Presented at the COMSOL Conference 2008 Hannover

Linear water wave propagation around structures

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COMSOL CONFERENCE HANNOVER 2008

Outline

- Motivations: Wave Energy Converters
- Objectives: Mild Slope Equations
- Methods
  - Equations
  - Boundary conditions
  - Dispersion relationship
  - Wave direction
- Model validation
  - Pure diffraction
  - Diffraction and Refraction
  - Natural frequencies
- Application
- Conclusions

Objective

Develop a tool for designing a port layout with energy sinks that minimise wave disturbance elsewhere.

Tool characteristics should be:

- ability to predict wave height transformation within a closed basin with partially absorbing walls
- ability to parametrize geometry

In this initial phase, the objective is merely to develop the tool.

Methods

- Treat problem with 2D mild slope equations. Solve equations with "Comsol Multiphysics Software" using PDEs in general form.

- Treat open boundary condition problem using the "Internal generation of waves" system according to Bellotti at al. (Ceng, 2003)

- Treat other boundary conditions according to Beltrami et al. (J Wat. Port. Coastal & Ocean Eng. 2001)

Governing equations

Domain:

Ψ is potential of velocities **U=(U,V)**

(i.e. **U=∇ Ψ**)

$$\Psi(x,t) = \mathrm{Re}[\psi(x)e^{i\omega t}]$$

$$\nabla \cdot \left(c c_g \nabla \psi \right) + k^2 c c_g \psi = 0$$

Internal generation of waves

Add a **line** in the domain where the RHS is S:

$$\nabla \cdot \left(c c_g \nabla \psi \right) + k^2 c c_g \psi = S$$

Across the line,
continuity is assured by: n Γ_1= n Γ_2
and a dweak term S is added

$$S = 2 g c_g a \delta(x)$$

a is amplitude of generated waves
(Bellotti at al., Ceng 2003)

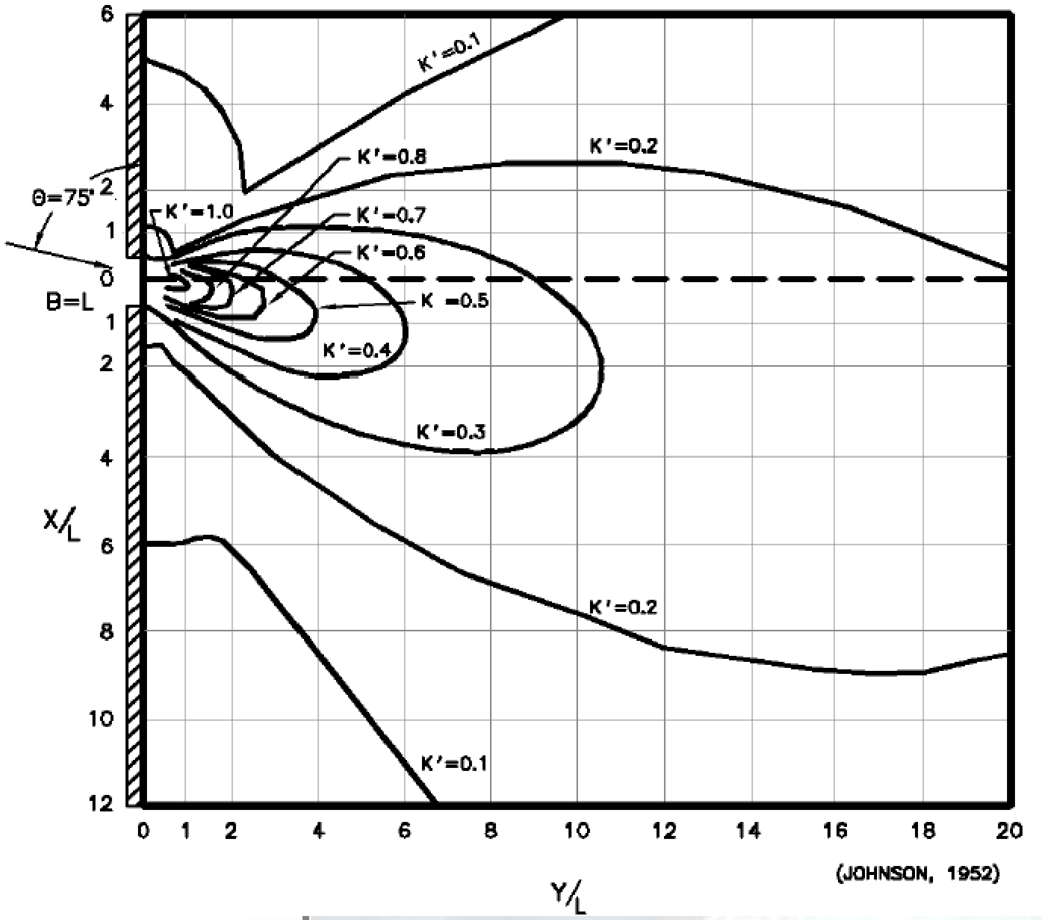
Dispersion relationship

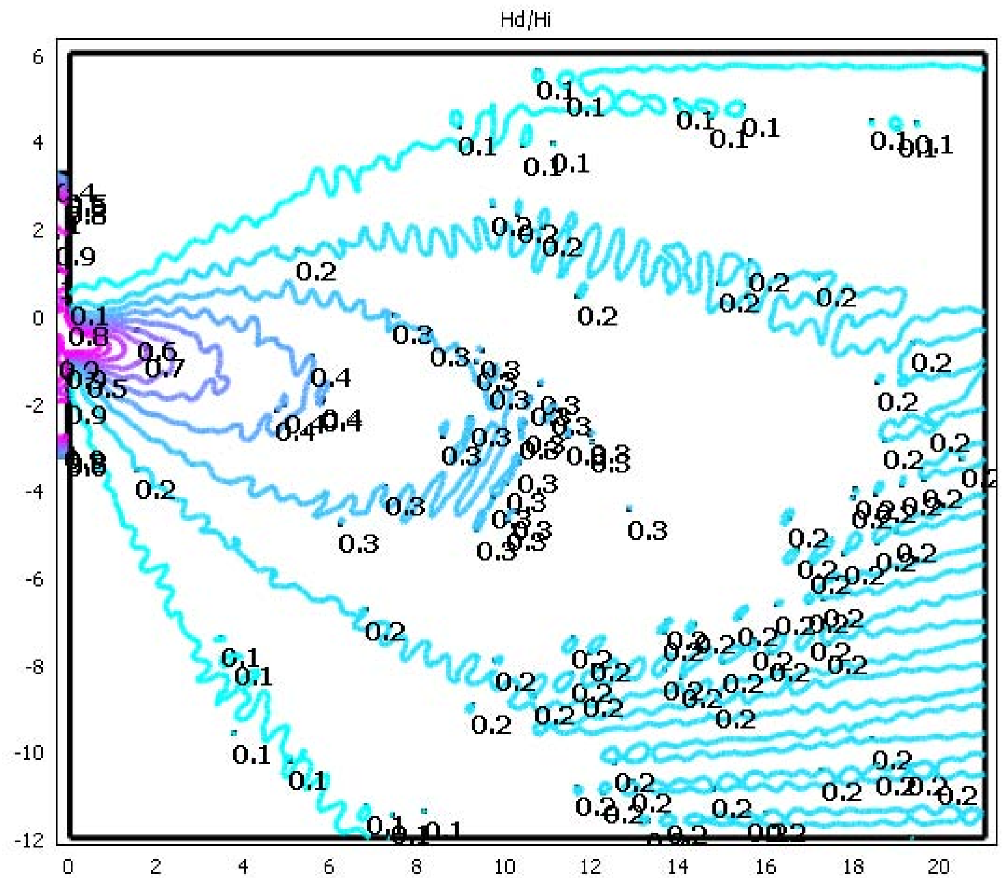
$$\omega^2 = kg \tanh(kh)$$

Algebraic equation, implicit in the unknown k (modulus, not direction!!)

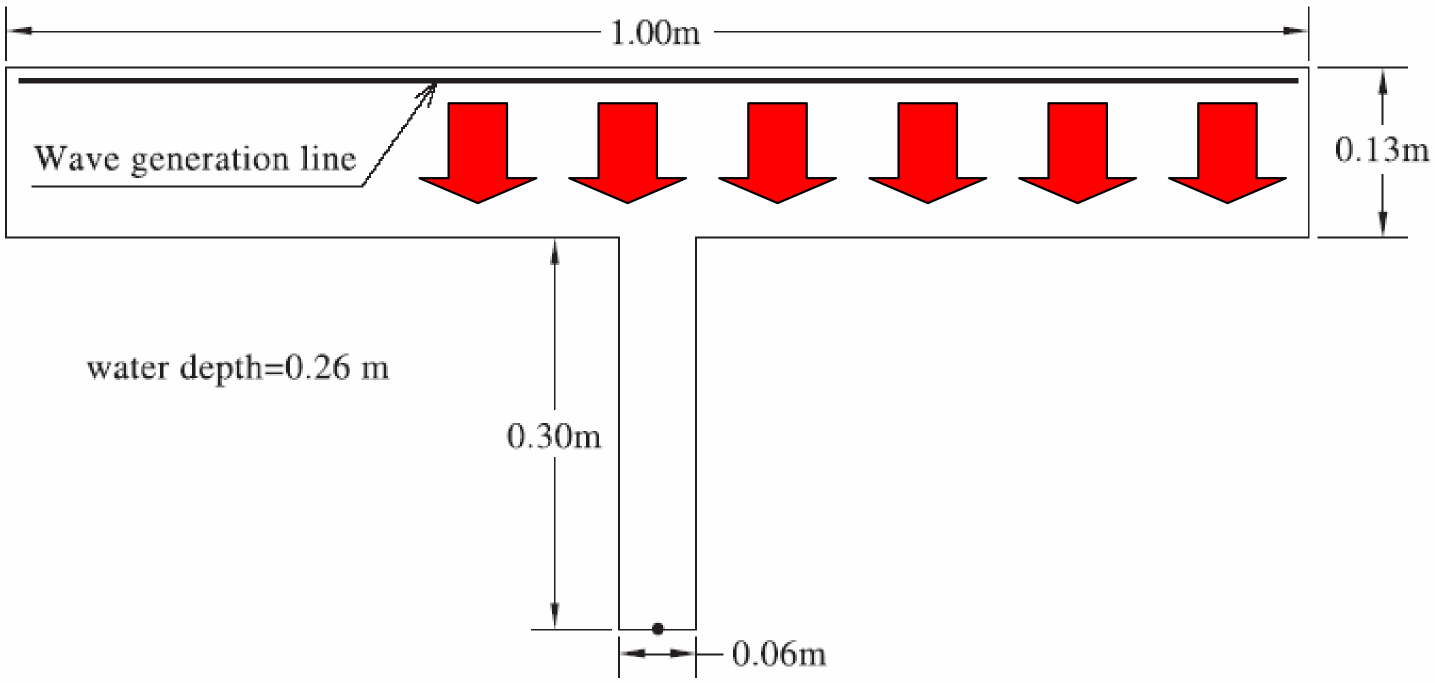
→ k is solved using a 5th order polynomium which is continuous and very accurate

Model validation: pure diffraction

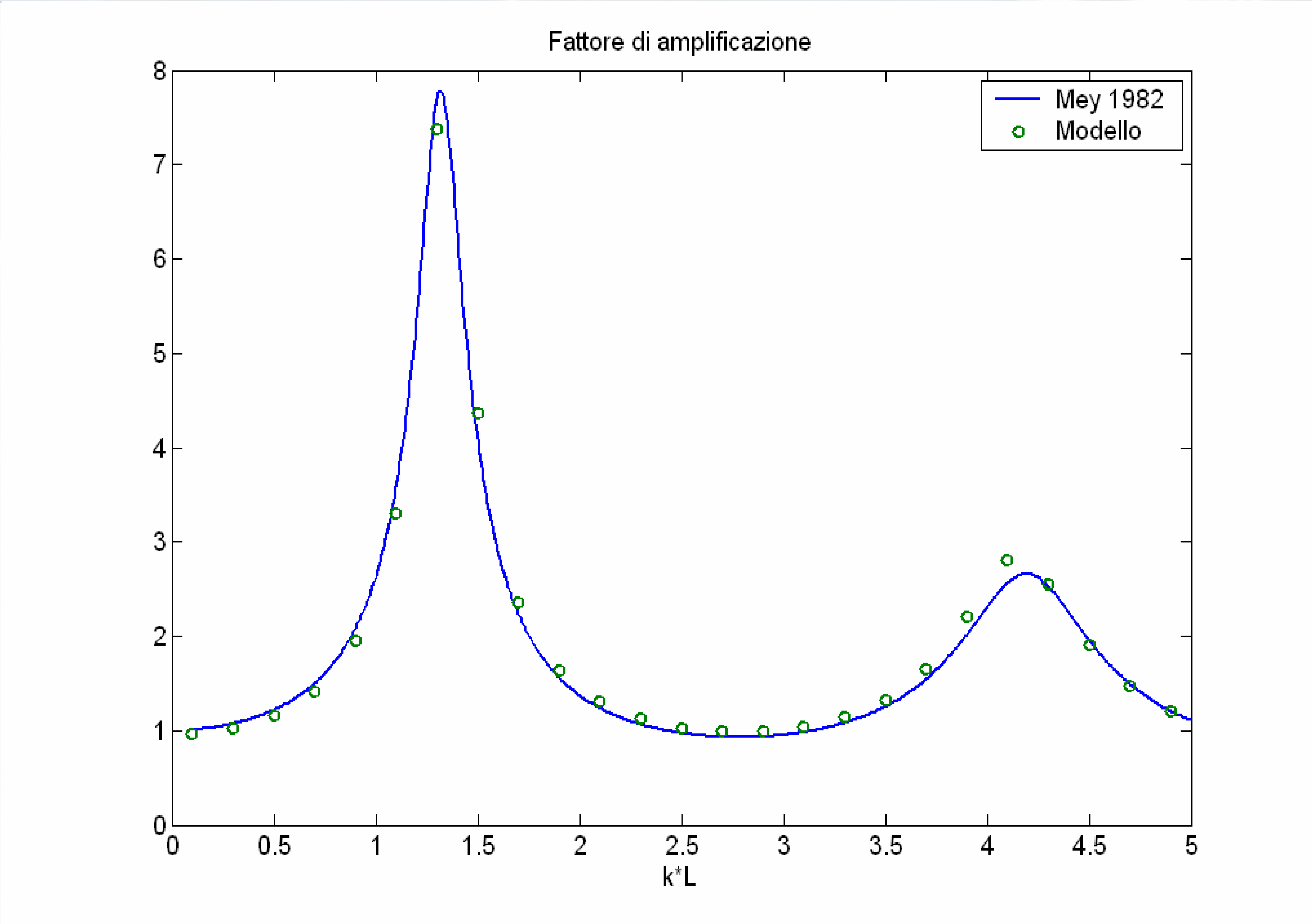




Checking resonance conditions



H_o/H_{inc}: theoretical Vs simulated



OUTLINE

MOTIVATIONS

OBJECTVES

METHODS

MODEL VALIDATION

APPLICATIONS

CONCLUSIONS

Boundary conditions

All boundary condition are of Noimann type and involve wave direction β

$$\Gamma \cdot \mathrm{n} = cc_{\mathrm{g}} \frac{\partial \psi}{\partial n} = icc_{\mathrm{g}} k \cos \beta \frac{1-R}{1+R} \psi$$

(except for full reflection, R=1, where the equation degenerates into a 0 flux condition)

→ boundary condition depends on the solution!

The wave direction

$\psi = A \exp(i\,\chi)$

since $\chi = \mathbf{k} \bullet \mathbf{x}$

$\mathbf{k} = \nabla\chi = \nabla(\text{-i phase}(\psi))$ (*)

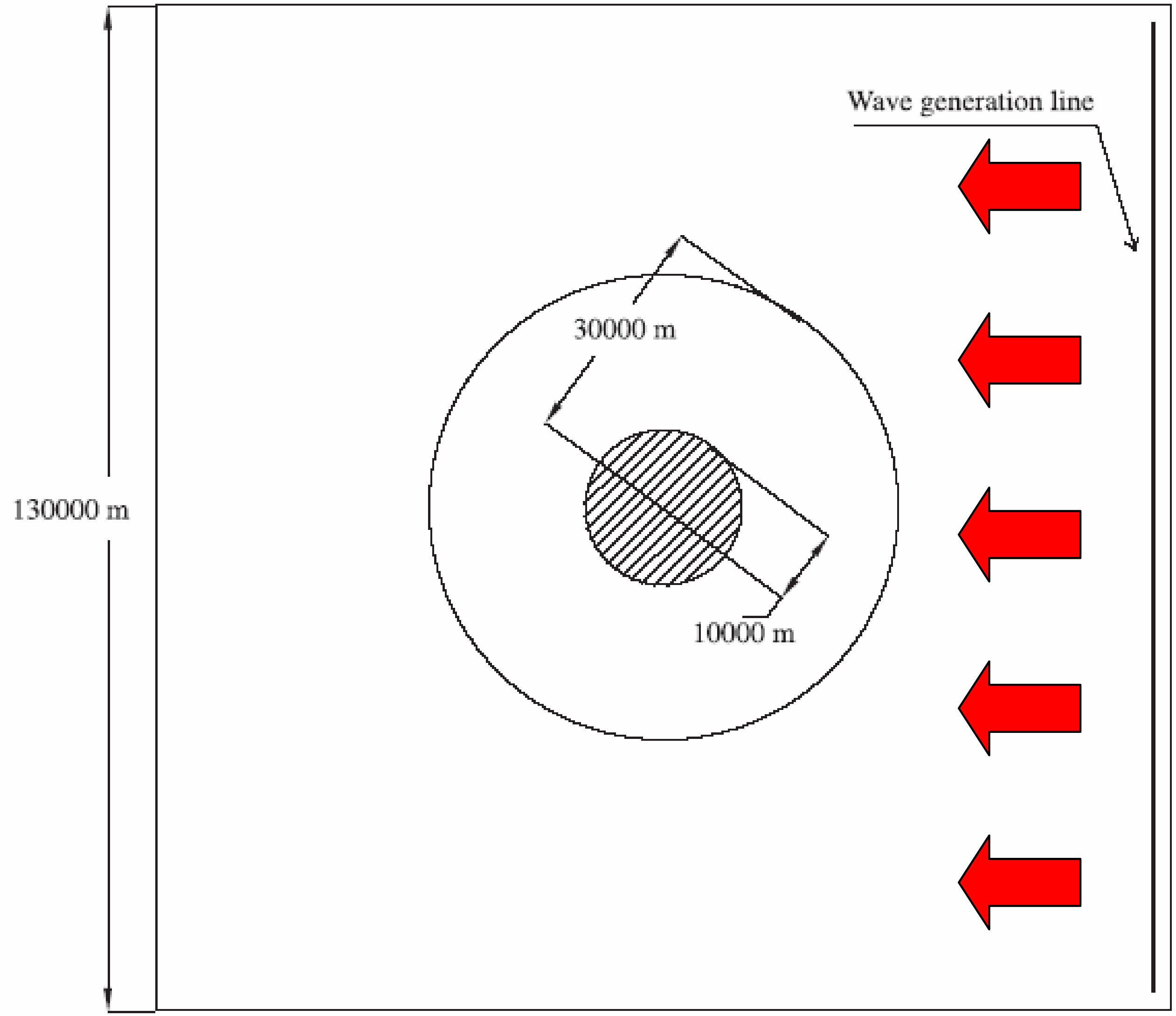
β is the direction of vector k with respect to the boundary

k is not computed correctly by eq. (*)
(spatial derivative!)

$$\nabla\psi = \nabla A e^{i\chi} + i\bar{\mathbf{k}} A e^{i\chi}$$

$$\Rightarrow \bar{\mathrm{k}} = i\left(\frac{\nabla A}{A} - \frac{\nabla\psi}{\psi}\right)$$

Model validation



OUTLINE

MOTIVATIONS

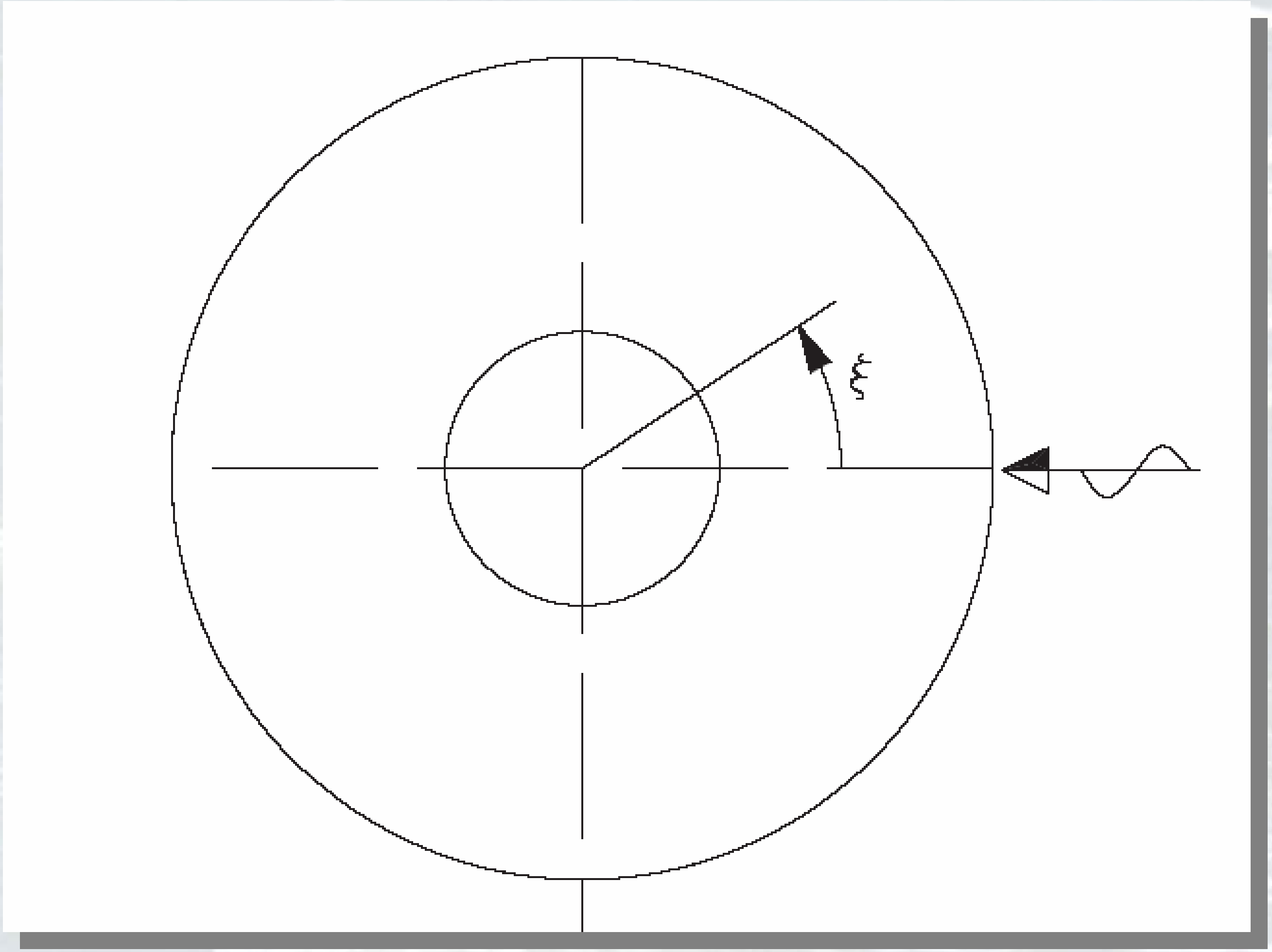
OBJECTVES

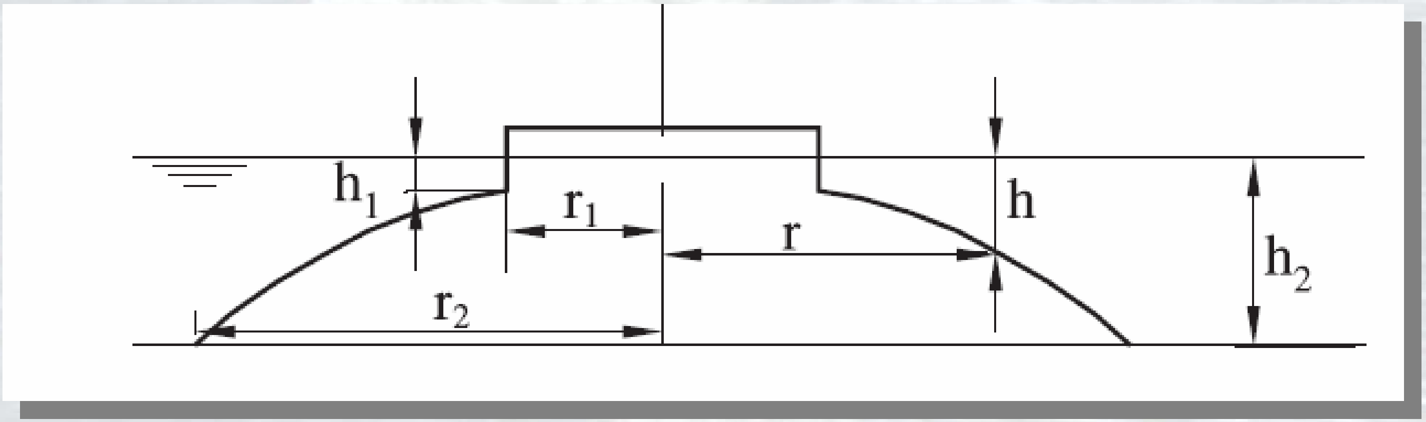
METHODS

MODEL VALIDATION

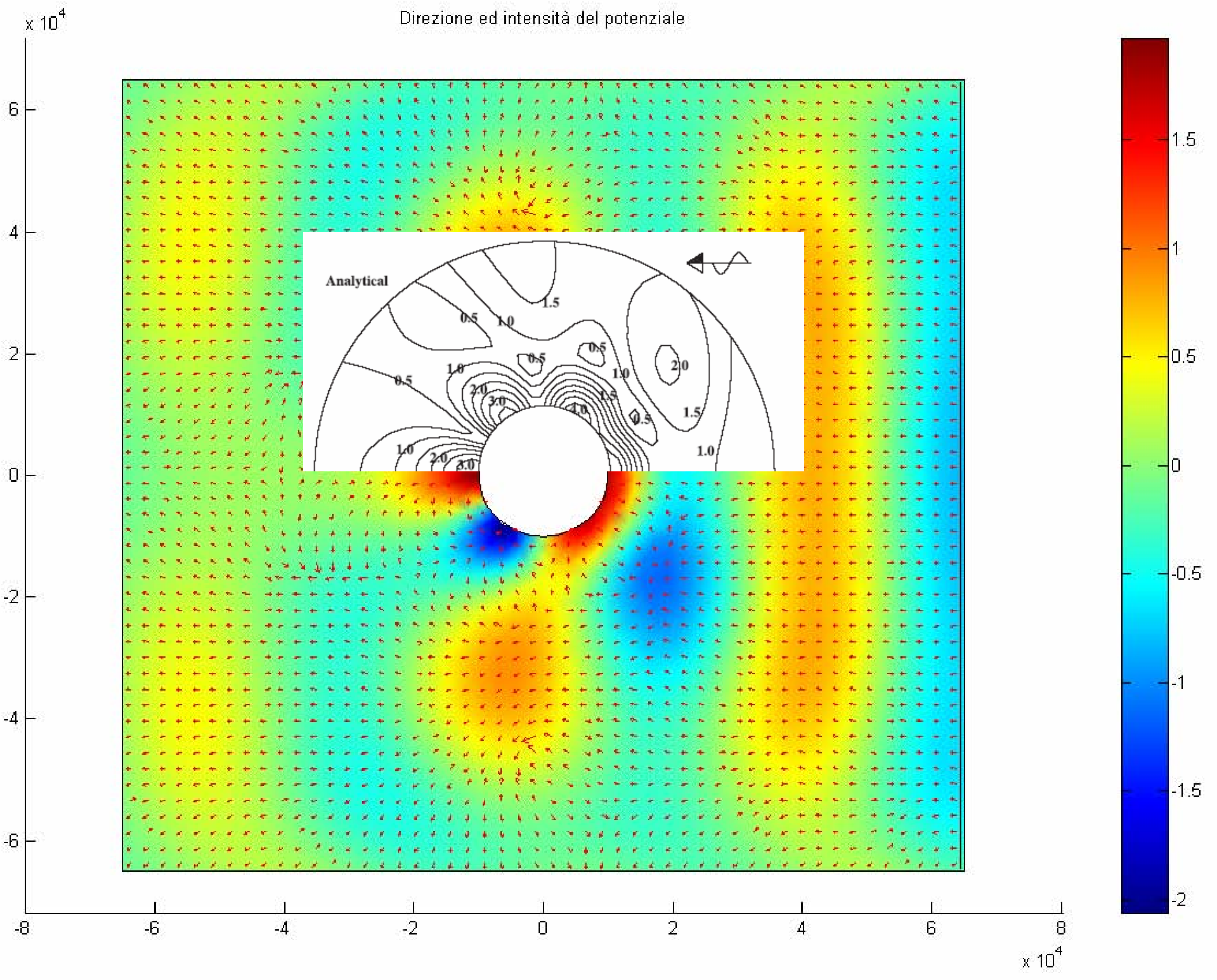
APPLICATION

CONCLUSIONS





Model validation: refraction/diffraction



Application: Casal Borsetti marina

OUTLINE

MOTIVATIONS

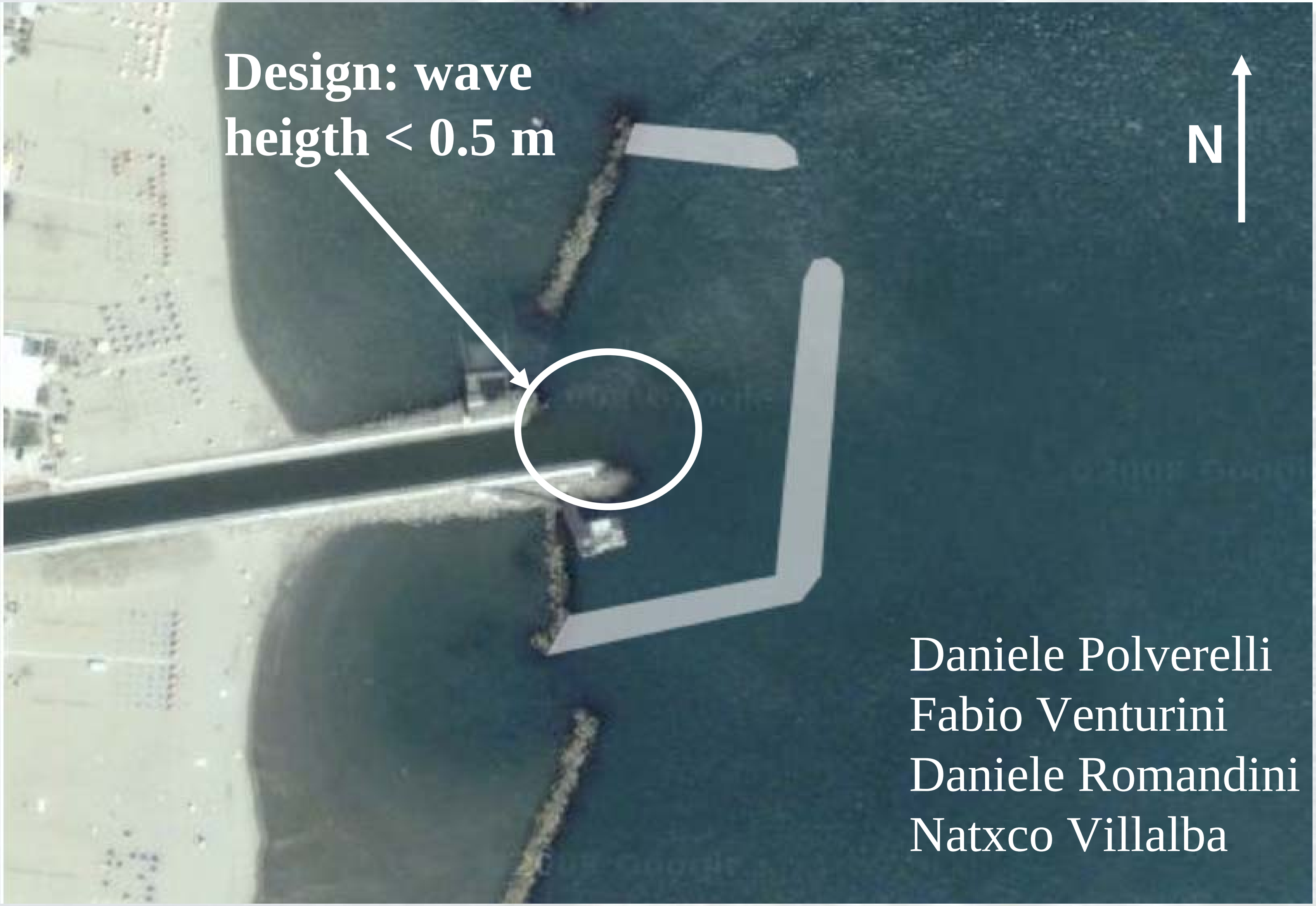
OBJECTVES

METHODS

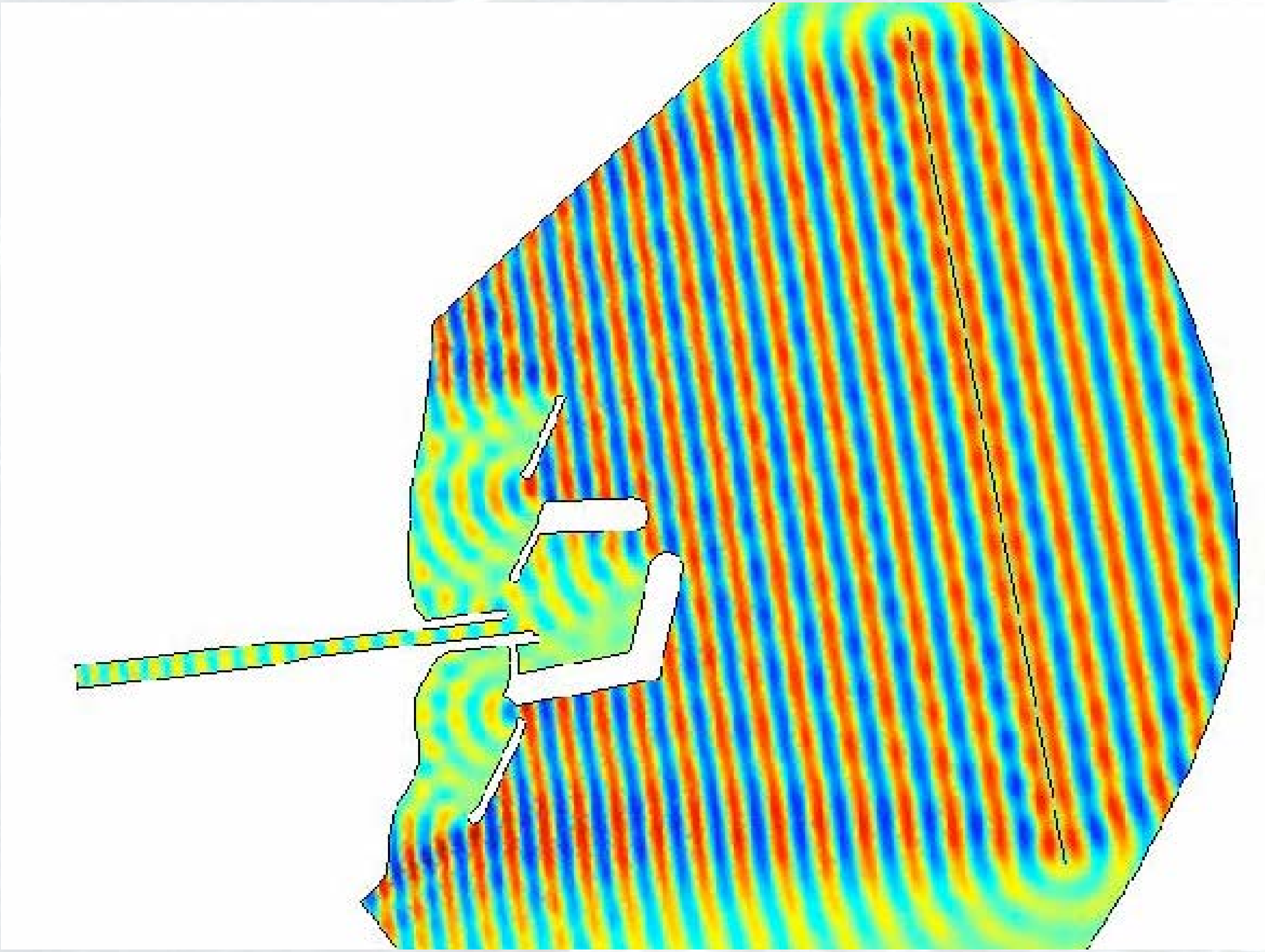
MODEL VALIDATION

APPLICATION

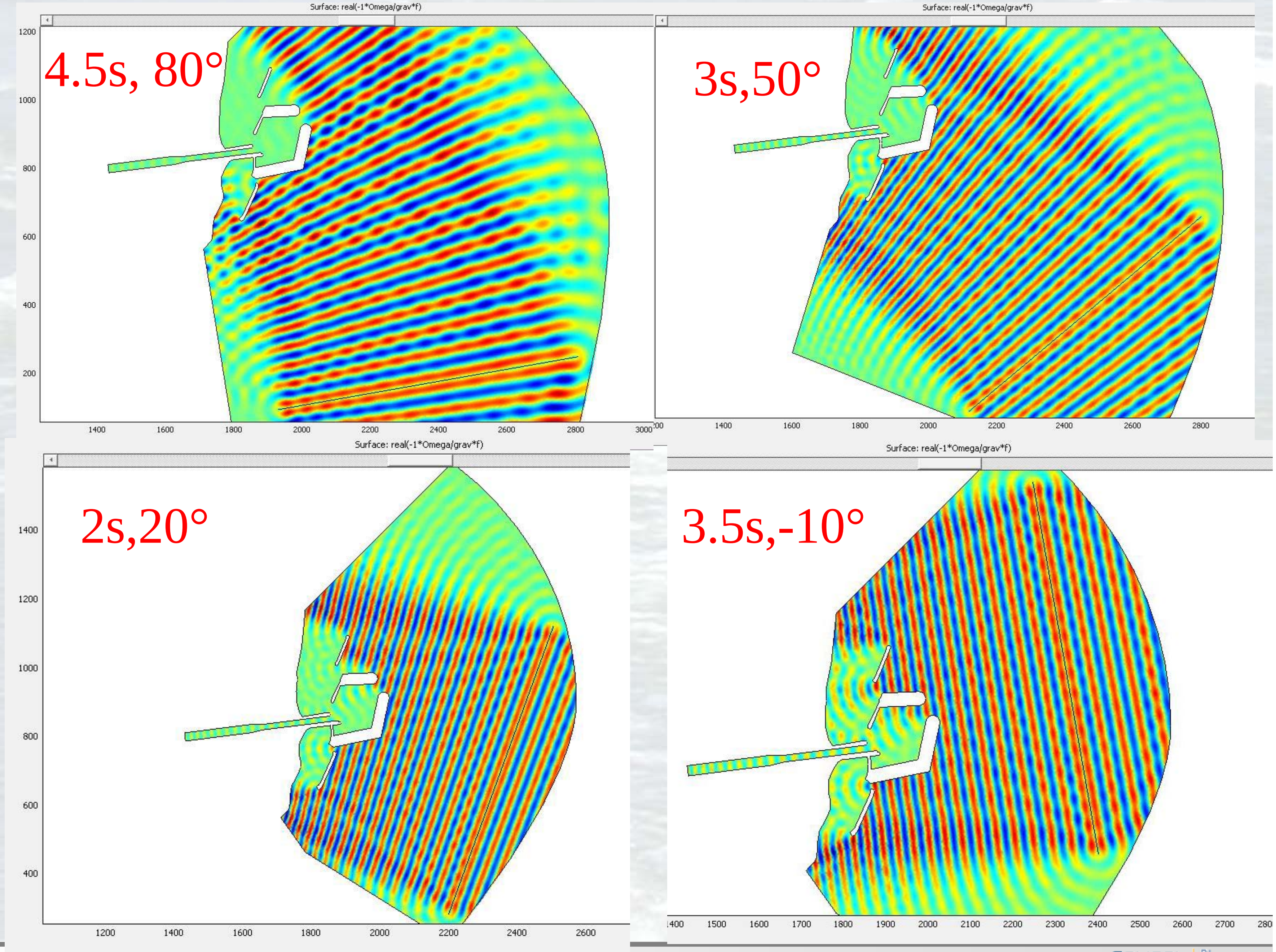
CONCLUSIONS



Example: -30°, 4 s



All direction and periods tested



Max wave in Casal Borsetti marina

	Ho	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
An-golo	θrel										
30	-70	Ver.	0,343	0,567							
60	-40	Ver.	Ver.	Ver.	0,387						
90	-10	Ver.	Ver.	Ver.	Ver.	0,344					
120	20	Ver.	Ver.	Ver.	Ver.	0,254	0,39				
150	50	Ver.	Ver.	Ver.	Ver.	Ver.	Ver.	Ver.	Ver.	Ver.	0,199

Blue shaded cells: waves break at the outer port entrance (navigation not allowed)

Gray shaded cells: breakwater does not protect inner entrance: this occurs 2.17% of the year

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OUTLINE

MOTIVATIONS

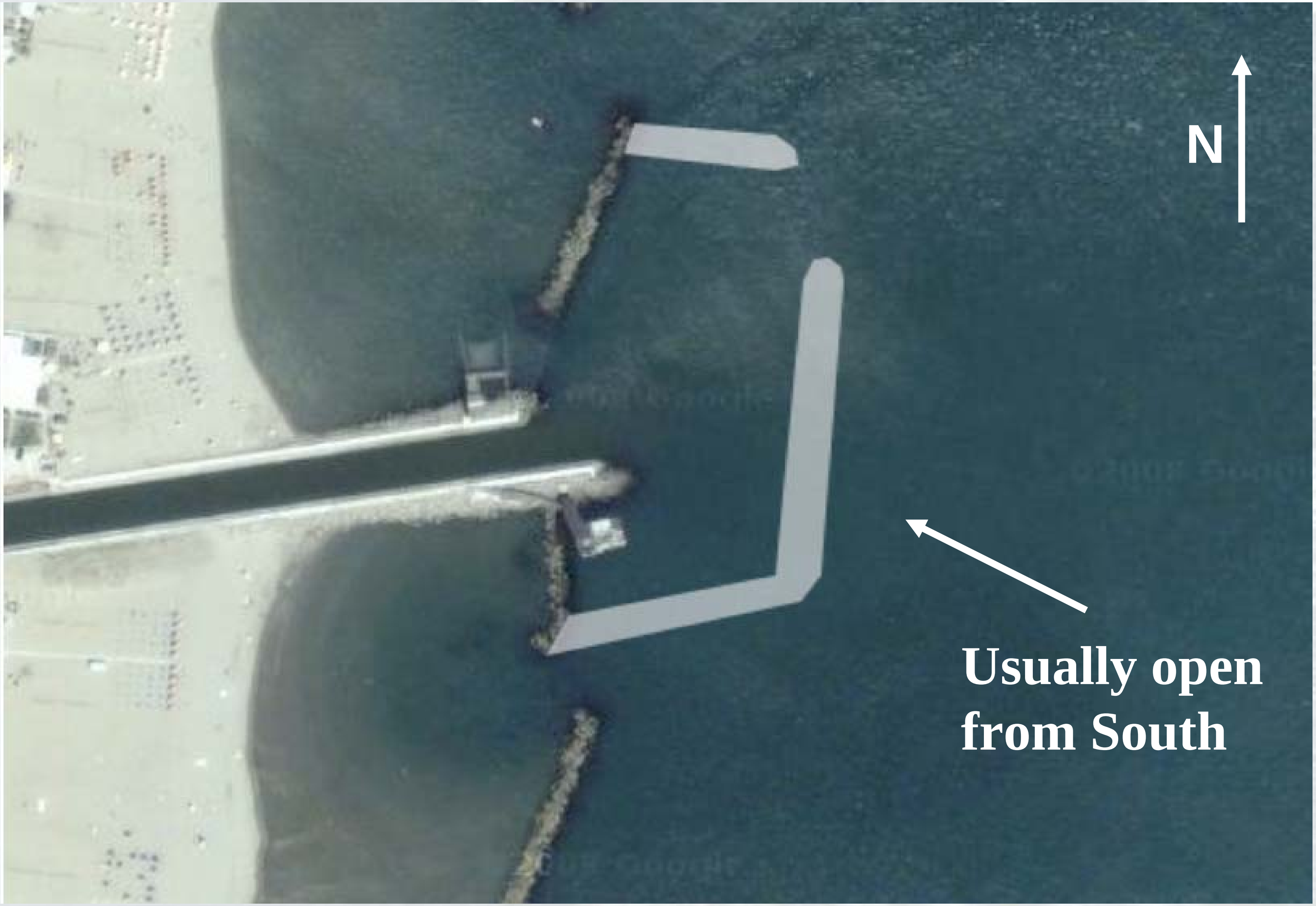
OBJECTVES

METHODS

MODEL VALIDATION

APPLICATION

CONCLUSIONS



OUTLINE

MOTIVATIONS

OBJECTVES

METHODS

MODEL VALIDATION

APPLICATION

CONCLUSIONS

Conclusions

- The MSE can be easily programmed in Comsol multiphysics.
- Some suggestions are given to walk around minor difficulties in the programming phase:
  - how to generate internal waves (with a weak term);
  - how to solve the implicit wave dispersion relationship;
  - a robust way to define wave direction, necessary for the iterative procedure.
- The method is validated against several benchmarks and applied to a realistic case: the design of Casal Borsetti marina.