Coupled Structural and Magnetic Models: Linear Magnetostriction in COMSOL

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Abstract: Accurate modeling of magnetostrictive materials and devices requires coupling of electrical, magnetic, mechanical, and possibly acoustic domains. There are relatively few finite element software packages that include all these physical models and even fewer that include magnetostrictive models. Comsol Multiphysics was used to create linear magnetostrictive models with fully coupled physics. Two-dimensional and three-dimensional models were generated using the AC/DC and structural mechanics modules. Coupling between the domains was implemented by adding appropriate terms to the subdomain variables for stress and magnetic field. Results from models compare favorably with experimental data for a Terfenol-D transducer. Impedance, phase, and displacement data showed very good agreement between COMSOL and experiments.

Keywords: magnetostriction, Terfenol-D, Galfenol, magnetomechanical coupling, transducer

1. Introduction

Materials which exhibit magnetostriction inherently couple magnetic and mechanical domains. Some materials, such as Terfenol-D and Galfenol, exhibit extraordinary amounts of magnetomechanical coupling and are used in applications ranging from sonar transducers and machine tools to MEMS sensors. The Joule effect$^1$ also known as the direct effect is the change in shape of the material in response to an external magnetic field. The Villari effect$^1$ is the inverse where an applied stress or strain causes a change in the magnetic state of the material. A fully coupled magnetostrictive model captures both types of behaviors. In order to evaluate and improve transducer designs, multiphysics models which accurately represent the coupled behavior of magnetostrictive materials are needed.

Typical magnetostrictive transducers consist of the active material (e.g. Terfenol-D, Galfenol, and nickel), coils, magnets, and magnetic flux carrying materials all comprising the magnetic circuit. In addition, there may be mechanical components which do not contribute to the magnetic performance. In order to optimize a transducer design, the entire magnetic circuit and coupling between the magnetic and mechanical domains must be included in models.

A typical starting point for modeling magnetostrictive materials are linear coupled magnetostrictive equations in terms of stress, strain, magnetic field, magnetic flux density, and constant material properties. These equations provide the basis for the coupled multiphysics model implemented in COMSOL.

There are commercial finite element packages which include magnetostriction, for example Atila. However, the magnetic and mechanical modeling capability of these software packages is somewhat limited. The use of COMSOL allows for detailed nonlinear magnetic analysis and detailed mechanical (stress and modal) analysis with subsequent coupled modeling using linear magnetostriction.

The models presented here are not limited to linear material behavior if equations or tables of properties are included that capture the nonlinear coupled behavior. While some authors$^2$ have implemented nonlinear magnetostrictive models in COMSOL, extremely long computation times and issues with full magnetostrictive coupling have limited the usefulness of these models. The linear coupled model provides a quick and efficient method of analyzing transducers and can be expanded to include nonlinear effects as the design optimization progresses.

2. Linear Magnetostrictive Equations

Reviewing the stress and magnetic variables for the structural and magnetic modes, the coupling terms can easily be implemented into the stress and magnetic field variables. In order to determine the correct terms to add into these variables, the linear magnetostrictive equations are written in the form:
These are full 3D equations where $T$ is stress, $c^B_S$ is the compliance matrix with constant magnetic flux density, $S$ is strain, $h$ and $h_t$ are the appropriate magnetostrictive coupling coefficients, $H$ is magnetic field, $B$ is flux density, and $\gamma^S$ is the inverse of permeability. More details on the specific terms can be found in the IEEE standard on piezomagnetic nomenclature.3

3. Use of COMSOL Multiphysics

The coupled magnetostriction model uses the Structural Mechanics module (SM) and the AC/DC module, although it could also be implemented with the AC/DC module and the Acoustics module. Magnetostriction is incorporated by modifying the stress and magnetic field variables with the appropriate coupling terms as shown in Equation 1. Terms including magnetic flux density, $-h_t B$, are added to stress variables and terms including strain, $-h_S$, are incorporated into the magnetic field variables.

For implementation in a 2D axisymmetric problem, the Axial Symmetry, Stress-Strain option in the SM module is used as the ruling application mode. The Azimuthal Induction Currents, Vector Potential option in the AC/DC module is used in order to include eddy current effects and provide for current input to a coil. Figure 1a shows a simple annular cylinder of magnetostrictive material surrounded by a coil and air moving in response to current in the coil. Figure 1b shows flux density induced in the material caused by an applied stress. These simple models show that the coupling works for both the Joule effect and the Villari effect. The problem has also been implemented in 3D, although these results are not discussed.

For the model shown in Figure 1, the coil and air are not active in the structural model; however, all three subdomains are active in the magnetic model. In order to identify resonance and characterize the response across a frequency band, a harmonic analysis is used to “sweep” the transducer. Impedance of a transducer can be calculated with the use of additional constants, expressions, and variables.

Figure 1. Simple annular ring of magnetostrictive material surrounded by a coil and air response to (a) current in the coil (Joule effect) and (b) stress applied to the material (Villari effect).
Two basic approaches for calculating impedance are: input a constant voltage or input a constant current. Table 1 shows the additional constants, expressions, and variables for both methods in a 2D axisymmetric model.

**Table 1:** Additional constants, expressions, and variables for constant voltage or constant current input for calculating impedance.

<table>
<thead>
<tr>
<th>Item</th>
<th>Constant voltage</th>
<th>Constant current</th>
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</table>
| **Constants** | V0 – input voltage  
N – number of coil turns  
R – DC resistance | I – input current  
N – number of coil turns  
R – DC resistance |
| **Global Expressions** | I= -(V0-Vi)/R, current in coil  
Z= V0/I, impedance of coil | V0= Vi-I*R, voltage in coil  
Z= V0/I, impedance of coil |
| **Subdomain Integration Variables** (coil subdomain) | Vi=2*pi*r*Ep_hi_emqa*N/Ac, induced voltage in coil | Vi=2*pi*r*Ep_hi_emqa*N/Ac, induced voltage in coil |

### 4. Transducer Model Results

A existing Terfenol-D transducer was modeled with a 2D axisymmetric representation and a 1V input to the coil. Geometry of the transducer is shown in Figure 2. A harmonic solution from 10-20 kHz was performed in order to capture the resonance around 15.5 kHz. The magnetic flux density and flux lines are shown in Figure 3 for 15.5 kHz. The magnitude of B-field is on the order of 0.2 mT, which meets the small signal requirement of the linear magnetostrictive equations.

Experimental data from the transducer in the form of impedance and displacement per unit input were compared with results from COMSOL. Figure 4a compares the magnitude of impedance, 4b compares the phase of the impedance, and 4c compares the displacement per input volt. The impedance, phase, and displacement show quite good agreement between the experiment and model. While there are some discrepancies in magnitude, the overall transducer behavior was captured remarkably well.

Discrepancies between the model and experiment are likely due to several causes. Material property data for Terfenol-D is based on specific preload and magnetic bias conditions which do not match the experimental conditions. Damping in the model is another factor that must be estimated from experimental data and cannot be known *a priori*. Given these limitations, the results are very promising.
Figure 4. Comparison of experimental results and COMSOL models of a Terfenol-D transducer (a) magnitude of impedance, (b) phase of impedance, and (c) displacement per input volt.

5. Conclusions

The linear coupled magnetostrictive modeling methods that have been developed provide a good modeling tool for development of Terfenol-D transducers. Comparisons between experimental and model results were remarkably good with only some small discrepancies in magnitude. These modeling techniques can readily be used to optimize magnetostrictive transducer designs. Using the 2D models presented in this paper, further enhancements including thermal effects can easily be implemented. By formulating the magnetostrictive coefficients as tables or equations, nonlinear behavior of magnetostrictive materials could also be added. Three-dimensional models have also been implemented using the same techniques used in this paper. Full 3D models allow analysis of much more complex geometry such as that found in Galfenol transducers.

Future efforts will focus on extending the existing models to other transducers, including thermal effects, and using nonlinear properties. A magnetostrictive “template” model will be developed to ease implementation of the coupled terms in the stress and magnetic field variables.

6. References


7. Acknowledgements

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