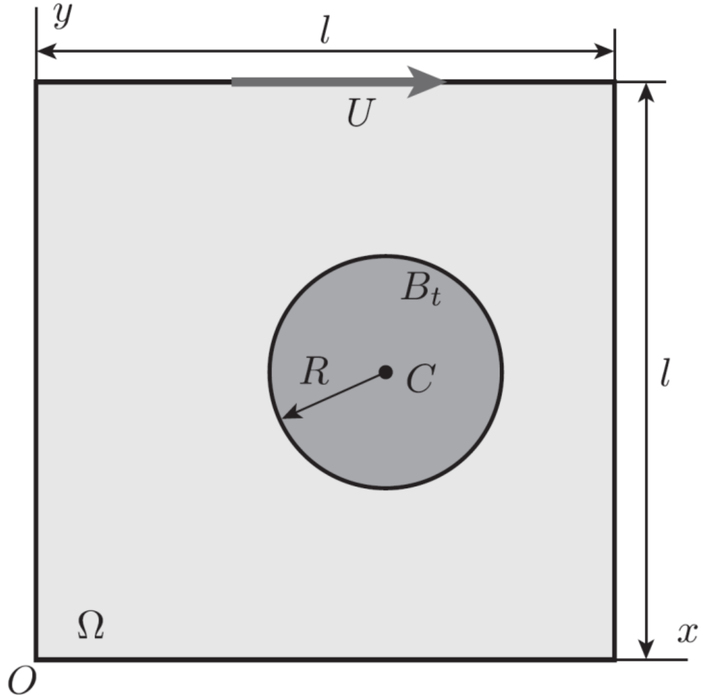
```
uEqIWCOmega 2*uxx*muF*test(uxx)+(uxy+uyx)*muF*test(uxy)+(uxy+uyx)*muF*test(uyx)+2*uyy*muF*test(uyy)-p*
(test(uxx)+test(uyy))+test(ux)*(-fOmegax+rF*(uxx*ux+uxt+uxy*uy))+test(uy)*(-fOmegay+rF*(uyx*ux+uyy*uy
+uyt))
pEqIWCOmega -((uxx+uyy)*test(p))
pEqIWCB0 (umxx+umyy+c1*(1+wxx-wxy*wyx+wyy+wxx*wyy)*pm)*test(p)m
uEqIWCB0 (muS*(wxx^2*test(umxx)+wxy^2*test(umxx)+wyx*(test(umxy)+test(umyx))+wxy*(1+wyy)*(test(umxy)
+test(umyx))+wxx*(2*test(umxx)+wyx*(test(umxy)+test(umyx)))+(wyx^2+wyy*(2+wyy))*test(umyy)))/(1+wxx-
wxy*wyx+wyy+wxx*wyy)
wEqIWCB0 test(wx)*(-umx+wxt)+test(wy)*(-umy+wyt)
```

- Mathematica handles the algebra (tensor multiplication, dot product with test function etc.)
- Going from PDEs to results in a matter of few hours for typical problems!

Test Case 1: Turek-Hron Benchmark



Test Case 2: Lid-Driven Cavity



Initial Configuration of the immersed disk.

First Piola-Kirchoff Stress $\mathbf{P_s} = -\mathbf{p_s}\mathbf{I} + \mu^{\mathbf{e}}\mathbf{F}$

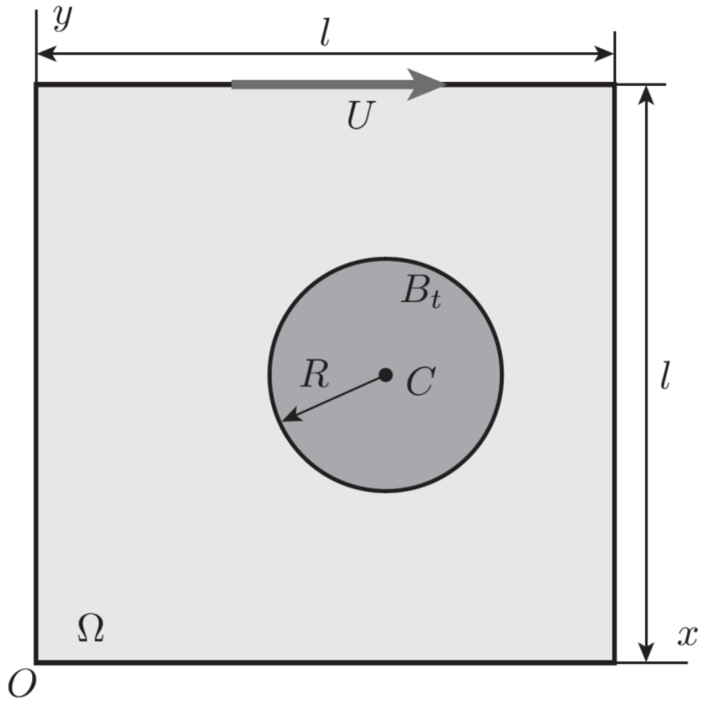
$$R = 0.25\, m$$

$$l = 1\, m$$

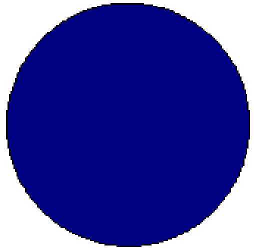
$$\mu^e = 0.1\, Pa$$

$$\rho_f = \rho_s = 1.0\, kg/m^3$$

Test Case 2: Lid-Driven Cavity



First Piola-Kirchoff Stress $\mathbf{P_s} = -\mathbf{p_s}\mathbf{I} + \mu^{\mathbf{e}}\mathbf{F}$



Initial Configuration of the immersed disk.

$R = 0.25\,m$

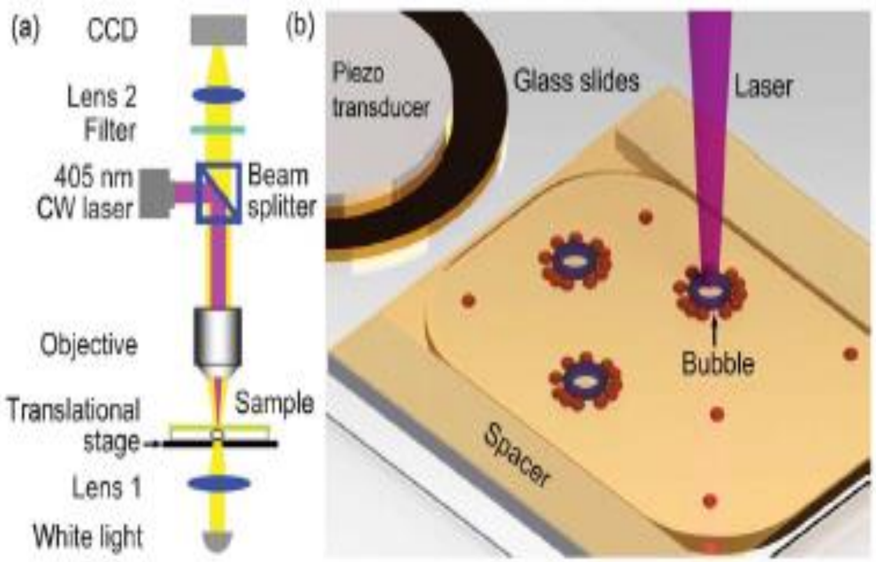
$l = 1\,m$

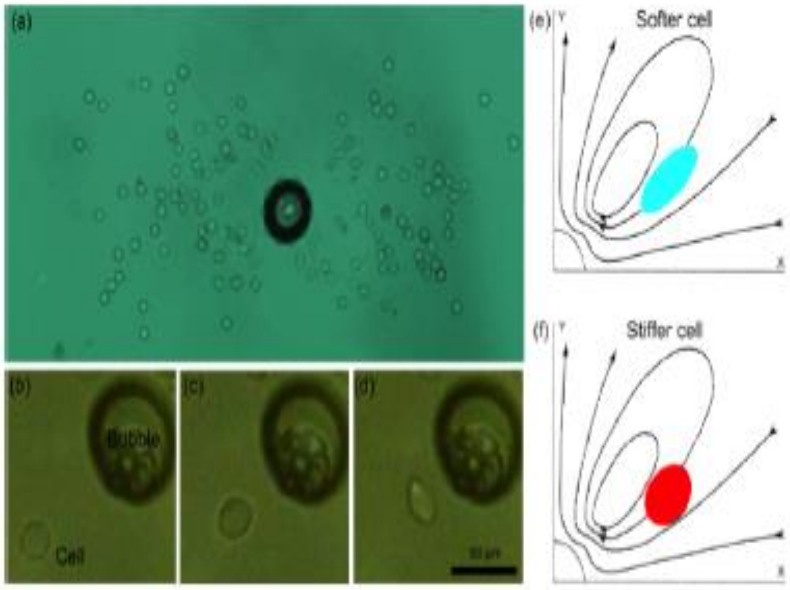
$\mu^e = 0.1\,Pa$

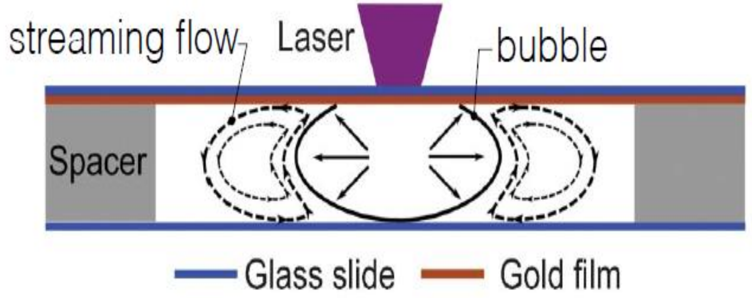
$\rho_f = \rho_s = 1.0\,kg/m^3$

Future Work: Opto-Acoustic Tweezers

Applications to cancer diagnostics







- Cell mechanical properties are indicators of state of disease.
- Precise measurements would enable early diagnosis and therapeutic intervention.

Need for predictive capability!

Xie et al, Lab on a Chip, 2013.

Summary

- Fully variational formulation of immersed finite element method using Comsol as a finite element library.

- Use of Mathematica and Comsol for significant time saving in implementation.

- Advantages of IFEM
  - Single velocity field ensures continuity of velocity across the interface automatically.
  - Interface conditions are satisfied implicitly.
  - No re-meshing required at all.

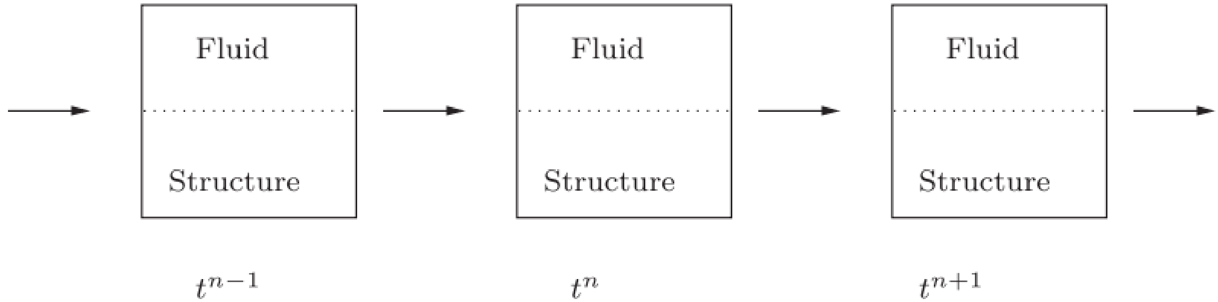
- Two test cases are reported
  - Lid Driven Cavity Problem
  - Turek Hron Benchmark

- Applications to the study of opto-acoustic tweezers are underway.
  - Comparisons with experimental results is expected to reveal the mechanical properties of the cancer cell, which indicates the stage of the cancer.

Questions?

Solution Approaches

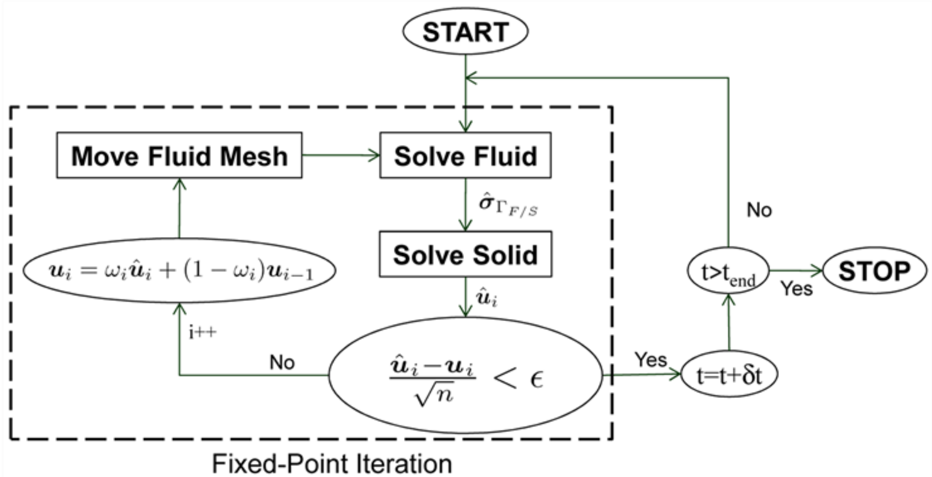
- Monolithic Approach: Fluid and solid equations are solved simultaneously with a single solver.



- Partitioned Approach: Fluid and solid equations are solved separately using two distinct solvers.

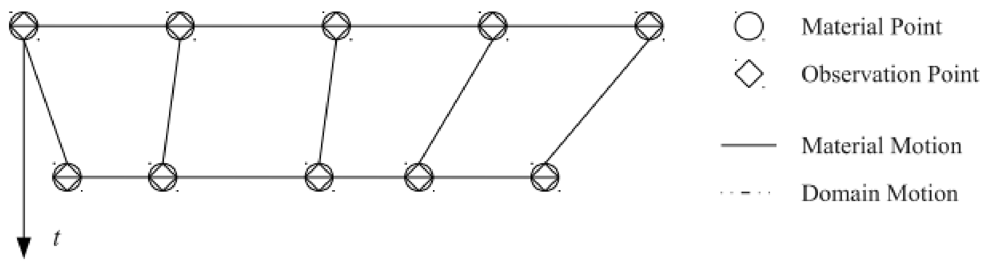
Pros and Cons:

- Standard solvers can be used for fluid and solid equations separately.

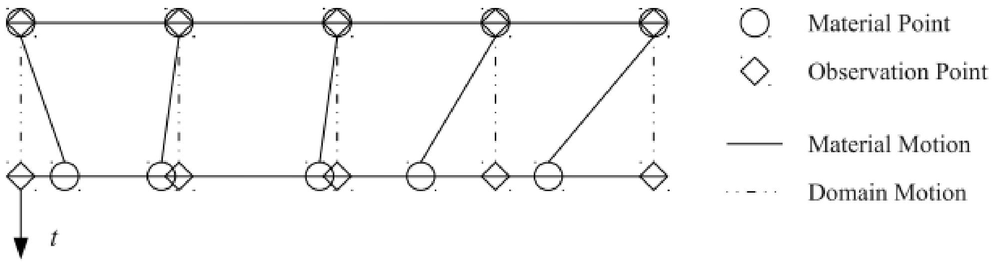


Typical Approach: Arbitrary Lagrangian Eulerian(ALE)

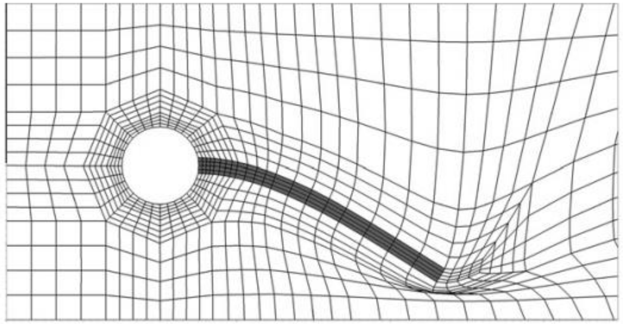
- **Lagrangian Approach:** Following the materials particles
  - Typical for study of solids

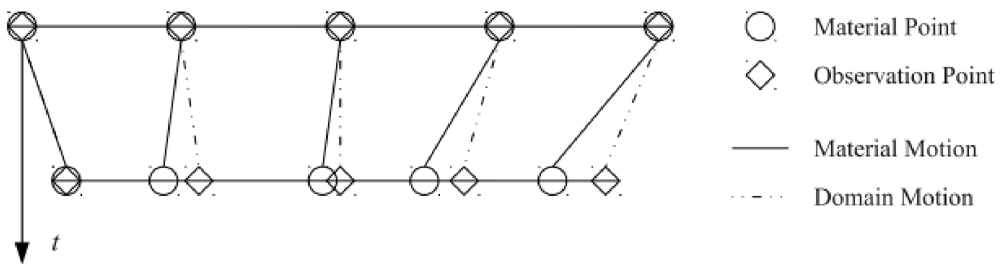


- **Eulerian Approach:** Fixed observation domain
  - Typical for study of liquids



- **Arbitrary Lagrangian Eulerian Approach:**
  - The observation domain (mesh) is also considered to be moving.

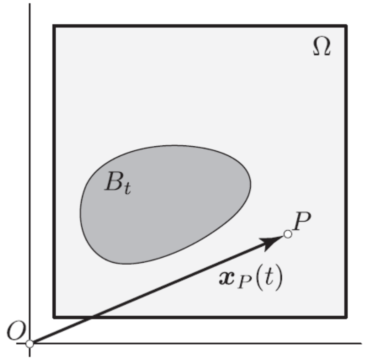




Overall system to implement

Balance of momentum

$$\int_{\Omega} \rho(\dot{u} - b) \cdot v \, dv + \int_{\Omega} \hat{\mathsf{T}}_{\mathrm{f}} \cdot \nabla_x v \, dv + \int_{B_t} (\hat{\mathsf{T}}_{\mathrm{s}} - \hat{\mathsf{T}}_{\mathrm{f}}) \cdot \nabla_x v \, dv - \int_{\partial\Omega_N} \tau_g \cdot v \, da = 0 \quad \forall v \in \mathscr{V}_0$$



Balance of mass

$$\int_{\Omega} q \, \mathrm{div}\, u \, dv = 0 \quad \forall q \in \mathscr{Q}.$$

Immersed body velocity

$$\Phi_B \int_B \left[\dot{w}(s,t) - u(x,t)\big|_{x=\zeta(s,t)} \right] \cdot y(s) \, dV = 0 \quad \forall y \in \mathscr{Y},$$

- Single velocity field ensures continuity of velocity across the interface automatically.
- Not assembling any term over the interface would ensure continuity of traction automatically.
- The position of solid is tracked via a mapping, which is used to query whether a particular point lies on the solid or not.
- No re-meshing required at all.