Definition of Optimization Problem for Electromagnetic Linear Actuator

P. Piskur*, W. Tarnowski, K. Just
Koszalin University of Technology, Poland
*Corresponding author: Department of Mechatronics, Nanotechnology and Vacuum Technology; Koszalin University of Technology; 75-620, Koszalin, Poland; email address: pawel.piskur@interia.pl

Abstract: In this paper a poly-optimization of the design of the electromechanical actuator is presented [5]. The shape of the actuator is defined by the decision variables. The number of decision variables under consideration is up to ten but in the next step while the multi-coils system will be analyzed the number of decision variables will increase up to hundred, so the genetic algorithm has been used.

The genetic algorithm program has been implemented in the Matlab program which operates together with the Comsol Multiphysics package. For each decision variable vector generated in Matlab program the electromagnetic force has been calculated in Comsol Multiphysics by using FEM (finite element method).

The aim of the optimization process [1] is to find the maximal electromagnetic force and the minimal mass of all the device. The shape of the coil and the current density in the coil are taken to be constant. Three kind of the construction ferromagnetic case have been taken under consideration. The final result is a set of the Pareto optimal solutions, which makes possible to draw out some more general conclusions on a design of the actuator.

Keywords: electromagnetic linear actuator, optimization, Pareto-optimal solution, genetic algorithm.

1. Introduction

Electromagnetic actuators are commonly used for various purposes, the main drawback however is the low energy efficiency and the large dimensions and mass of the device. In the case of an electromagnetic linear launcher, there is a set of coils, displayed in series. However, in this paper for a preliminary analysis there is only one coil system under consideration (Figure 1). The system consists of one cylindrical coil and a ferromagnetic plunger. The coil is supplied by a constant voltage impulse of a finite period of time. The electromagnetic field is pulling the plunger inside the coil.

Figure 1. The cross section view of the electromagnetic linear actuator

A former analysis of the electromagnetic linear actuators without ferromagnetic case [6] shows that the electromagnetic force is ten times lower than for the electromagnetic linear actuators with ferromagnetic case, and is to small to proposed application. In Figure 2 the electromagnetic force is depicted as a function of the plunger displacement for both solutions. The maximal value of the ferromagnetic force is about 1 Newton for coil surrounded by air (brown color line) and about 24 Newtons for coil covered by the ferromagnetic case (green color line).

Figure 2. Comparison between the electromagnetic force for coil with and without ferromagnetic case (see text)
In the optimization process three kinds of the device shape have been taken under consideration.

The first one is shown in Figure 3 and consists of three ferromagnetic rings which thickness is defined as decision variables \( p(1) \div p(3) \). The length of the ferromagnetic plunger is defined as \( p(4) \).

![Figure 3. Cross sectional view of the half of the ferromagnetic actuator with four decision variables](image)

The second version of geometry is defined by the decision variables \( p(1) \div p(5) \) presented in Figure 4. The length of the ferromagnetic plunger is defined as \( p(6) \).

![Figure 4. Cross sectional view of the half of the ferromagnetic actuator with six decision variables](image)

The second version of geometry is defined by the decision variables \( p(1) \div p(5) \) presented in Figure 4. The length of the ferromagnetic plunger is defined as \( p(6) \).

![Figure 5. Cross sectional view of the half of the ferromagnetic actuator with nine decision variables](image)

In Figure 5 another example of electromagnetic linear actuator with seven decision variables is shown. The eight decision variable defines the length of ferromagnetic plunger.

There is much more combinations of shape of the ferromagnetic linear actuator, but the main ones have been chosen to determine the relationship between the shape and the ferromagnetic force. Another question was the device manufacturing process difficulties.

2. Governing Equations

The problem of electromagnetic analysis is to solve the Maxwell’s equations subjected to certain boundary conditions. Maxwell’s equations are written in a differential or integral form, defining the relationship between the fundamental electromagnetic quantities. The differential form are presented here, because it leads to differentials equations that the finite element method can handle [2]. Here only the second and the fourth Maxwell’s equation have been used:

\[
\nabla \times H = J + \frac{\partial D}{\partial t} \quad (1)
\]

\[
\nabla B = 0 \quad (2)
\]

Where:
$B$ – is the magnetic flux density [T];
$H$ – is the magnetic field intensity [A/m];
$D$ – is the electric flux density [C/m$^2$];
$J$ – is the current density in the coil [A/m$^2$].

The quasi-static analysis is made under the assumption that:

$$\frac{\partial D}{\partial t} = 0 \quad (3)$$

This implies that the equation (1) can be rewritten in the following manner:

$$\nabla \times H = J \quad (4)$$

If we assume the electromagnetic field is stationary then:

$$\frac{\partial B}{\partial t} = 0 \quad (5)$$

That is, if the field is varying so slowly that we can neglect the contribution from induced currents. We also assume that the modeled object is not moving: $v = 0$, so that there is no contribution from Lorentz forces. This assumption is appropriate also, if the ferromagnetic part of the device is made in that way, that the eddy currents are low.

It can be helpful to formulate the magnetic field intensity generated by the current in the coil in terms of the magnetic vector potential $A$ [3]. It is given by the equation:

$$B = \nabla \times A \quad (6)$$

where the relation between the magnetic field intensity and the magnetic flux density is given by the equation:

$$B = \mu \cdot H \quad (7)$$

where:

$\mu = \mu_0 \cdot \mu_r$;
$\mu_0 = 4 \cdot \pi \cdot 10^{-7} \ [H/m]$ is the permeability of vacuum.
$\mu_r = f\left(\frac{B}{H}\right)$ – is the relative permeability of the material.

Assuming static currents and fields, the magnetic vector potential $A$ must satisfy the following equation:

$$\nabla \times (\mu^{-1} \nabla \times A) = J \quad (8)$$

In Figures 6÷9 magnetic potential $A$ is depicted as a result of the finite element method computation process.

The force is computed on the electromagnetic energy $W$ analysis of the system with respect to the small displacement. The method of a virtual work utilizes the fact that under constant magnetic flux condition, the total magnetic force on a system is computed as: $Fe = -\nabla W$

### 3. Theory

**The independent variable (an operand):** in the optimization process it is the displacement of the ferromagnetic plunger.

**Values to be searched:** dimensions variables $p(1)\ldots p(n)$.

**Given data:** The dimensions of coil (length and width = 2 x 2 [cm]) and the inner radius equal to 1 [cm]. The outer radius of a plunger equals to 0.9 [cm]. The difference between both radii is the air gap. The coil is supplied with 3 [A] current.

**The main optimization criteria:** the first is a kinetic energy of the plunger to be maximal and the other is a total mass of the device to be minimal, with an assumption of the constant electric energy delivered to the device.

The kinetic energy of the plunger is:

$$E_k = \frac{1}{2} m_p v^2$$

where:

$m_p$, $v$ – is the mass and the velocity of the plunger, respectively.

The velocity depends on the electromagnetic force:

$$v = \int \frac{Fe}{m_p} \, dt$$

The mass of the plunger varies depending on the decision variable defining the length of the plunger.

The mass of the device is to be directly proportional to the volume and density ($\rho = \text{const.}$) of the material.

$$m = \rho V$$

where:

$\rho$ – is the density of the material;
$V$ – is the device volume.

**Global criterion:**

$$P = (1 - w)Fe + s(1 - w)m_p + w \cdot m$$

where:

$w$ – is a weigh coefficient
$s$ – is a scaling factor.

**Limitation range:**

The mass of the device: $m < m_{\text{max}}$;
4. Numerical model

The electromagnetic actuator model has been implemented in the Comsol Multiphysics package. Because of the model symmetry, only a half of the device has been taken under consideration.

In Figures 6÷9 the lines of the magnetic potential and arrows of the magnetic flux density have been presented for the various solution of the device shape.

One of the Pareto-optimal solution is presented in the Figure 7, for the dimension (see Figure 3):

- \( p(1) = 0.0040 \) [m]
- \( p(2) = 0.0055 \) [m]
- \( p(3) = 0.0075 \) [m]
- \( p(4) = 0.0575 \) [m]

Electromagnetic force achieves the value:

\( F_e = 39 \) [N]

The total mass of the device:

\( m = 0.89 \) [kg]

Another Pareto-optimal solution is presented in Figure 8, for the following dimensions (see Figure 4):

- \( p(1) = 0.0045 \) [m] \( p(4) = 0.0135 \) [m]
- \( p(2) = 0.0025 \) [m] \( p(5) = 0.0130 \) [m]
- \( p(3) = 0.0035 \) [m] \( p(6) = 0.0475 \) [m]

Electromagnetic force achieves the value:

\( F_e = 46.24 \) [N]

The last one of the Pareto-optimal solutions is presented in the Figure 9, for the following dimensions (see Figure 5):

- \( p(1) = 0.0055 \) [m] \( p(5) = 0.0045 \) [m]
- \( p(2) = 0.0035 \) [m] \( p(6) = 0.0010 \) [m]
- \( p(3) = 0.0115 \) [m] \( p(7) = 0.0030 \) [m]
- \( p(4) = 0.0035 \) [m] \( p(8) = 0.0425 \) [m]
Electromagnetic force achieves the value:
\[ F_e = 41.34 \text{ [N]} \]

The total mass of the device:
\[ m = 0.75 \text{ [kg]} \]

In Figure 10 all the Pareto-optimal solutions are presented for the three kinds of construction shapes. It can be found that the simplest kind of ferromagnetic case (with 3 decision variables) would be enough for the small value of the electromagnetic force (data 1). Then the mass of the device reaches the lowest value. Another advantage is an easy construction.

If the electromagnetic force is the most important optimization criterion (a coefficient weight \( w < 0.5 \)) then the geometry specified by nine decision variables should be applied (Figures 5 and 9, and data 2 in Figure 10).

The design presented in Figure 4 and 8 has the poorest parameters (data 3 in Figure 10).

5. Conclusions

The poly-optimization of electromagnetic devices requires a very accurate simulation model, which has been built in the Comsol Multiphysics program and presented in this paper. A combination of the two programs (Comsol and Matlab) gives the powerful opportunity to analyze the object without building the prototype. Although the Genetic Algorithm needs the high computing power of a computer, it is able to find the global minima.

6. References