Analytical Solution for the Steady Poroelastic State under Influence of Gravity

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Introduction

Poroelastics combines the fields of hydraulics and mechanics. Hydraulics and mechanics are linked in a two-way coupling, also referred to as HM coupling. If gravity as outer force is considered, the zero deformation does not match with the hydrostatic no deformation state anymore. Instead a quadratic solution for the vertical deformation is obtained [1]. Here the vailidity of the solution within a usual geomechanical parameter set is discussed.

Analytical Solution

The 1-dimensional description of the geostatic state, using the common linear constitutive stress-strain relationship, is given by (see: [2,3]):

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\cdot\frac{\partial^2 w}{\partial z^2}=\alpha\frac{\partial p}{\partial z}+\rho g$$

with parameters Young modulus *E*, Poisson ratio *v*, the Biot coefficient *α,* density of the fluid/solid porous system *ρ* and acceleration due to gravity *g*. The dependent variable *w* is the deformation in vertical direction. The solution for w that corresponds with the linear hydrostatic state is given by (see [1]):

$$w(z)=\frac{(1+\nu)(1-2\nu)}{2E(1-\nu)}\left(-\alpha\rho_f+\rho\right)gz\left(z-2H\right)$$

with $\rho=\theta\rho_f+\rho_s$.

The analytical solution can be utilized in various ways in numerical modeling of poroelastic systems. Here we explore the applicability of the solution in comparison to the reference geomechanical parameter set, given in Table 1.

Parameter	**Value [unit]**	**Parameter**	**Value [unit]**
Young modulus *E*	100 [MPa]	Biot parameter *α*	1
Poisson ratio *v*	0.25	Porosity *θ*	0.25
Fluid density ρ_f	1000 [kg/m^3]	Gravity acceleration *g*	9.81 [m/s^2]
Bulk density ρ_b	2500 [kg/m^3]	Thicknesses H_0	variable

Table 1. Reference geomechanical parameter set

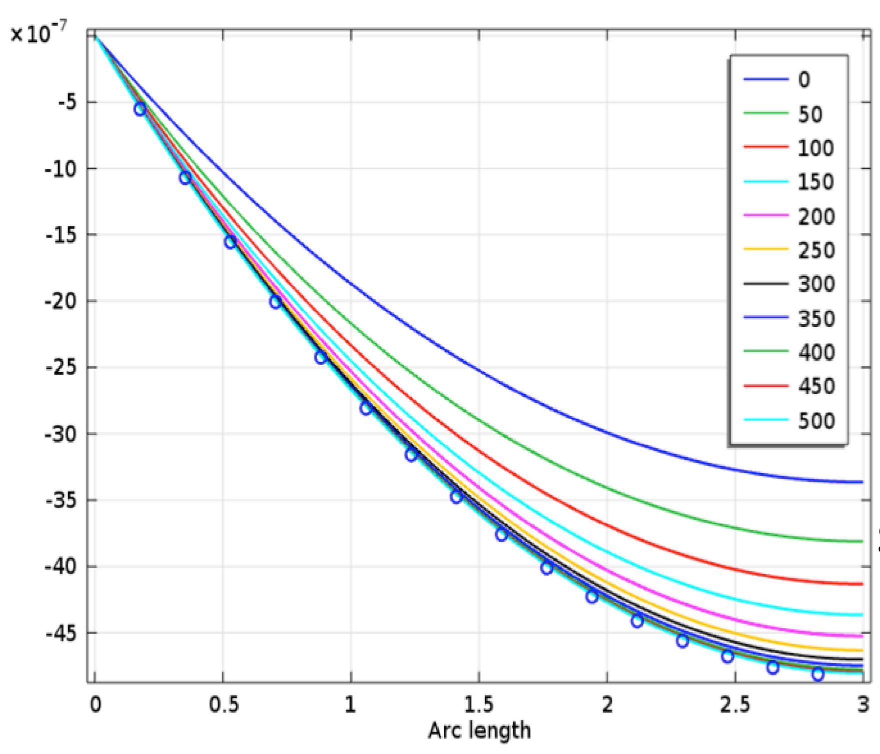

Figure 1. Simulation of transient poroelastic solutions (lines) converging to the analytical solution (circles); calculated using COMSOL Multiphysics

Results: Figure 2 shows that for higher values of the geostatic thickness the relative maximum deformation increases. For layer thicknesses under influence of gravity up to 3500 m the analytical formula delivers reasonable values. If the maximum de-formation exceeds the layer thickness the solution delivers complex values.

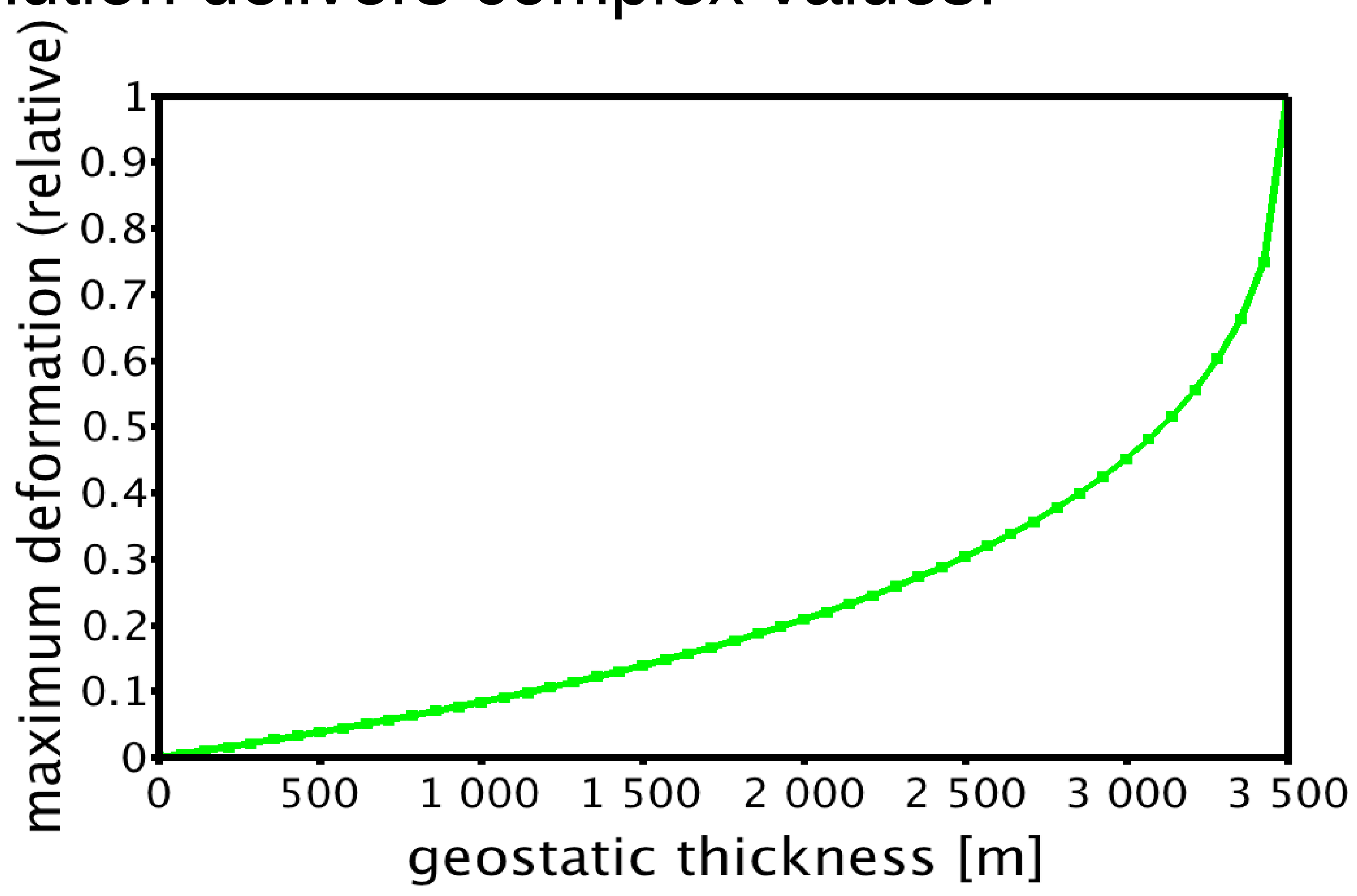

Figure 2. Maximum deformation (normalized) in dependence of thickness H_0 for the reference parameter set

In the corresponding conference paper I examine the change of the graph in Figure 2 in dependence of *v, E* and *θ.*

Conclusion

The analytical solution for the geostatic steady state under influence of gravity delivers a deformation profile that correspondents with the hydrostatic profile of pore pressure. The presented formula be utilized for various purposes, such as

- analytical solution for steady states
- initial state in unsteady simulations
- boundary condition at vertical edges
- comparison of steady states
- comparison of steady and unsteady states
- benchmark for code developers

It it shown that the proposed approach is valid for the usual parameter range of real geological applications.

References

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3. H. Wang, *Theory of Linear Poroelasticity*, Princeton Univ. Press, Princeton (USA) (2000)