Analyzing Muffler Performance using the Transfer Matrix Method

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Exhaust Systems in General

- Exhaust gas transportation
- Noise reduction
- NOx, HC, PM reduction
The Transfer Matrix Method
The Transfer Matrix Method

The Transfer Matrix

- The acoustical transfer properties of a system
- Plane wave decomposition in the connecting pipes
The Transfer Matrix Method

The Transfer Matrix Extraction

1. Symmetric matrix
2. Reciprocity requires $\det(T) = 1$

Advantage:
Solving the FEM problem only once
The Transfer Matrix Method

Evaluation Parameters

Transmission loss (source independent)

Insertion loss (source dependent)
The Transfer Matrix Method

**Evaluation parameters**

Transmission loss (source independent)

\[ TL \propto \frac{W_{in}}{W_{tr}} \]

Insertion loss (source dependent)
The Transfer Matrix Method

Evaluation parameters

Transmission loss (source independent)

\[ TL = 10 \log \left( \frac{1}{4} \left| T_{11} + T_{12} \frac{S}{\rho c} + T_{21} \frac{\rho c}{S} + T_{11} \right|^2 \right) \]

Insertion loss (source dependent)

\[ TL \propto \frac{W_{in}}{W_{tr}} \]
The Transfer Matrix Method

**Evaluation parameters**

**Transmission loss (source independent)**

\[ TL = 10 \log \left( \frac{1}{4} T_{11} + T_{12} \frac{S}{\rho c} + T_{21} \frac{\rho c}{S} + T_{11} \right)^{2} \]

\[ TL \propto \frac{W_{in}}{W_{tr}} \]

**Insertion loss (source dependent)**

\[ IL \propto \frac{W_{ref}}{W_{muffler}} \]
The Transfer Matrix Method

**Evaluation parameters**

**Transmission loss (source independent)**

- Incident sound power: $W_{in}$
- Transmitted sound power: $W_{tr}$

$$TL \propto \frac{W_{in}}{W_{tr}}$$

$$TL = 10 \log \left( \frac{1}{4} \left| T_{11} + T_{12} \frac{S \rho_c}{S} + T_{21} \frac{\rho_c}{S} + T_{11} \right|^2 \right)$$

**Insertion loss (source dependent)**

- Radiated sound power with reference system: $W_{ref}$
- Radiated sound power with muffler system: $W_{muffler}$

$$IL \propto \frac{W_{ref}}{W_{muffler}}$$

$$IL = 10 \log \left( \frac{|T_{11}Z_{a,r} + T_{12} + Z_{a,s} + (T_{21}Z_{a,r} + T_{22})|^2}{|T_{11}'Z_{a,r} + T_{12}' + Z_{a,s} + (T_{21}'Z_{a,r} + T_{22}')|^2} \right)$$
The Numerical Model
The Numerical Model

Boundary Conditions

- **Solid walls (sheet metal)**
- **Coupling boundaries conditions (wave propagation from one medium to another)**
- **Radiation conditions (reflection free ends)**
- **Impedance conditions (perforated plates)**
Subdomain Conditions

- **Air**
  - Defined by the speed of sound and the density

- **Absorptive material**
  - Defined by the apparent density and average fiber diameter
  - Based on theory by Delany and Bazley, Bies and Hansen

- **Ceramic structure (Diesel Particulate Filter)**
  - Preliminary described by general damping
Simulation Setup

- Maximum element size = $\lambda/5 = 34$ mm
- 24,000 elements, 38,000 DOF
- PARDISO solver
- 100 discrete frequencies
The Results
The Measurement Setup

- The two source method
  - Up and down stream source direction

- Flow speed up to 30 m/s (cold air)
  - Corresponds to 160 kW engine @ rated speed
The Test Objects

- Reflection muffler
  - Simple expansion chamber
  - Quarter wave resonator

- Absorption muffler

- Perforated muffler
  - Hole size: Ø3, Ø4, Ø8, Ø12
  - Porosity: 10 – 40 %

- Automotive exhaust
  - Diesel Particulate Filter
  - Hybrid muffler
The Reflective Muffler
Comparison

- Good correlation
- Peak offset due to inaccurate lengths, temperatures, densities
- First axisymmetric higher-order mode will propagate above 1400 Hz.
- First TL peak corresponds to a quarter muffler length

The Results
The Results

The Quarter Wave Resonator Comparison

- Again good correlation

- The first peak corresponds to a quarter pipe length.

- The 500 Hz minima could be eliminated by a pipe of 1/8 of the muffler length.
The Absorption Muffler
Comparison

- Good low frequency correlation

- Mid & high frequency differences
  - Inaccurate Delany & Bazley model
  - Too large sub-domain

The Results

Absorption chamber – 5000 Rayls/m
The Results

The Plug Flow Muffler Comparison

- 0 m/s flow speed
  - Good correlation
  - 800 Hz peak due to 80 mm extended inlet

- 30 m/s flowspeed
  - Good correlation
  - 1350 Hz peak not affected in simulation
  - Peaks limited by losses due to flow

Transmission Loss

Transmission Loss

Transmission Loss

Transmission Loss

Measured

Simulated

Transmission Loss [dB]

Frequency [Hz]

Transmission Loss [dB]

Frequency [Hz]
The Plug Flow Muffler Simulations

- Flow speed variations (Ø3, 25%)
  - Flow smoothes the peaks and dips

- Porosity variations (Ø4, 30 m/s)
  - Same effect as changing the flow speed
  - Porosity is important, not hole size
The Hybrid Muffler
Comparison at 0 m/s

- Good correlation up to 700 Hz

- Difference due to
  - Inaccurate Delany & Bazley model
  - Slightly different lengths
  - Additional small features of in the real exhaust

The Results

Transmission Loss

Transmission Loss [dB]

Frequency [Hz]

Measured
Simulated

2 reflection chambers
Plug flow chamber – Ø3, 30 %
Absorption chamber – 5000 Rayls/m

going the extra mile

dinex
The Diesel Particulate Filter Comparison at 0 m/s

- No correlation (general damping)

- Additional simulations proved the model against measurements made by KTH

![Image of Diesel Particulate Filter Diagram]

**Transmission Loss**

- **Measured**
- **Simulated**

Clean DPF
Sic, 90 CPSI
11.25” x 12”
Conclusion

- Successful transfer matrix approach
  - One run
  - Insertion loss calculation possible

- Model validation
  - Reflective and plug flow muffler
  - Absorptive and ceramic

- Simulation approach
  - Frequency limitations by pipe diameter
  - Short setup time
  - Easy redesign
Future work

- Pressure loss and mean flow distribution simulation
  - backpressure result

- Source impedance measurements
  - Insertion loss results
Questions?
Appendix
Benefits

Of acoustic simulation of exhaust systems
- Reduced cost price and development time
- Increased performance and knowledge
- Minimizing material consumption
- Simplifying construction and production

Of using the Transfer Matrix approach
- Modular approach
- Transmission loss calculation
- Insertion loss calculation
Appendix

Limitations

- Upper frequency is 2 kHz: $f < \frac{1.84c}{\pi D}$
  - $D$ is the duct diameter
  - $f$ is the frequency
  - $c$ is the speed of sound
- Exhaust system length max 15 m
- Max 150 dB re 20 $\mu$Pa
- Constant temperature
- Zero mach number