Presented at the COMSOL Conference 2008 Boston

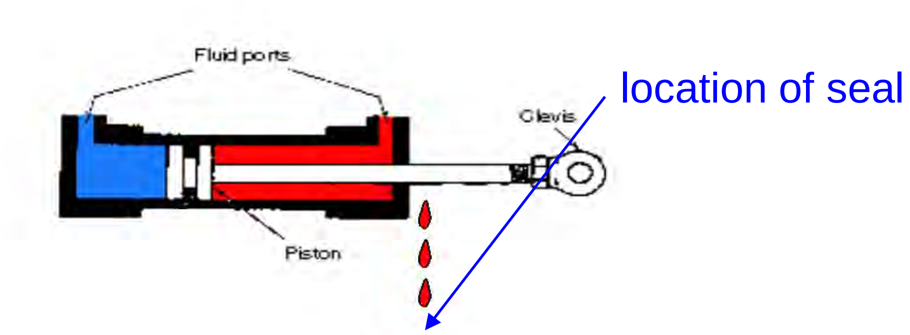
Hybrid Finite Element-Finite Volume Algorithm for Solving Transient Multi-Scale Fluid-Structure Interaction during Operation of a Hydraulic Seal

Azam Thatte
Richard Salant
Georgia Institute of Technology

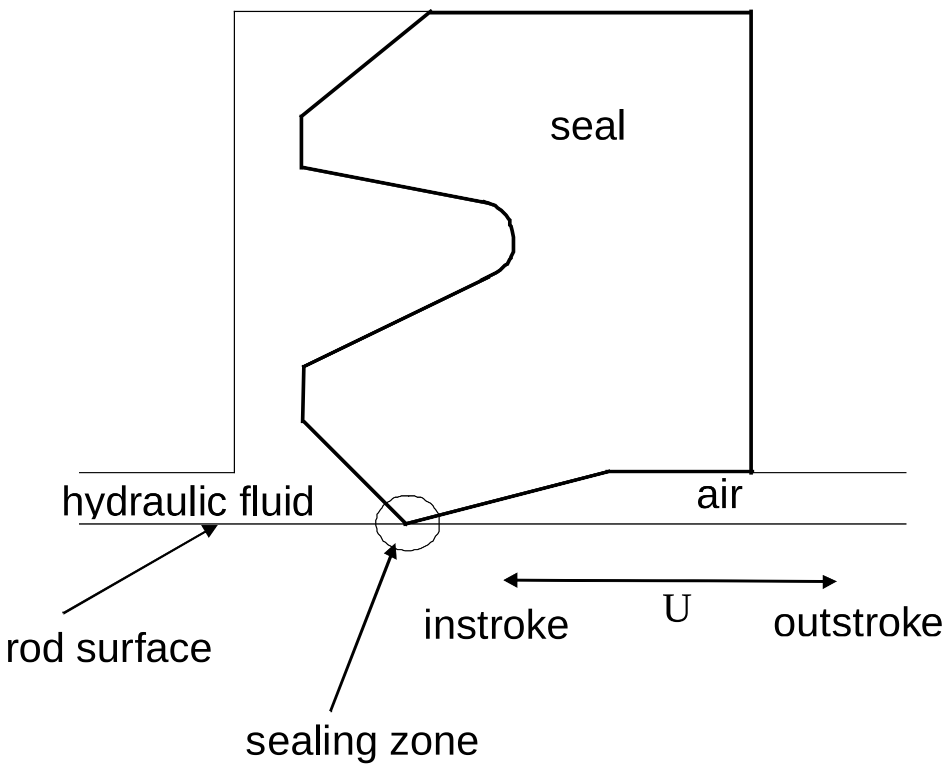
Background

Rod seals : among the most critical elements in hydraulic equipments

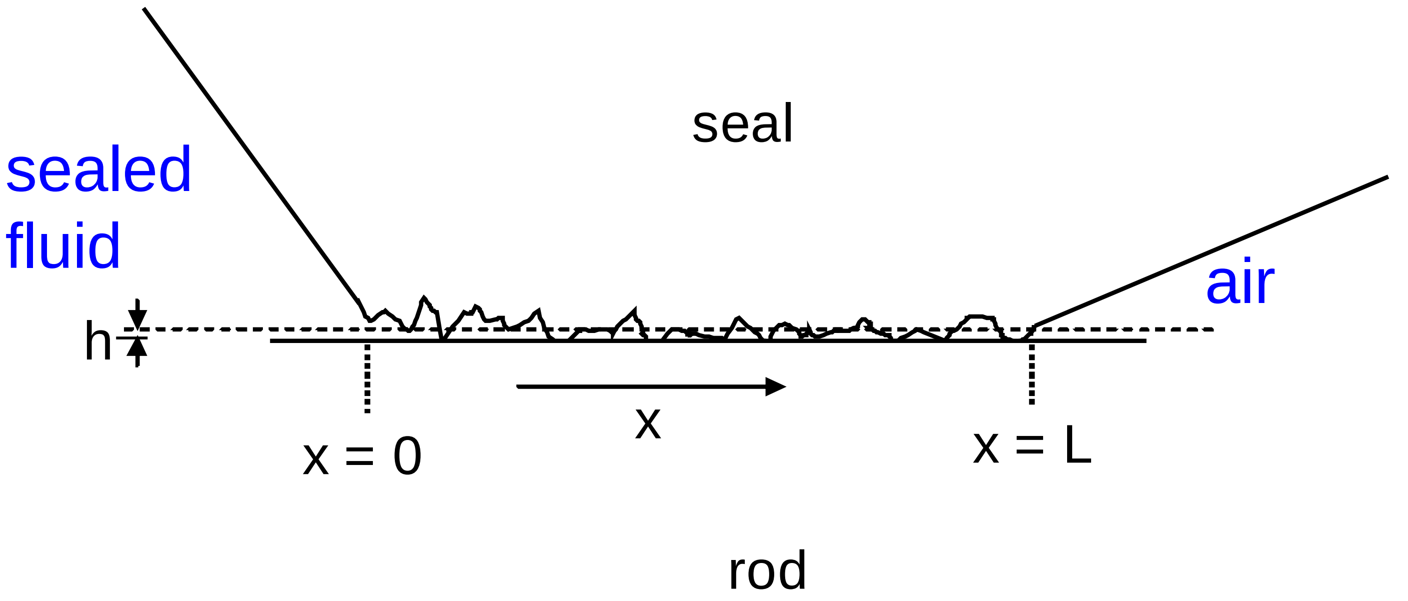
– prevent hydraulic fluid from entering the environment.

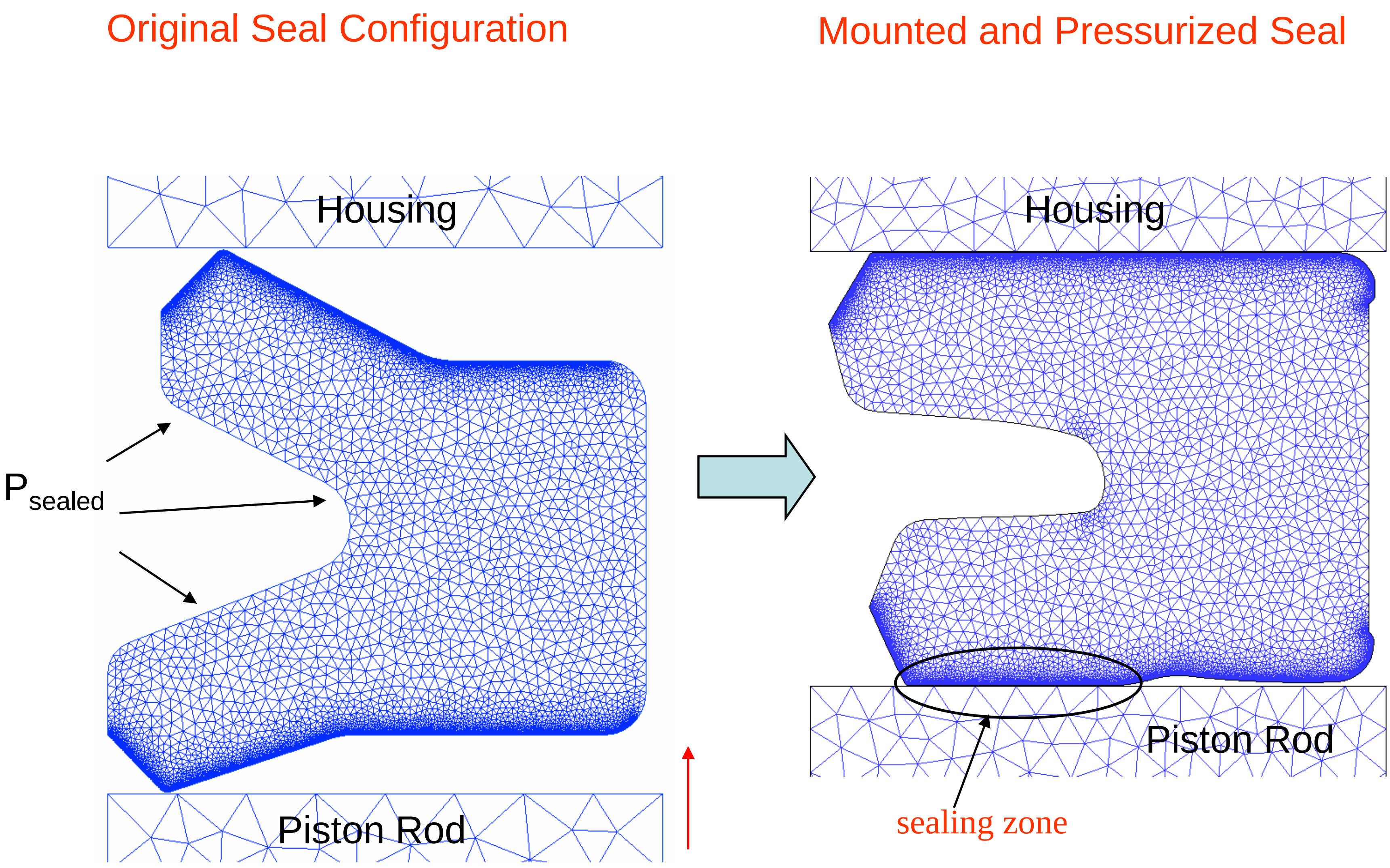


Schematic of Rod Seal



Sealing Zone





Original Seal Configuration
Mounted and Pressurized Seal
Housing
Housing
P sealed
Piston Rod
Piston Rod
sealing zone

Problem Decomposition
&
Hybrid Framework

Multi-scale FSI model consists of :

- Macro-scale structural mechanics for the seal deformations (F.E.) .
- Micro-scale fluid mechanics of the lubricating film in the sealing zone (F.V.) .
- Micro-scale statistical contact mechanics of the contacting asperities on the seal lip .
- Micro-scale elastic deformation mechanics of the sealing zone (F.E) .
- Macro-scale elastic contact mechanics at the seal-rod interface (F.E) .

A single hybrid finite element – finite volume framework, incorporating all these models, will solve these highly coupled nonlinear multiphysics equations simultaneously.

Macro-Scale Deformation Mechanics

- Macro-scale deformation mechanics for the seal mounted between the housing and rod, under pressurized conditions.

- Solved using an in-house MATLAB code coupled with COMSOL's finite element code.

- The seal is modeled as a nearly incompressible, linear, elastic and isotropic material with small deformation theory**.** The principle of virtual work for the axisymmetric system reads,

$$2\pi \int_A r \begin{pmatrix} -\varepsilon_{r\,test}\sigma_r - \varepsilon_{\theta\,test}\sigma_\theta - \varepsilon_{z\,test}\sigma_z \\ -2\varepsilon_{rz\,test}\tau_{rz} + r.uor_{test}F_r + w_{test}F_z \end{pmatrix} dA \; + \; 2\pi \int_l r\left(r.uor_{test}F_r + w_{test}F_z\right) dl$$

$$+ \; \left(r.uor_{test}F_r + w_{test}F_z\right) = 0$$

- Mixed formulation is used to model near incompressibility.

- Negative mean stress is added as a new dependent variable and the stress tensor is decomposed into a deviatoric part and a mean part.

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_d - m\,\tilde{p}$$

$$\tilde{p} = \tilde{p}_0 - K\,\mathbf{m}\boldsymbol{\varepsilon}^T\left(\boldsymbol{\varepsilon} - {}_0\right)$$

Micro-Scale Fluid Mechanics

- The micro-scale fluid mechanics in the sealing zone is governed by the transient Reynolds equation.
- Model takes into account cavitation and squeeze film effects.

$$\frac{\partial}{\partial \hat{z}}\left(\phi_{xx} H^3 e^{-\hat{\alpha} F \phi} \frac{\partial}{\partial \hat{z}}(F\phi)\right) = 6\zeta \frac{\partial}{\partial \hat{z}}\left(\left\{1+(1-F)\phi\right\}\left\{H_T + \phi_{s.c.x}\right\}\right) + 1 \ \varepsilon 2 \frac{\partial}{\partial \hat{t}}\left(\left\{1+(1-F)\phi\right\} H_T\right)$$

ϕ_{xx} , $\phi_{s.c.x}$: Flow factors to account for surface roughness effects of seal lip

In the liquid region:

$$\phi \geq 0, \ F = 1 \text{ and } P = \phi$$

In the cavitated region:

$$\phi < 0, \ F = 0, \ P = 0 \text{ and } \hat{\rho} = 1 + \phi$$

Boundary conditions :

$$P = P_{sealed} \qquad \text{at } \hat{z} = 0$$

$$P = 1 \qquad \text{at } \hat{z} = 1$$

Micro-Scale Fluid Mechanics continued…

- The average truncated film thickness is given by,

$$H_T = \int_{-H}^{\infty} (H+\delta) f(\delta) d\delta$$

- Gaussian distribution of asperities is assumed which yields

$$H_T = \frac{H}{2} + \frac{H}{2} erf\left[\frac{H}{\sqrt{2}}\right] + \frac{1}{\sqrt{2\pi}} e^{-H^2/2}$$

- Fluid equations solved for ϕ and F at each time step numerically with a finite volume formulation.

- Set of linear algebraic equations is solved using the tri-diagonal matrix algorithm (TDMA).

- Time integration is carried out using a fully implicit method giving an unconditional numerical stability to the procedure.

- Solution yields the fluid pressure distribution and location of cavitation zones at each time step.

Micro-Scale Statistical Contact Mechanics

- Significant asperity contact may occur during mixed lubrication.

- Necessity to add an micro-scale asperity contact pressure to the hydrodynamic pressure in computing radial seal deformations and local film thickness.

- Assuming Gaussian distribution of asperity heights, the micro-scale contact pressure is given by,

$$P_c = \frac{4}{3}\frac{1}{\left(1-\nu^2\right)}\hat{\sigma}^{3/2}\frac{1}{\sqrt{2\pi}}\int_H^\infty \left(\delta-H\right)^{3/2} e^{-\delta^2/2}\,d\delta$$

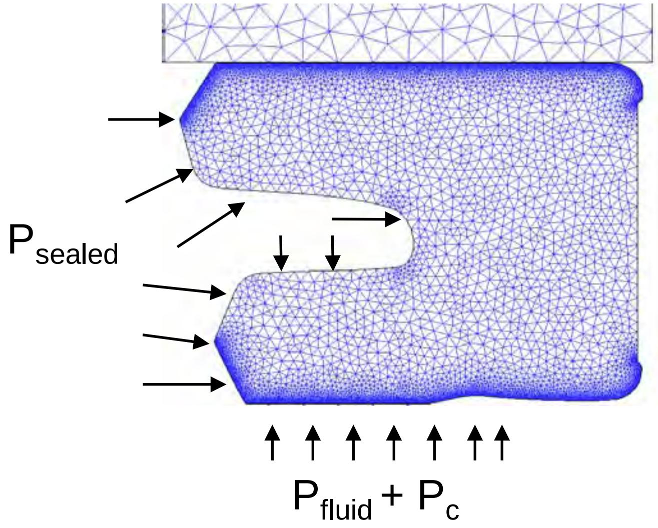
- Integral calculated using “Adaptive Gauss-Kronrod” quadrature.

Micro-Scale Deformation Mechanics

- Needed to get micro-scale film thickness distribution at each time step.
- Radial deformations of sealing element under combined action of sealed pressure, fluid pressure and contact pressure.
- In discretized form with *n* axial nodes along the sealing zone, the film thickness at the i th node can be expressed as,

$$H_i = H_d + \left(H_{def}\right)_i$$

- $\left(H_{def}\right)_i$: Obtained from F.E. calculations after applying a net pressure of $\left(P_t - P_{dc}\right)$ over the contact zone.
- H_d : Thickness that a hypothetical film would occupy under dry contact conditions.



Micro-Scale Deformation Mechanics continued..

- H_d Computed by equating dry contact pressure P_{dc} from the macro-scale F.E. analysis with the statistical micro-scale contact pressure P_c .

- Using a curve fit method to invert equation for P_c yields,

$$H_d = a + b \cdot \log(x) + c \cdot (\log(x))^2 + d \cdot (\log(x))^3 + e \cdot (\log(x))^4 + f \cdot (\log(x))^5$$

$$x = -\log_{10} |\, I \,| \qquad I = \frac{P_{dc}}{\frac{4}{3}\frac{1}{\left(1-\nu^2\right)}\hat{\sigma}^{3/2}}$$

Macro-Scale Contact Mechanics

- To obtain P_{dc} , the macro-scale F.E. contact mechanics is solved at the seal-rod interface and seal-housing interface.

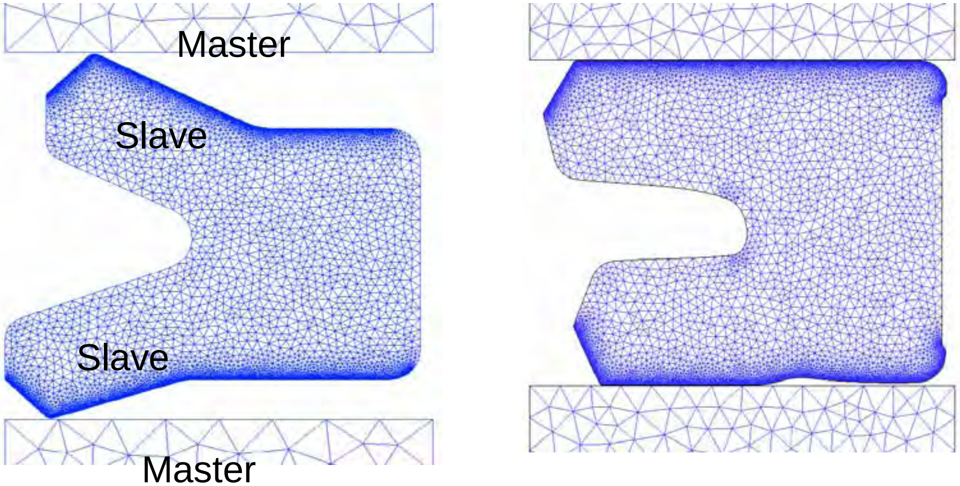
- Augmented Lagrangian method used. Augmentation component introduced for dry contact pressure.

- For each slave point, a corresponding master point is searched in the direction perpendicular to the slave boundary.

- The contact interaction gives the following contribution to the weak form on the slave boundary

$$\int_{slave} \left[\left(P_{dc,p} \delta\lambda \quad + \quad P_{dt,p} \cdot \Re(\mathbf{F}_D) \delta(\Re(X)) \right) \quad + \quad \left(C_1 \delta P_{dc} + C_2 \cdot \delta P_{dt} \right) \right] dA$$

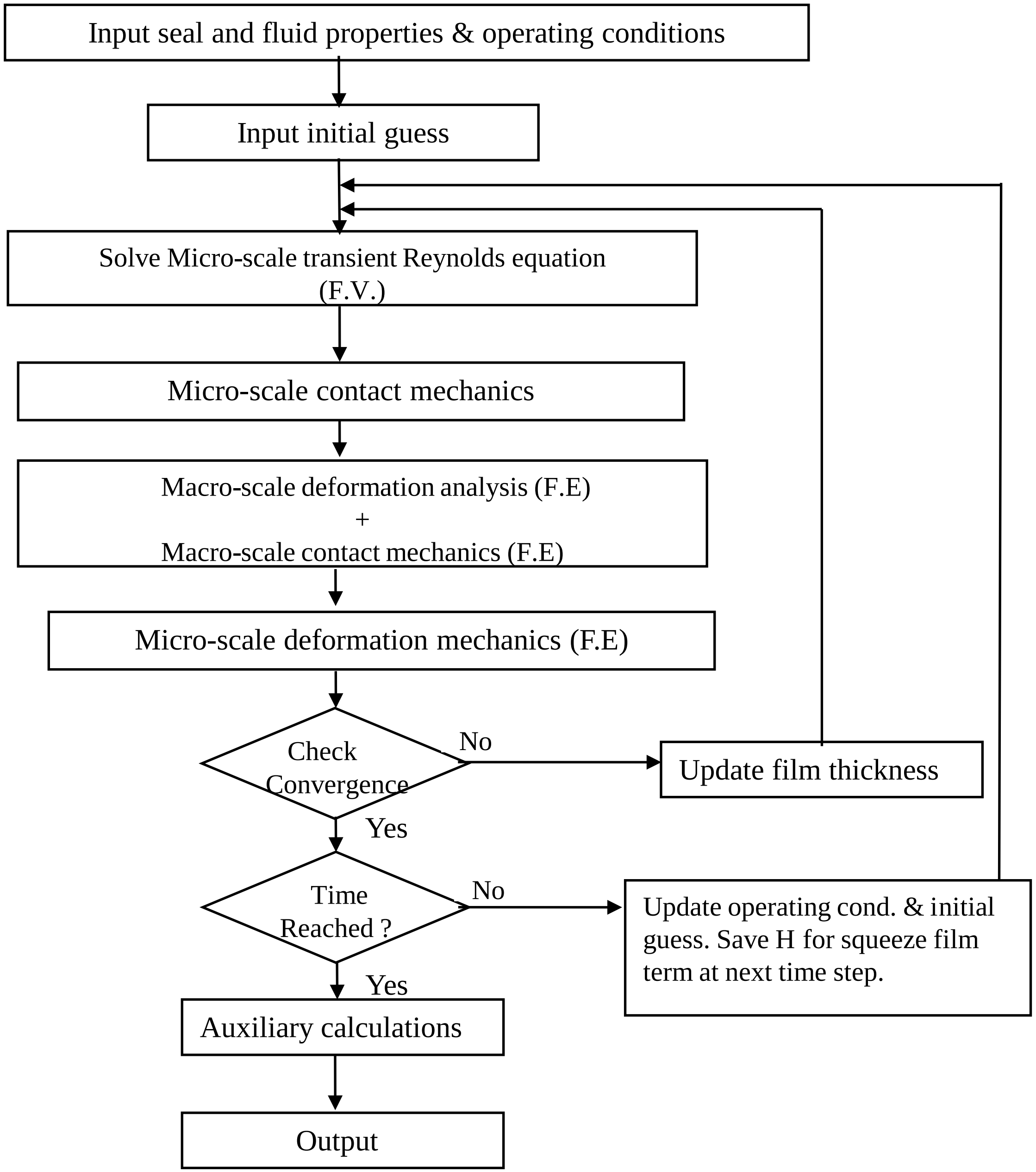
- The augmented dry contact pressure defined on the slave boundary is given by :



$$P_{dc,p} = P_{dc} - \varepsilon_n \lambda \quad ; \quad \lambda \le 0$$

ε_n : normal penalty factor λ : gap variable

Flow Chart for Hybrid Algorithm

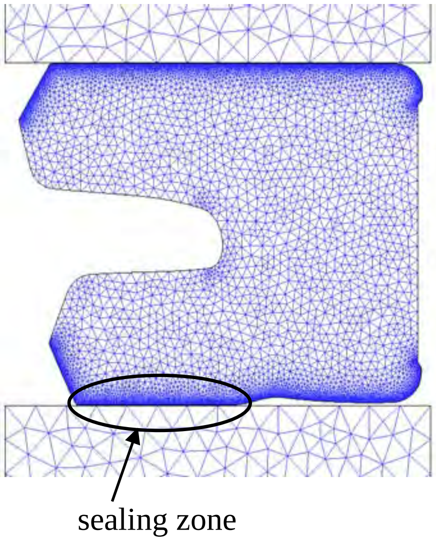


Base Parameters

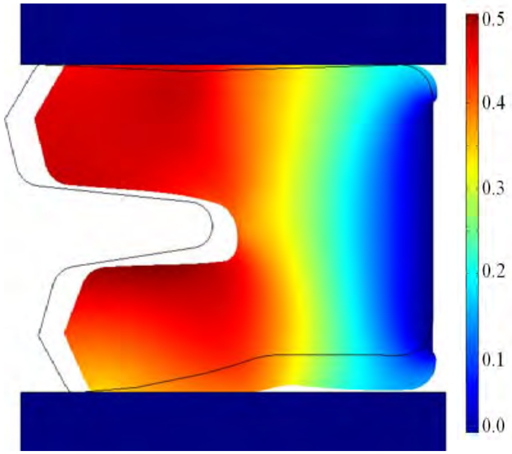
Elastic modulus	43 x 10^6 Pa
Poisson's ratio	0.49
Sealed pressure	6.90 MPa
Rod diameter	88.9 mm
Stroke	1.93 m
Reference viscosity	0.043 Pa-s
Pressure-viscosity coefficient	20 x 10^{-9} Pa^{-1}
Asperity radius	1 μm
Asperity density	10^{14} m^{-2}
Asperity contact friction coefficient	0.25

Results

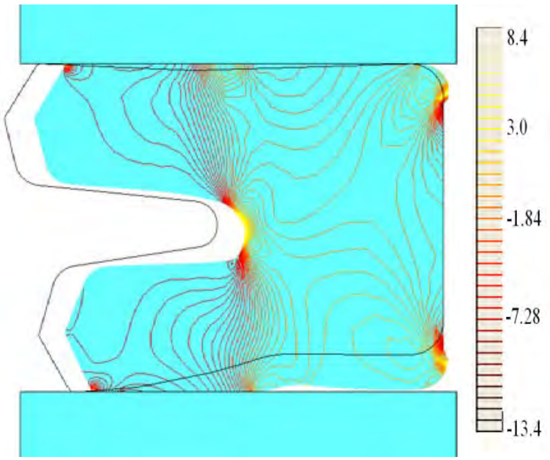
Macro-Mechanics



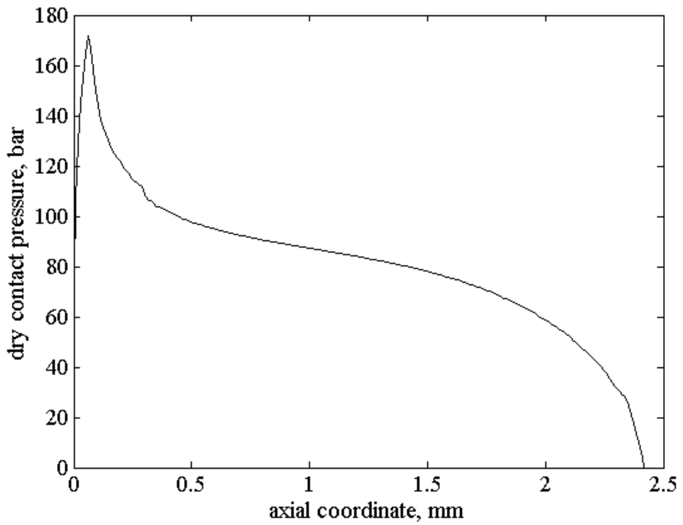
Deformed Mesh & Sealing Zone after macro-scale contact mechanics



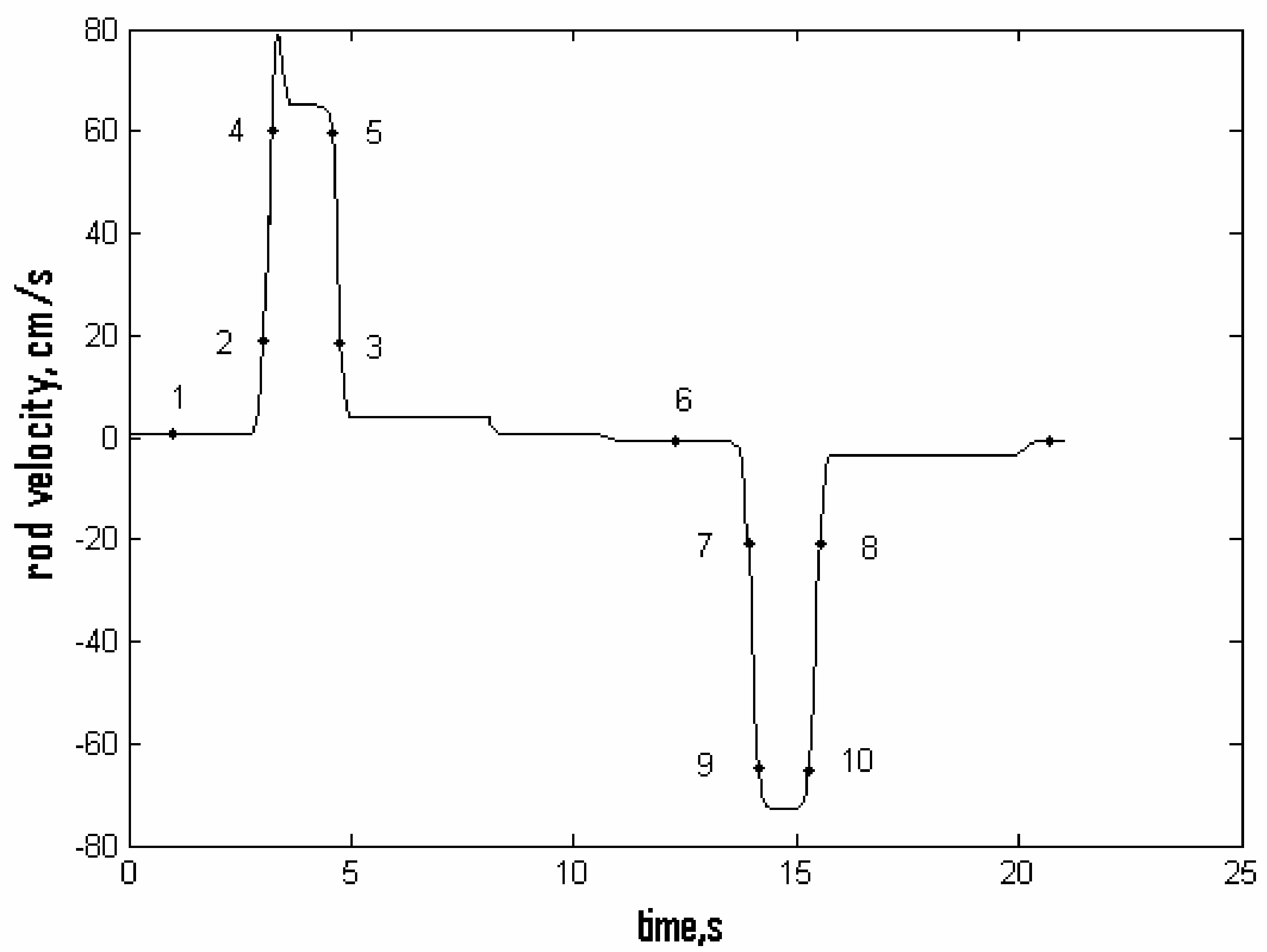
Axial Displacements (mm)



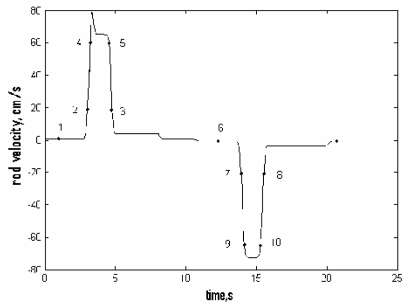
First Principal Stress in the Seal Body

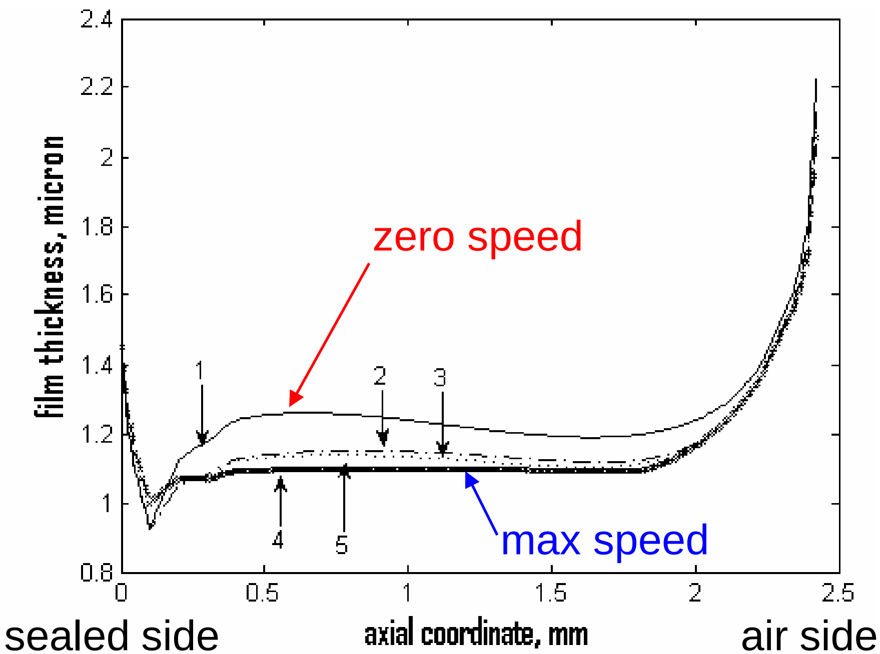


Rod Velocity vs. Time

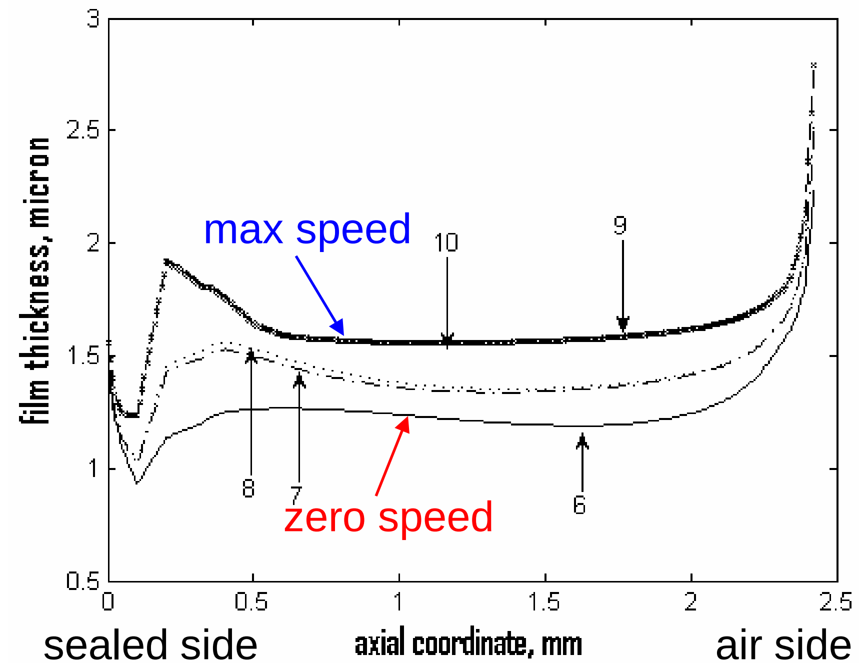


Film Thickness Distribution





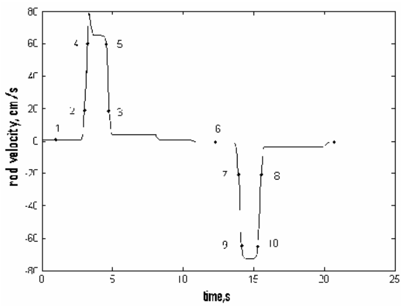
outstroke

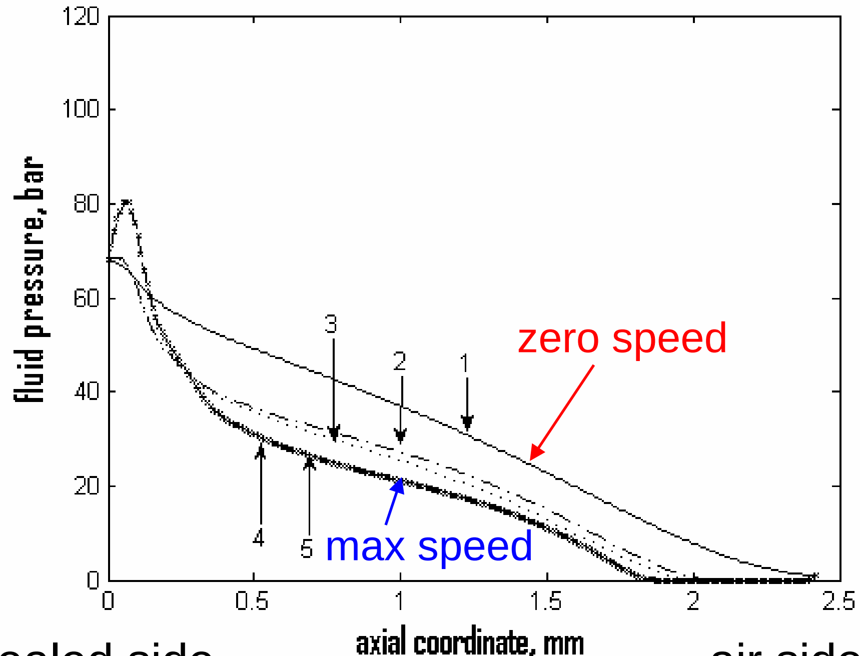


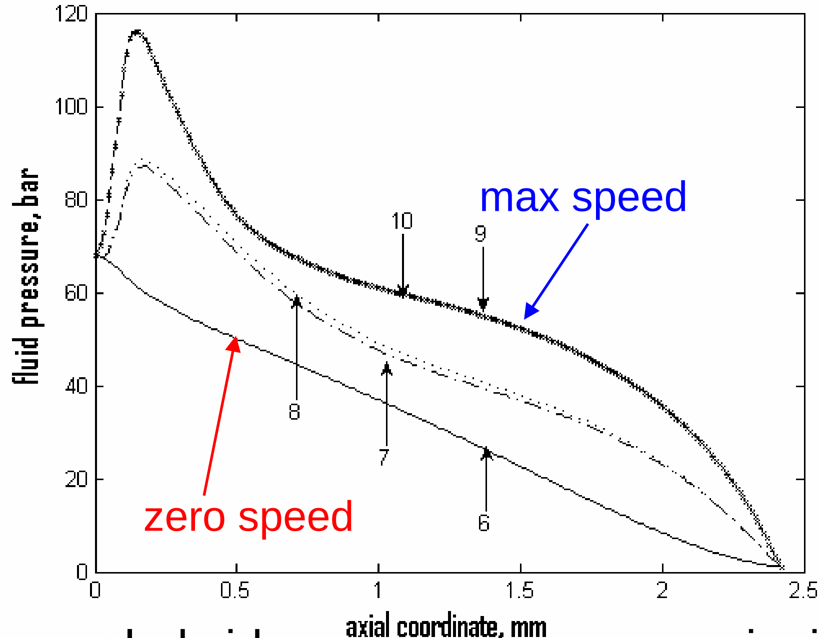
instroke

Time sequence: 1-2-4-5-3-6-7-9-10-8

Fluid Pressure Distribution



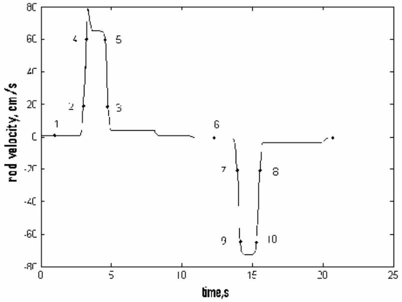




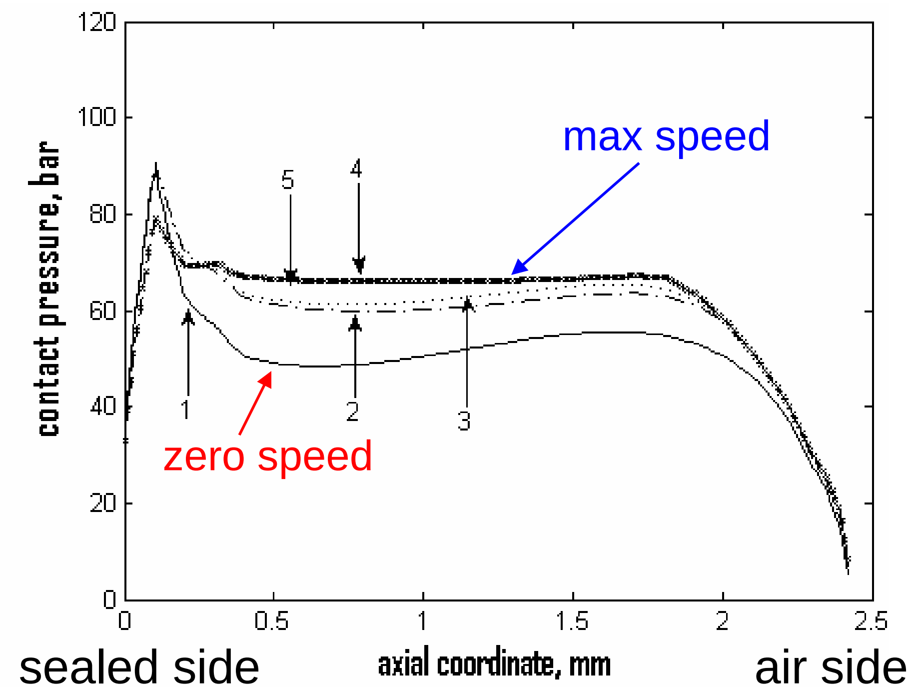
sealed side air side sealed side air side

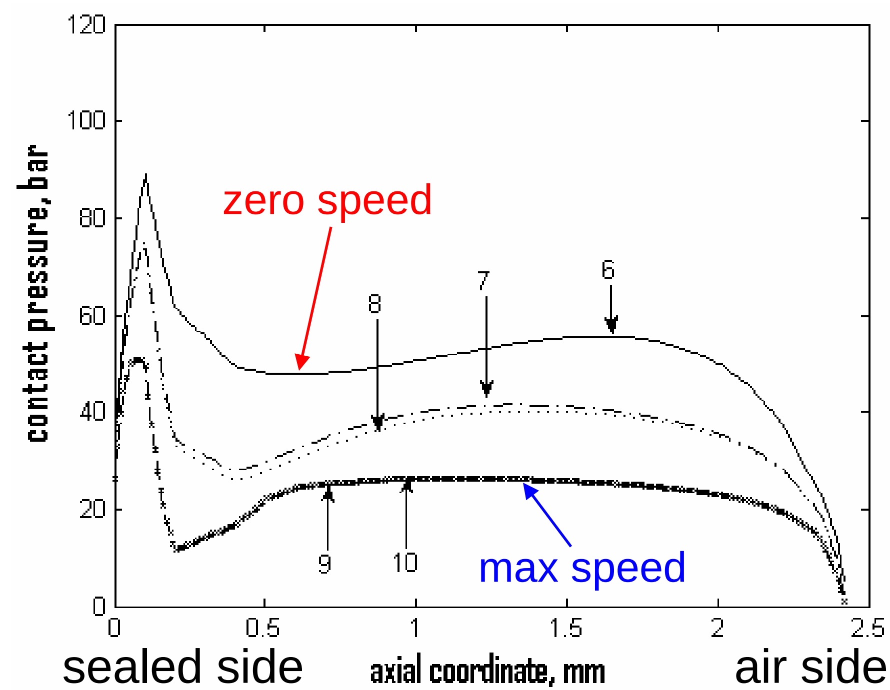
outstroke instroke

Time sequence: 1-2-4-5-3-6-7-9-10-8



Contact Pressure Distribution



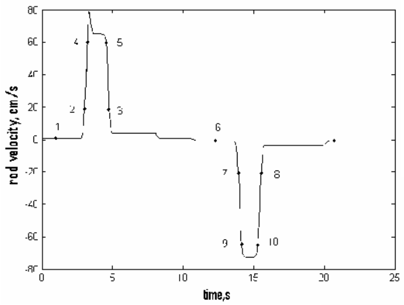


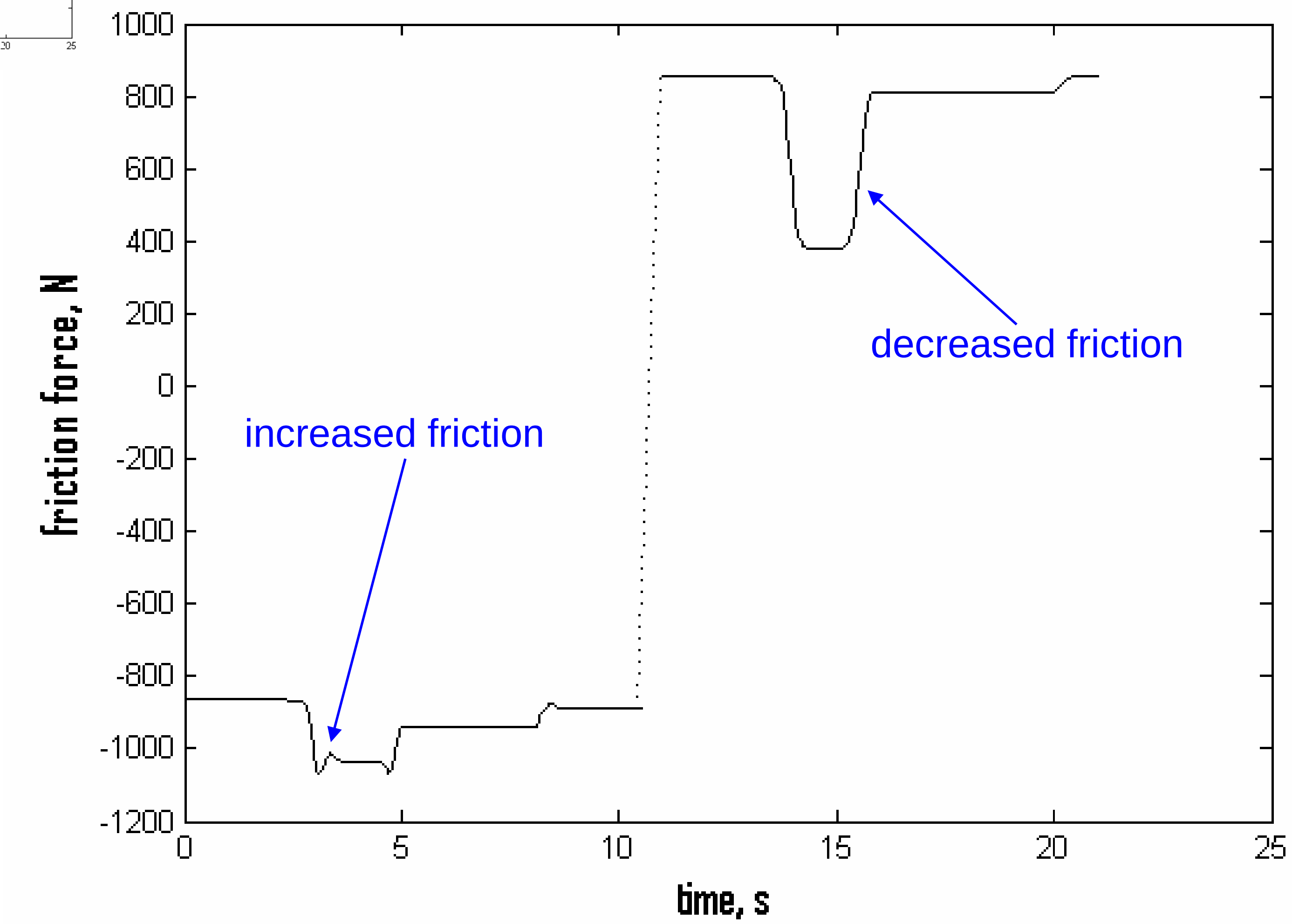
outstroke

instroke

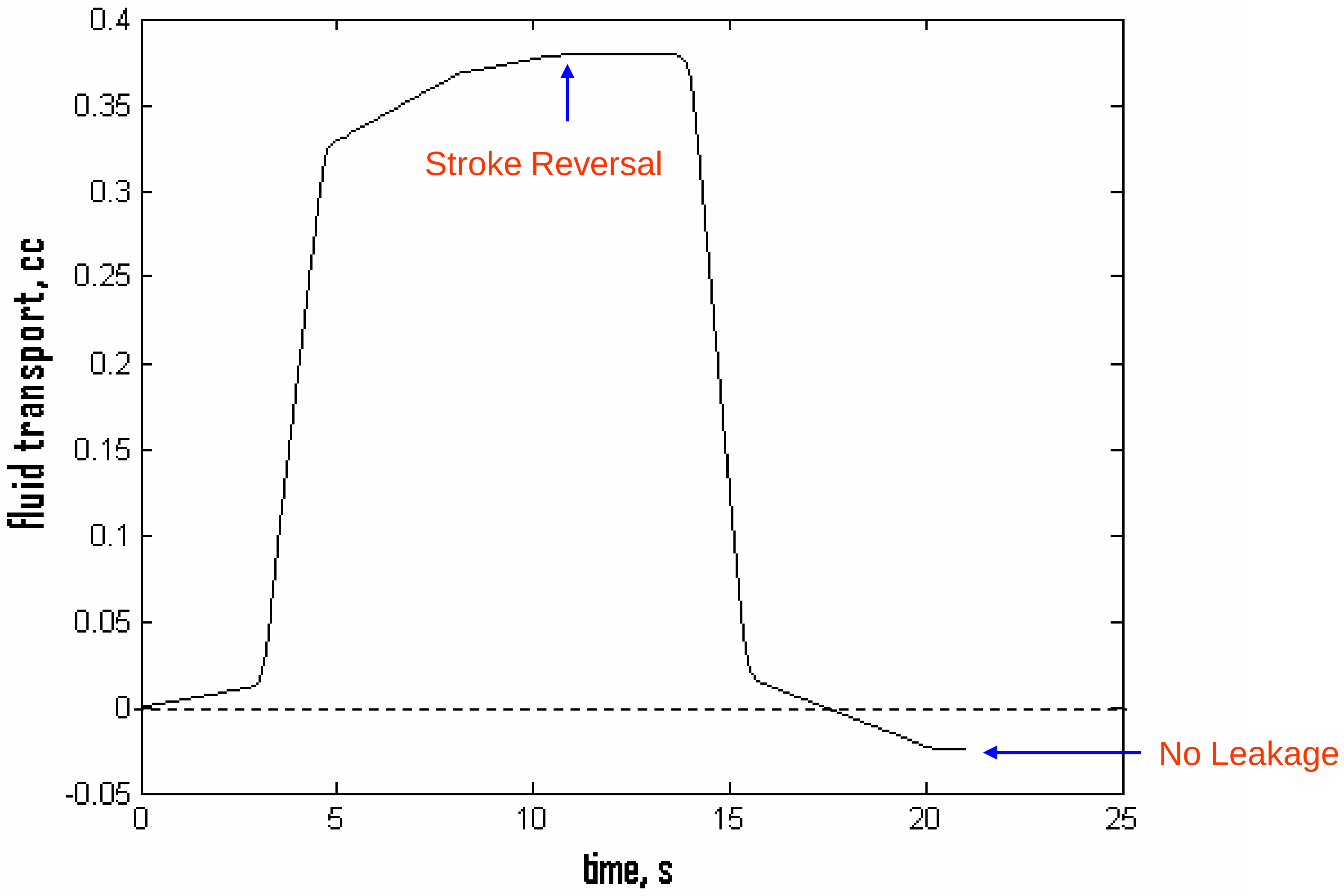
Time sequence: 1-2-4-5-3-6-7-9-10-8

Friction Force vs. Time

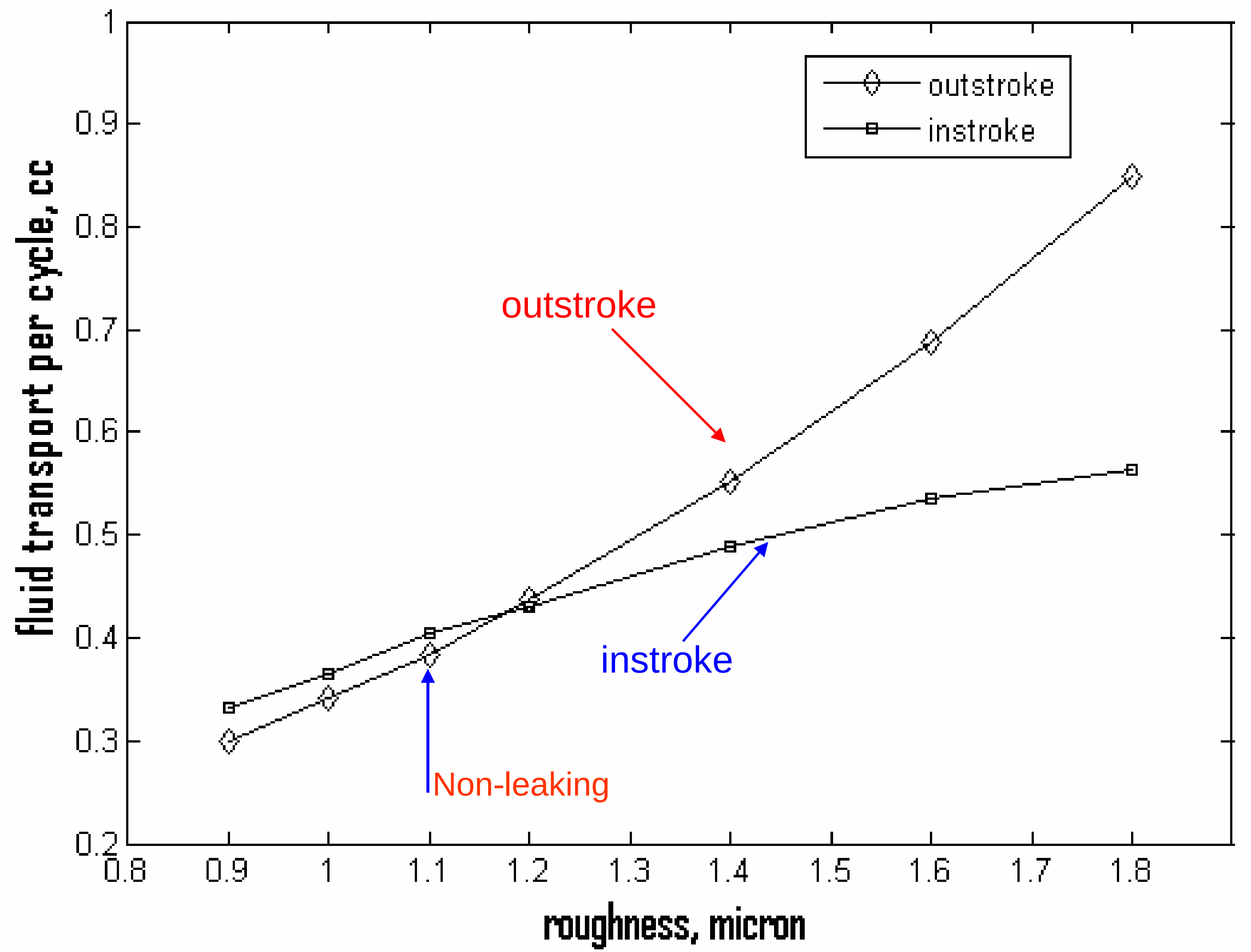




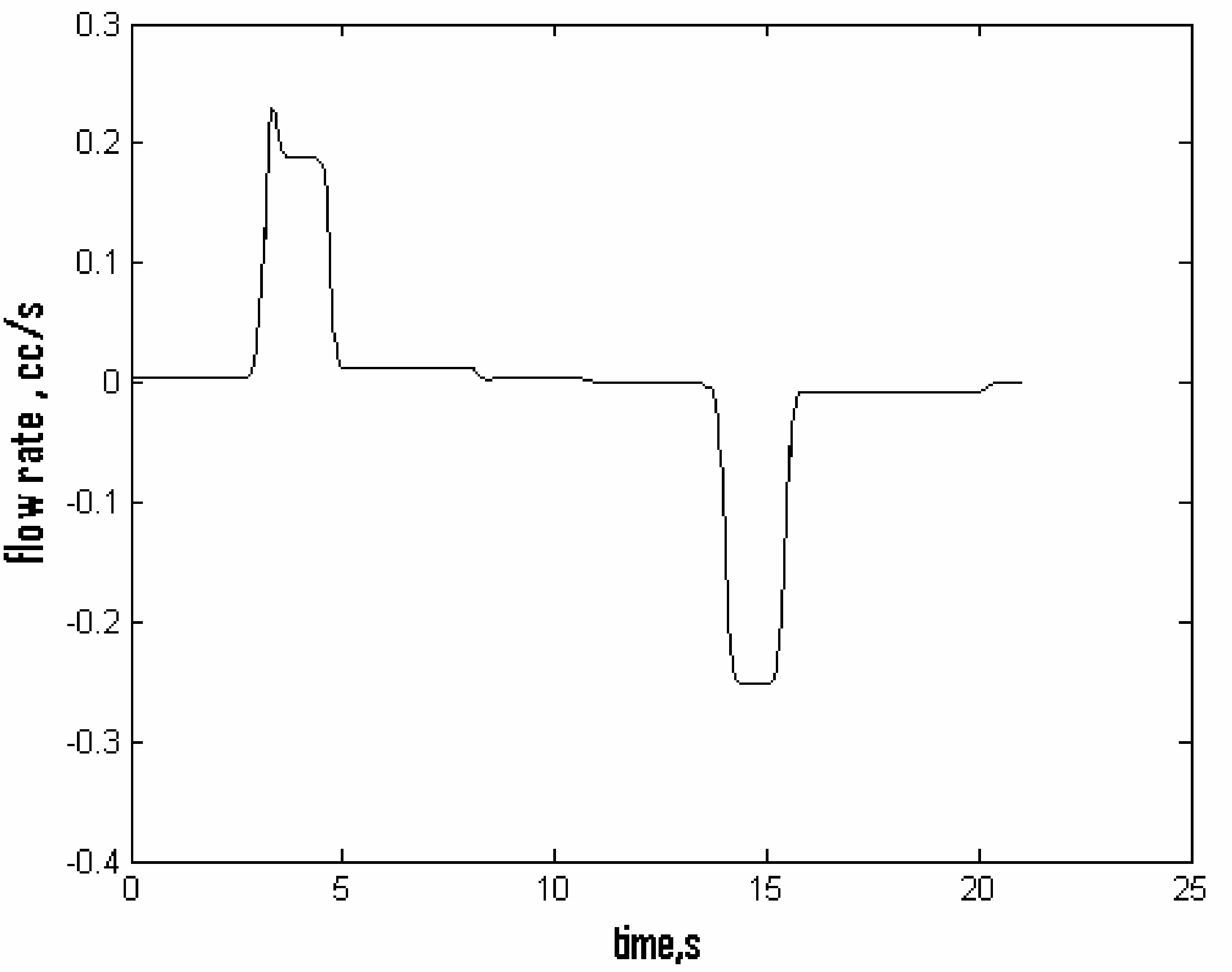
Net Fluid Transport vs. Time



Fluid Transport vs. Seal Roughness



Flow Rate vs. Time



Couette Flow Prominent

Conclusions

- Hybrid framework of finite element – finite volume solution algorithms for solving highly coupled, nonlinear, multiscale fluid-structure interaction is developed.
- Hybrid method facilitated an *online* calculation of micro-scale deformations necessary to model the transient seal response.
- Transient FSI solution revealed the history of a reciprocating seal's behavior over a cycle.
- Solution confirmed the presence of a "critical seal roughness" needed to prevent the leakage.
- Solution showed that thinner films during the outstroke than during the instroke, and cavitation during the outstroke, are characteristics of a non-leaking seal.
- It also provided insight into why the behaviors during outstroke and instroke differ.