Presented at the COMSOL Conference 2010 Paris

COMSOL CONFERENCE 2010
NOVEMBER 17-19 2010, PARIS, FRANCE

Karlsruhe Institute of Technology

Numerical Modeling of Thin Superconducting Tapes

Francesco Grilli[1], F. Sirois[2] and R. Brambilla[3]

[1]Karlsruhe Institute of Technology, Karlsruhe, Germany
[2]Ecole Polytechnique Montréal, Montréal, Canada
[3]Ricerca sul Sistema Energetico - RSE S.p.A., Milano, Italy

ÉCOLE POLYTECHNIQUE MONTRÉAL

RSE Ricerca Sistema Energetico

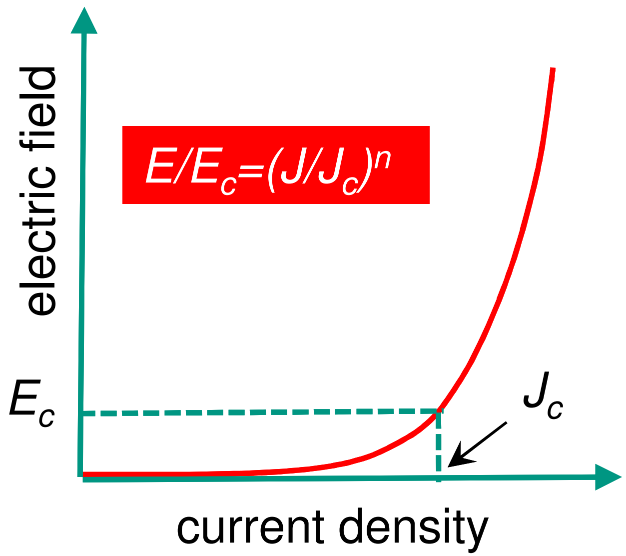
Goal of this presentation

- Present issues of
- Show results obta

superconductors

Superconducting film
Width: 4-12 mm
Thickness: ~1 μm
Aspect ratio: up to 10,000!

Superconductor material
Non-linear behavior
Power-law current-voltage characteristics



Constitutive equations

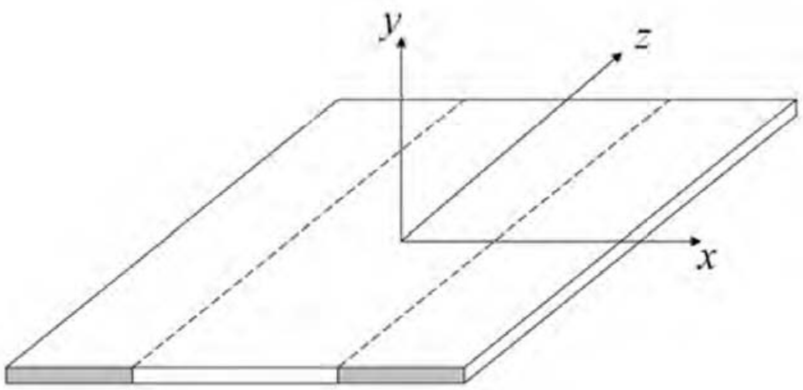
- Faraday's law $$\mu \frac{\partial B}{\partial t} + \nabla \times E = 0$$
- Material properties for HTS $$\begin{cases} \rho(J) = \dfrac{E_c}{J_c} \left| \dfrac{J}{J_c} \right|^{n-1} \\ B = \mu_0 H \end{cases}$$
- Current density $$J = \nabla \times H$$

Francesco Grilli
KIT-ITEP

Equations in 2D

- Faraday's law

$$\begin{cases} \mu \dfrac{\partial H_x}{\partial t} + \dfrac{\partial E}{\partial y} = 0 \\ \mu \dfrac{\partial H_y}{\partial t} + \dfrac{\partial E}{\partial x} = 0 \end{cases}$$



- Expressions for *J* and *E*

$$\begin{cases} J = \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} \\ E = \rho(J)J \end{cases}$$

Implementation in general PDE system

$$\cancel{e_a \frac{\partial^2 u}{\partial t^2}} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = \cancel{F}$$

$$u = \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

$$\mu \frac{\partial H_x}{\partial t} + \frac{\partial E}{\partial y} = 0$$

$$\mu \frac{\partial H_y}{\partial t} + \frac{\partial E}{\partial x} = 0$$

Very important feature: edge elements

- Physics: we want *div(B)=0*
- $B=\mu_0 H$ ➔ *div(H)=0*
- Comsol's curl elements *shcurl*
  - Not only do they impose *div(H)=H1x+H2y=0*
  - They also impose *H1x=0* and *H2y=0*
  - Much more stringent condition than that obtained with Lagrange elements

$$\begin{cases} \mu \dfrac{\partial H_x}{\partial t} + \dfrac{\partial E}{\partial y} = 0 \\ \mu \dfrac{\partial H_y}{\partial t} + \dfrac{\partial E}{\partial x} = 0 \end{cases}$$

Take *x*-derivative

+

Take *y*-derivative

$$\mu \frac{\partial}{\partial t}\left(\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} \right) = 0$$

$$\nabla \cdot H = \text{constant} = 0$$

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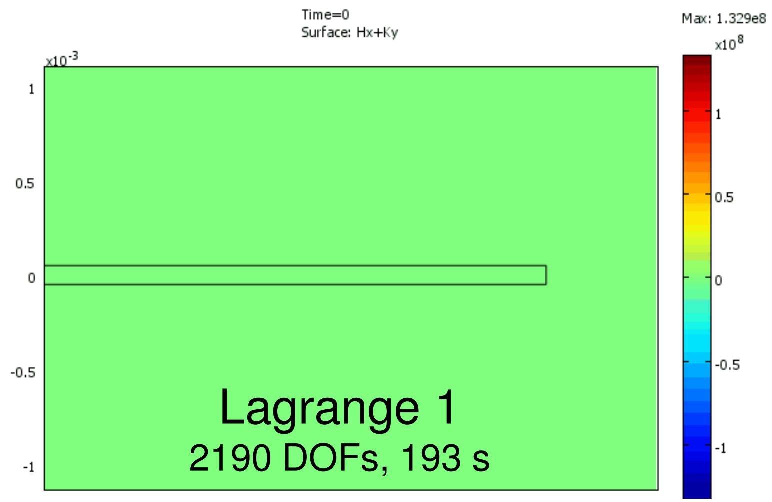
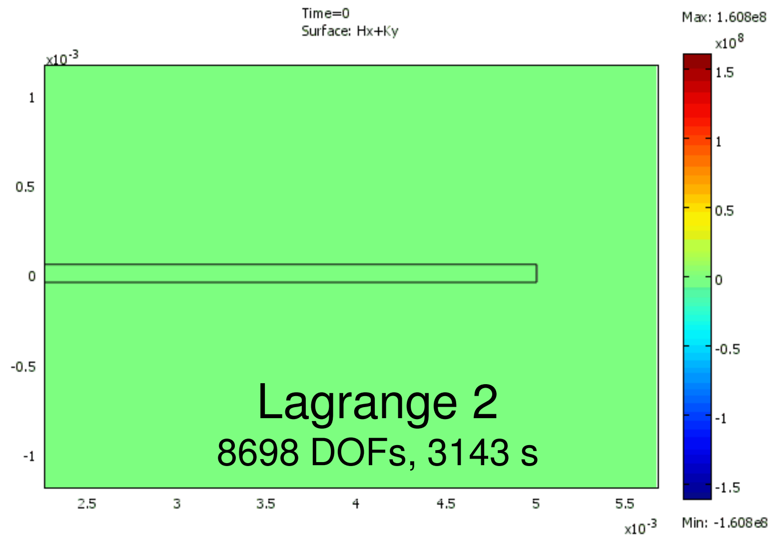
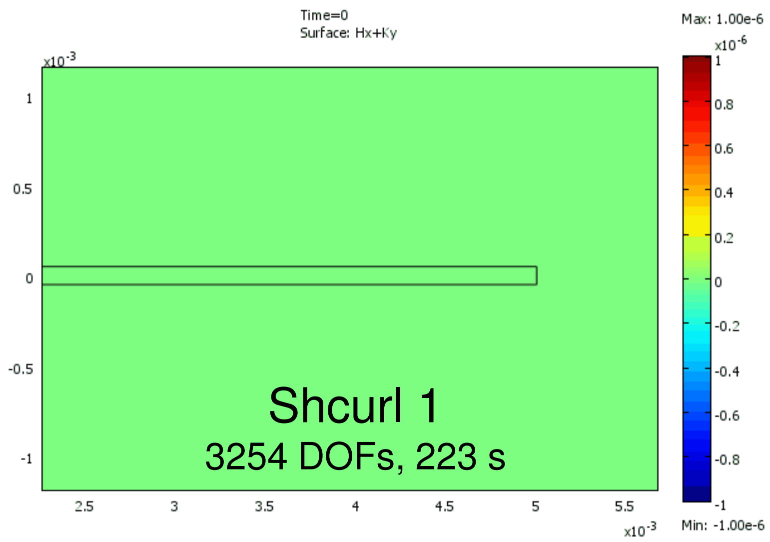
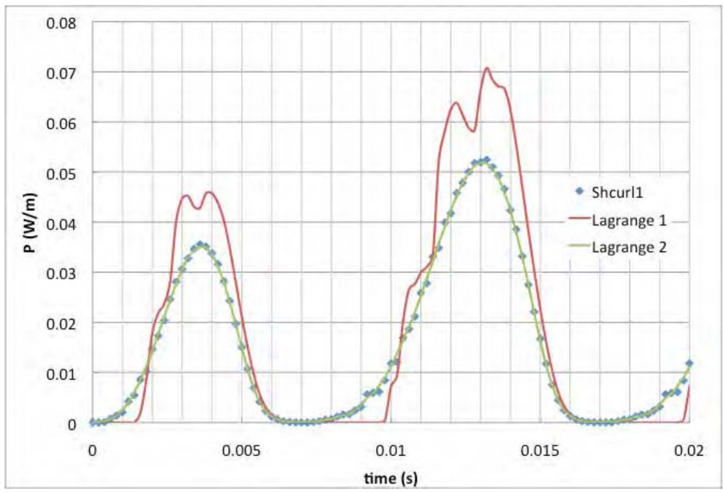
“Drift” of divH with Lagrange elements



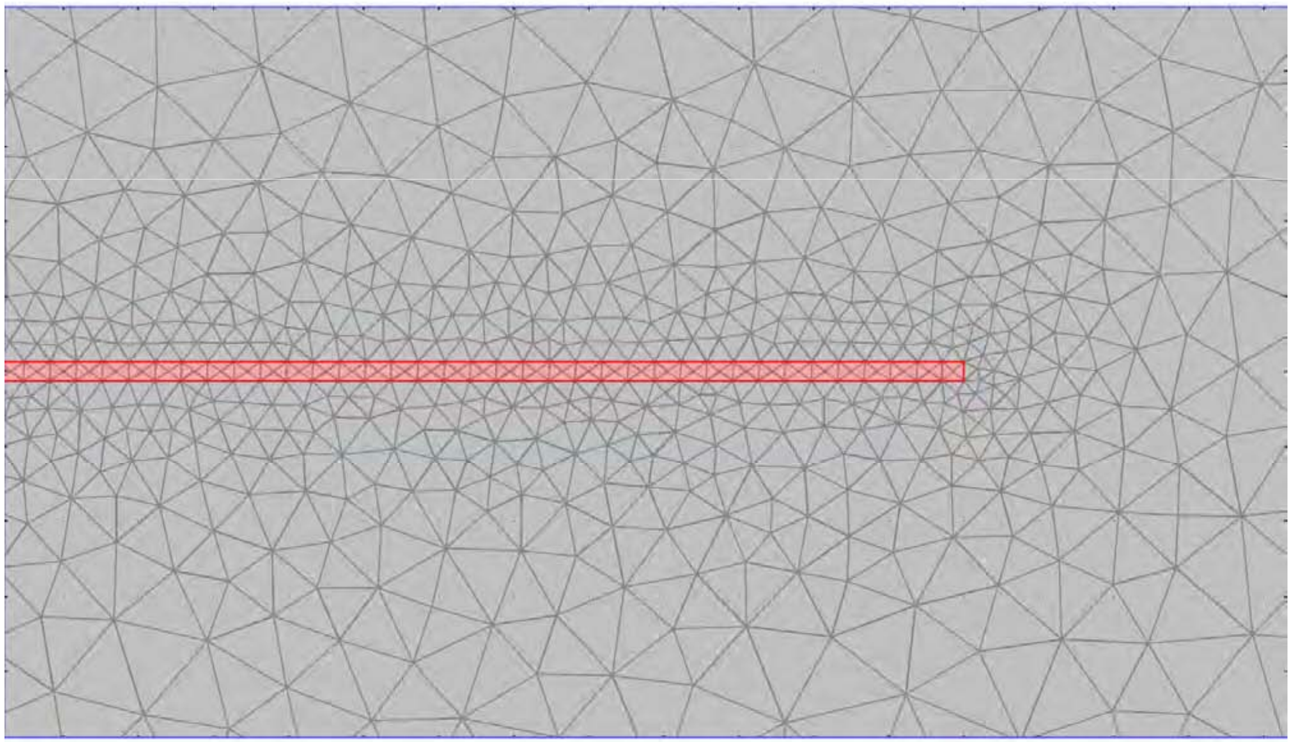






Mesh issues

- Aspect ratio 1,000-10,000
- Large number of mesh elements, even with a ‘coarse’ mesh



Possible solution: increase the thickness

- Increase the thickness, keep I_c constant
- Justification: flux penetration as in infinitely thin tape
- Tape behaves as a 1-D object

Expand thickness by a factor N

Critical current density=J_c
Width=w
Thickness=d
$I_c=J_c*w*d$

Critical current density=J_c/N
Width=w
Thickness=$d*N$
$I_c=J_c/N*w*d*N=J_c*w*d$

Possible solution: increase the thickness

- It works well for an isolated tape
- What happens in case of interacting conductors?
  - Top/bottom losses become important (depend on actual thickness)
  - Expanded thickness may become comparable with tape separation

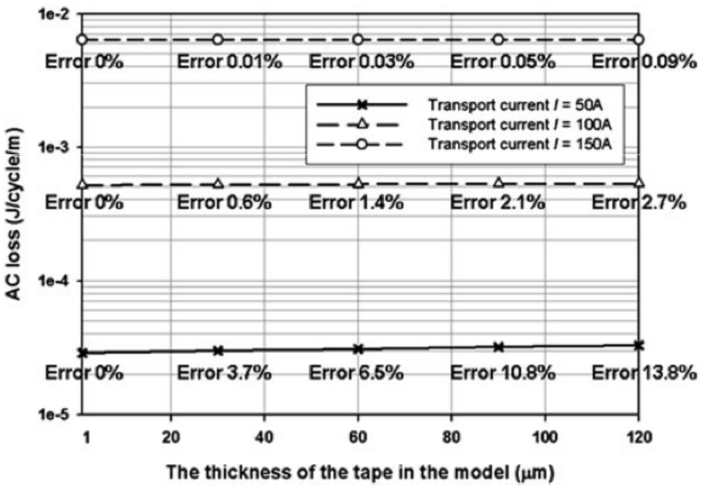


Fig. 4 Values of transport AC losses versus the thickness of the tape. Data shown with transport currents of 50 A, 100 A, and 150 A. The I_c in this case is 120 A. The first point on each line is calculated using the actual geometry of the tape (1 μm). The errors between the first point and other points in each line are shown

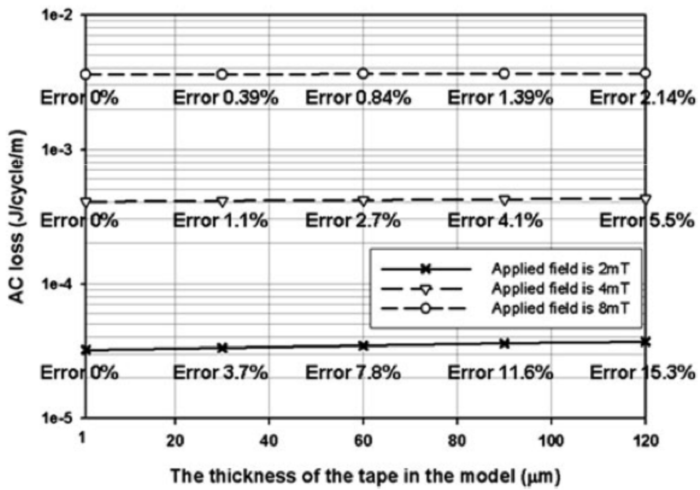
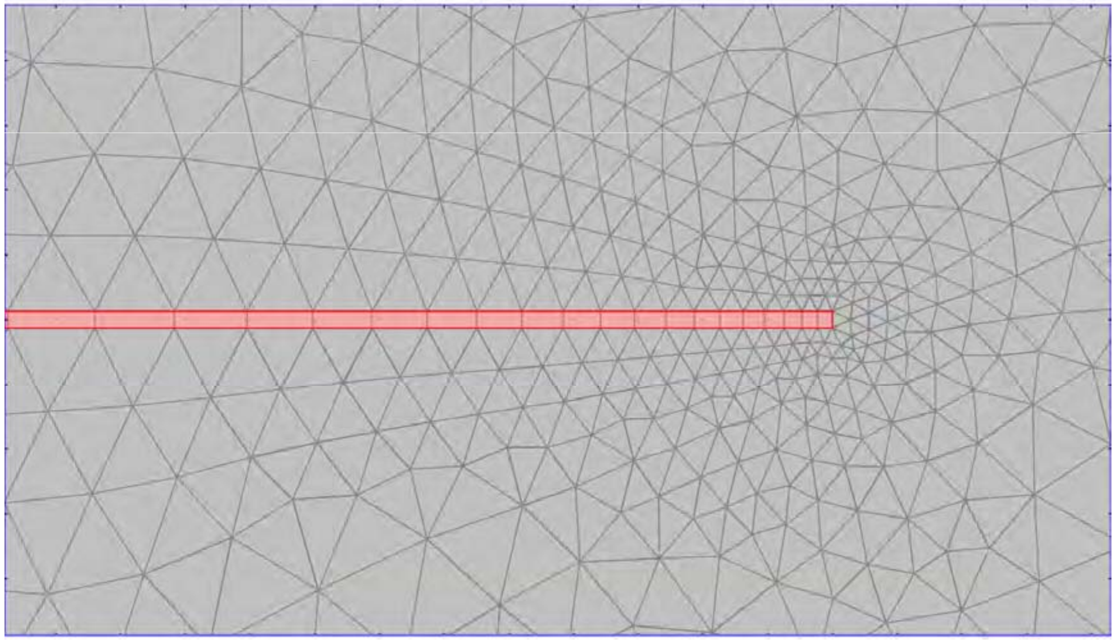


Fig. 6 Values of magnetisation AC losses versus the thickness of the tape. Data shown with the applied field of 2 mT, 4 mT, and 8 mT. The I_c in this case is 120 A. The first point on each line is calculated using the actual geometry of the tape (1 μm). The errors between the first point and other points in each line are shown

Figures taken from Hong and Coombs, J. Supercond. Nov.Magn., 2010

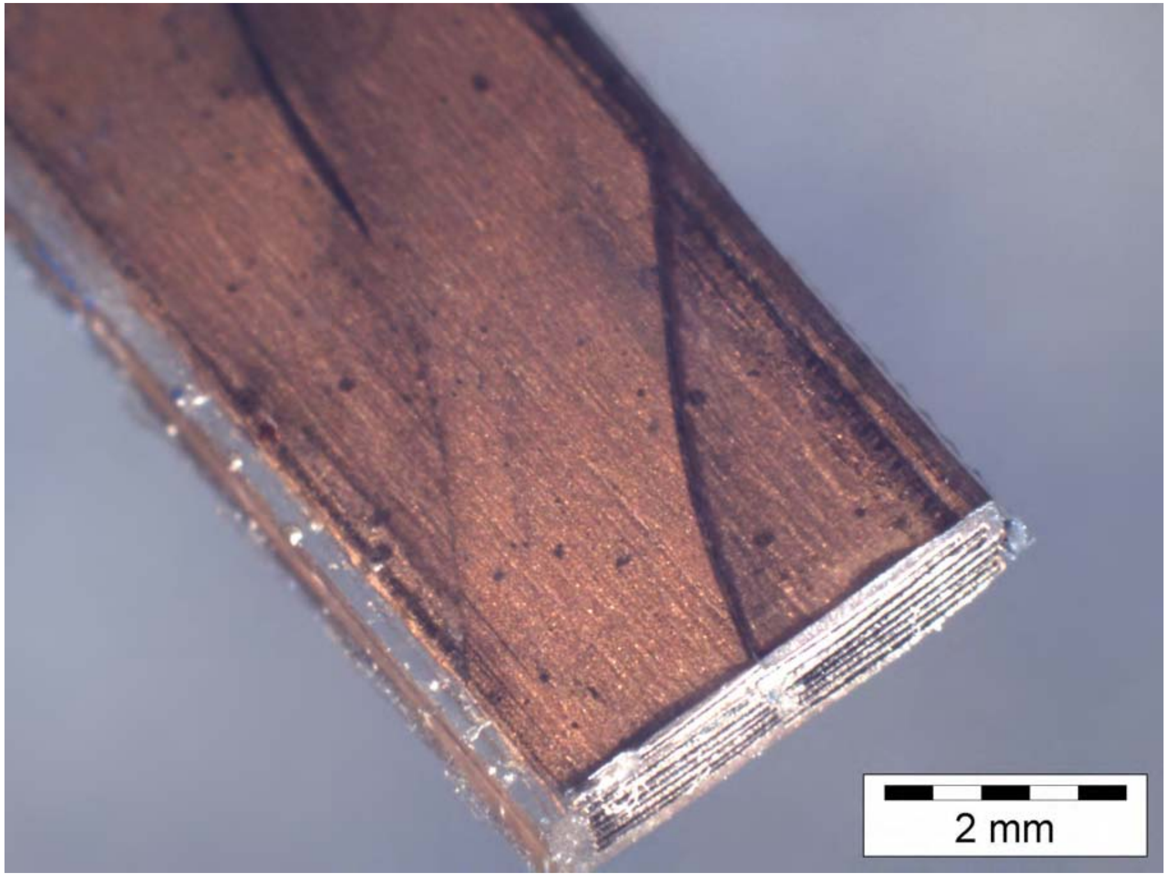
Another solution

- Use elongated quadrangular elements (Rodriguez-Zermeno, # 6765)
  - Elongation allows reducing number of DOFs
  - For a 1-D description of the tape, 1 element along the thickness is enough
- Successfully applied to the simulation of a Roebel cable



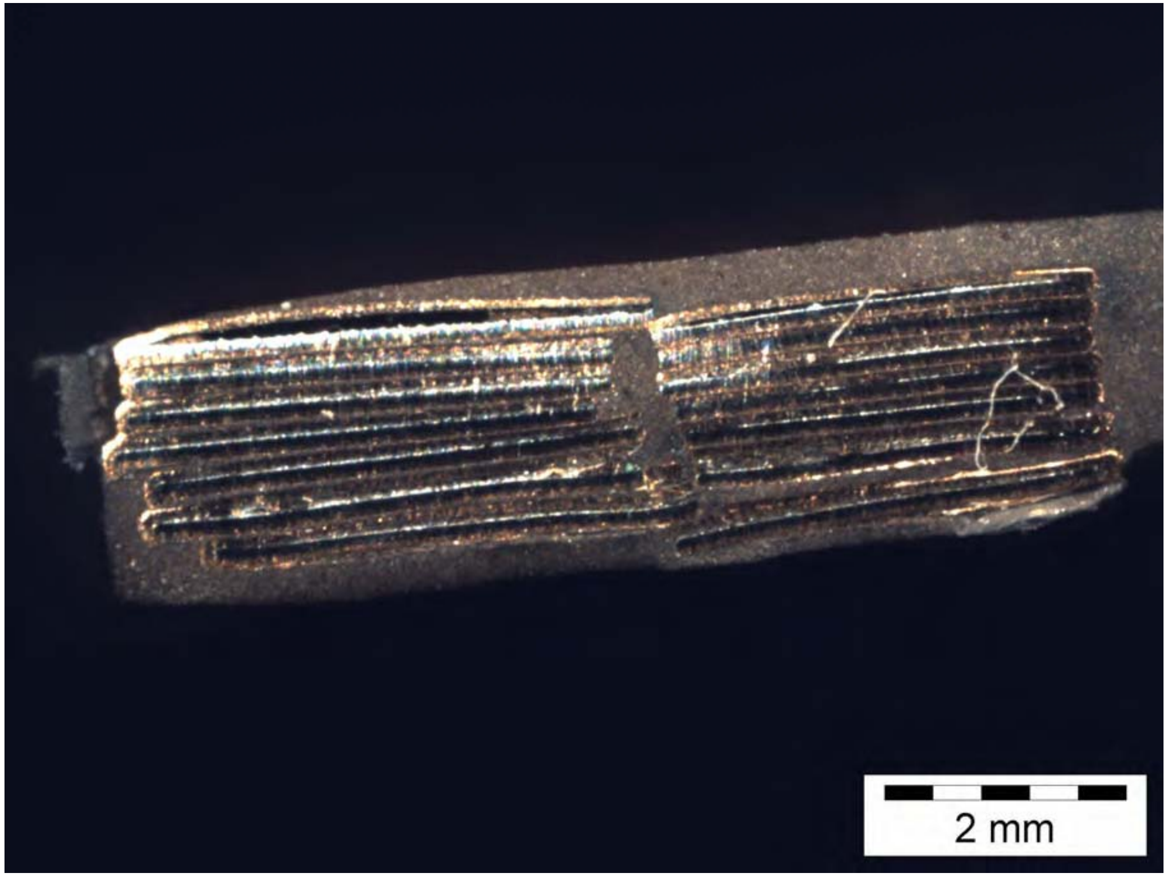
Simulation of a Roebel cable

- Magnetization losses



Simulation of a Roebel cable

- Magnetic field distribution in 2 stacks of 7 tapes
- Representative of the (2-D) cross section of a Roebel cable



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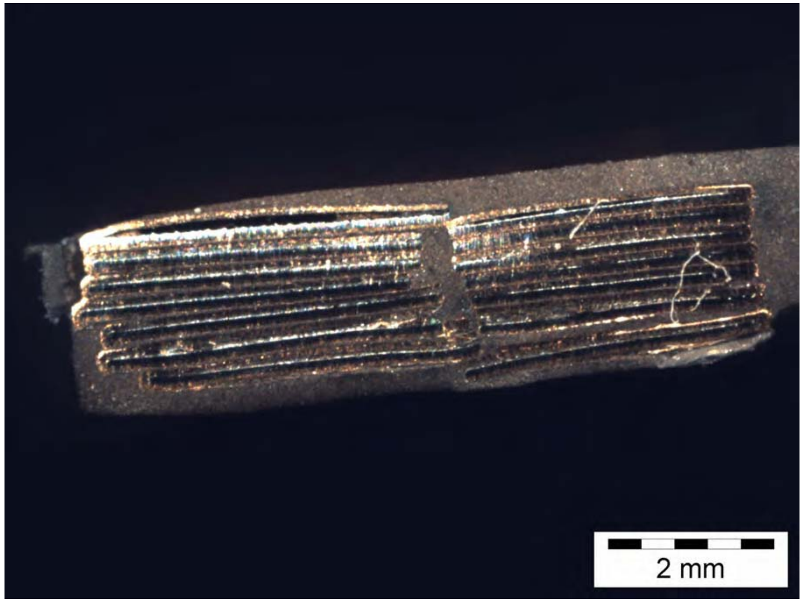
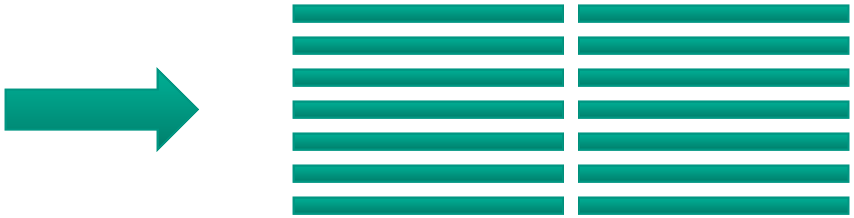


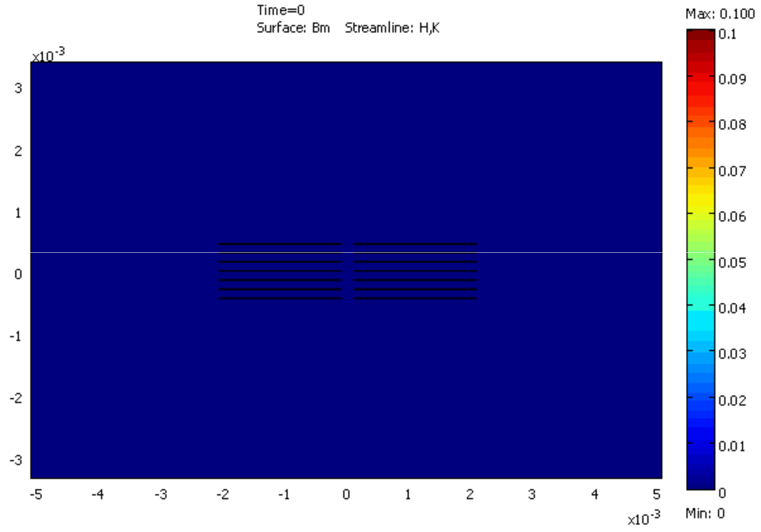


Simulation of a Roebel cable

- Standard triangular mesh
  - 280,000 elements, 450,000 DOFs, several days
- Elongated quadrilateral elements
  - 23,000 elements, 36,000 DOFs, a few hours
- Very important for design optimization

Simulation of a Roebel cable

- Magnetic field distribution in 2 stacks of 7 tapes
- Representative of the (2-D) cross section of a Roebel cable



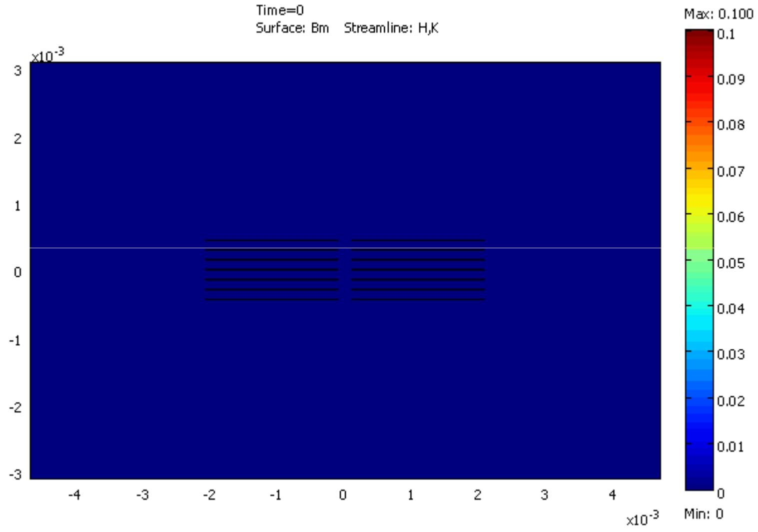
Uncoupled

$I_1=0$

$I_2=0$

…

$I_{14}=0$



Coupled

$I_1 + I_2 + \ldots + I_{14} = 0$

Conclusion

- H-formulation implemented in COMSOL's PDE General Form module to compute J and H profile and ac losses in HTS
- Very flexible
- Use 1st order curl elements
  - Ensure *divB*=0
  - Keep the number of DOFs at a reasonable level
- Use elongated quadrilateral elements for thin conductors
  - Lower number of mesh nodes with respect to standard triangular elements
- Method applied to simulate Roebel cable

Ac loss calculation

$$P(t) = \int_T \int_S J \cdot E dS$$

- How many cycles?