COMSOL CONFERENCE
STUTTGART 2011
OCTOBER 26-28

Improved finite element modeling of heat and mass transfers in single corn kernels during drying

Authors:

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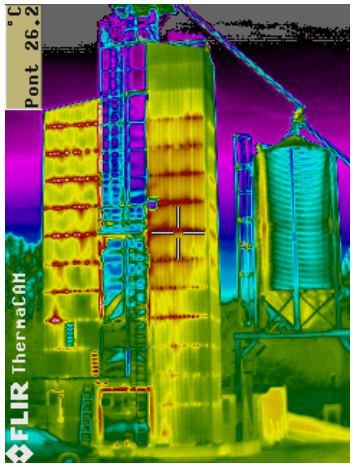
Main research area of our Institute is grain drying since the 1980's.



Thermographic image of the dryer

Testing of grain dryers

Experimental convective drying tunnel

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Besides experimental research drying modeling of a single kernel was initiated.

For example, the **mass diffusion** relationship was determined for different maize hybrids:

$$D = e^{a+bX+cT^{-1}}$$

where: D = moisture diffusion coefficient;
X = kernel average moisture content (dry basis);
T = average temperature of kernels;
a, b, c = constants.
(These constants for the Pioneer 3780 hybrid were found: a = -3.1411; b = 3.2159; c = -6696.6)

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Based on the measurements mathematical modeling studies of drying were carried out using Finite Element method:

$$\rho \cdot c \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + L \cdot \rho \frac{\partial X}{\partial t}$$

for heat transfer

$$\frac{\partial X}{\partial t} = D\left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2}\right)$$

for mass transfer

where: X is moisture content d.b. [kg/kg];
ρ is density [kg/m^3];
T is temperature [K];
K is thermal conductivity [W/mK];
L is latent heat of vaporisation of water [J/kg].

D is diffusion coefficient [m^2/s];
c is specific heat [J/kgK];
t is time [s];
x,y are directions;

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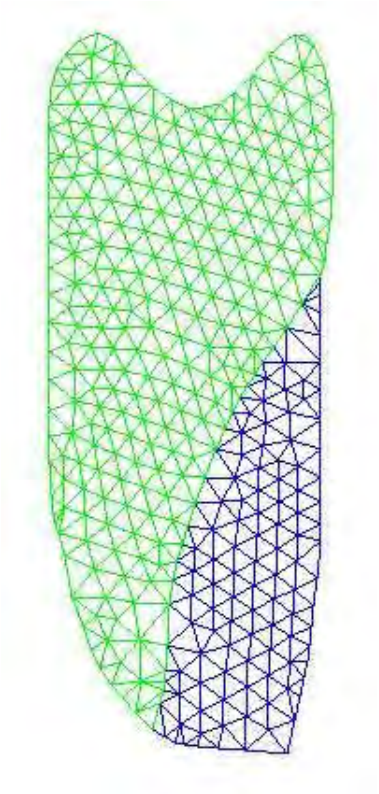
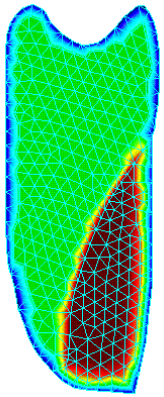
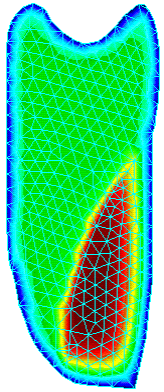
FE modelling with **COSMOS 1.71**



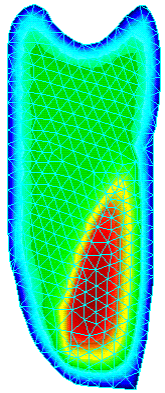
Mass transfer



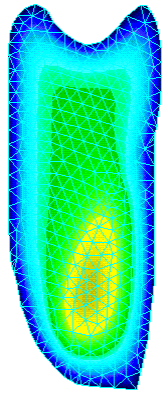
5 min



35 min



50 min



85 min of drying

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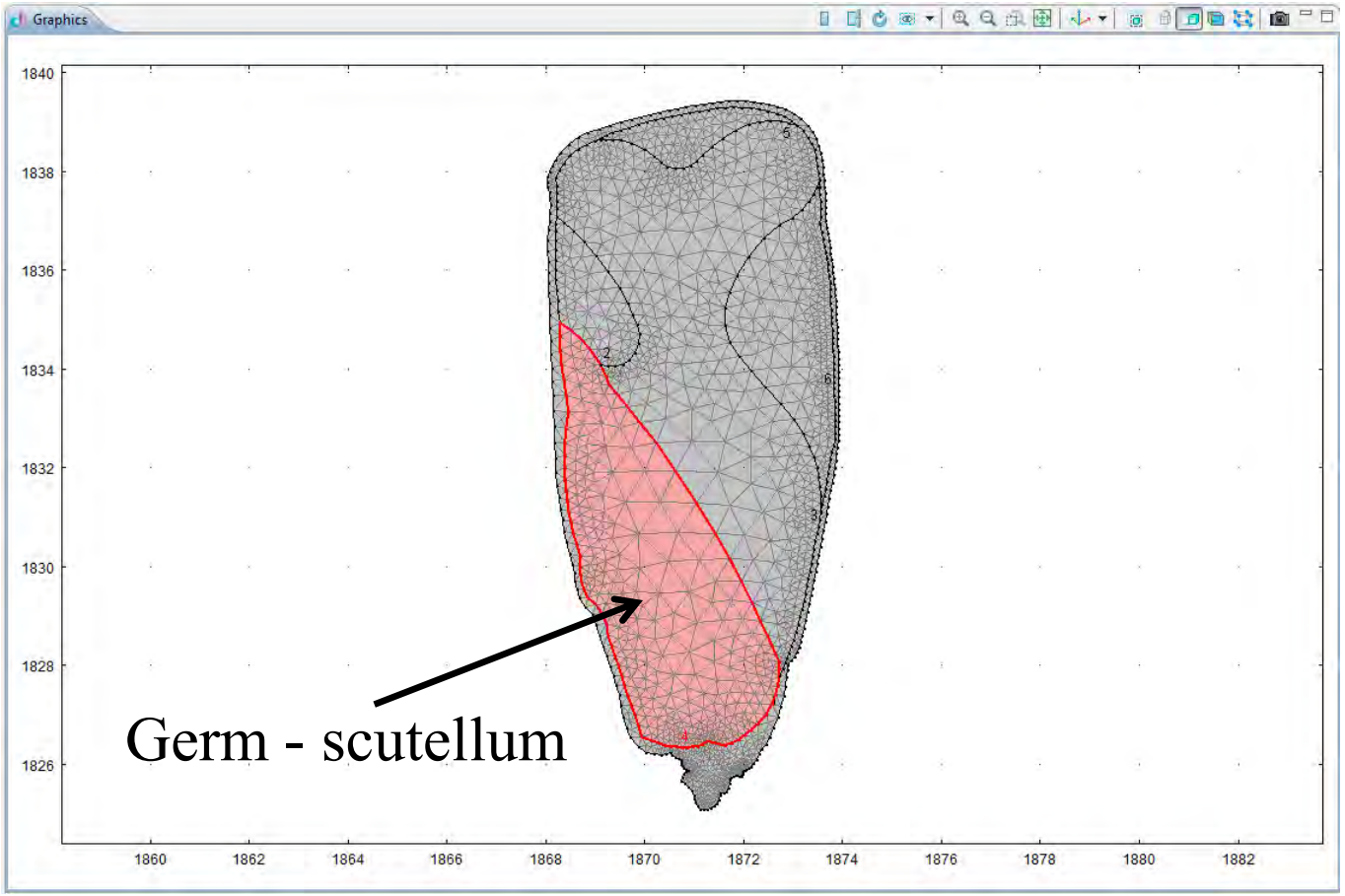
The structure of cross-sectioned maize and the FE mesh



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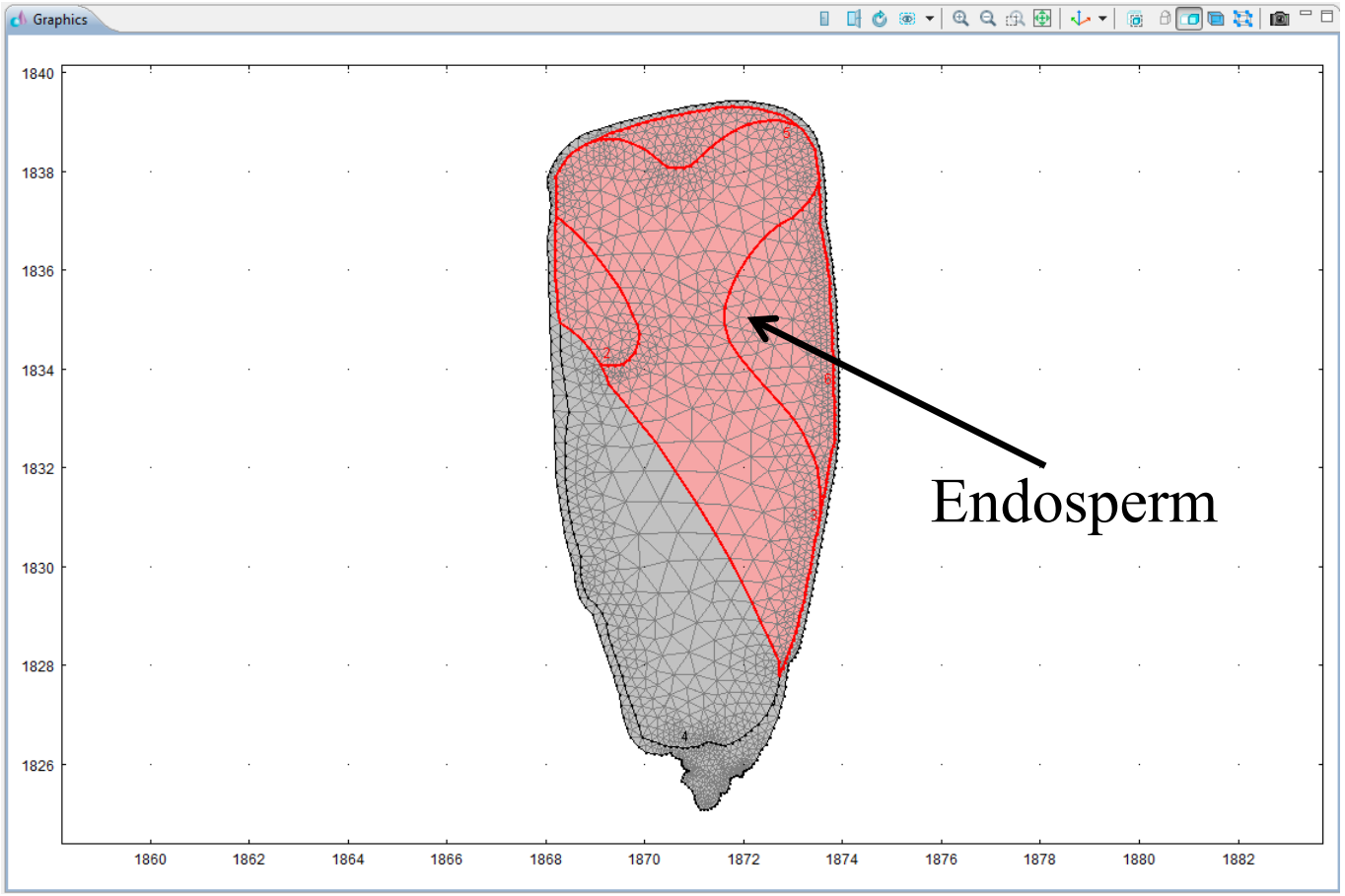

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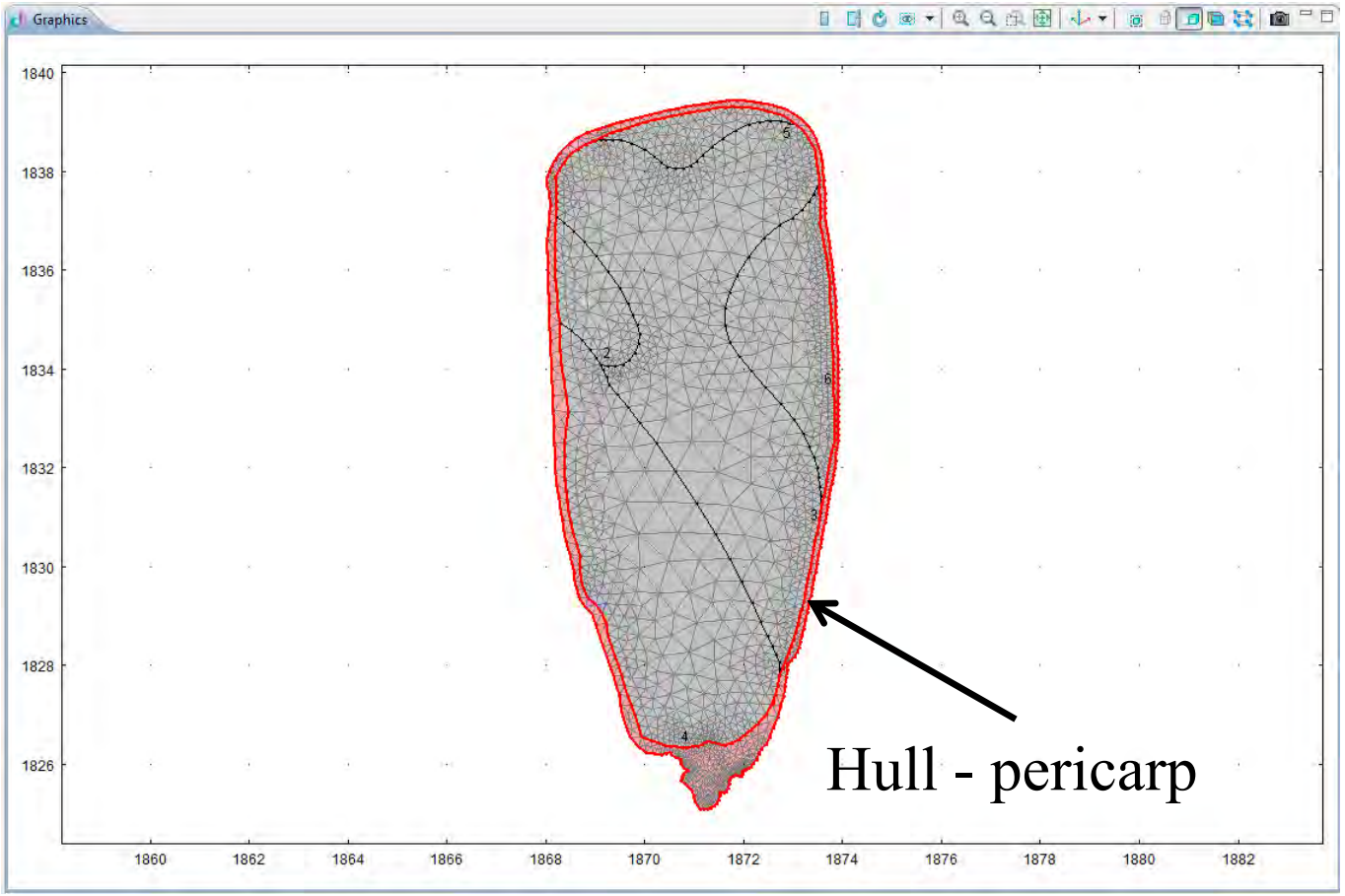

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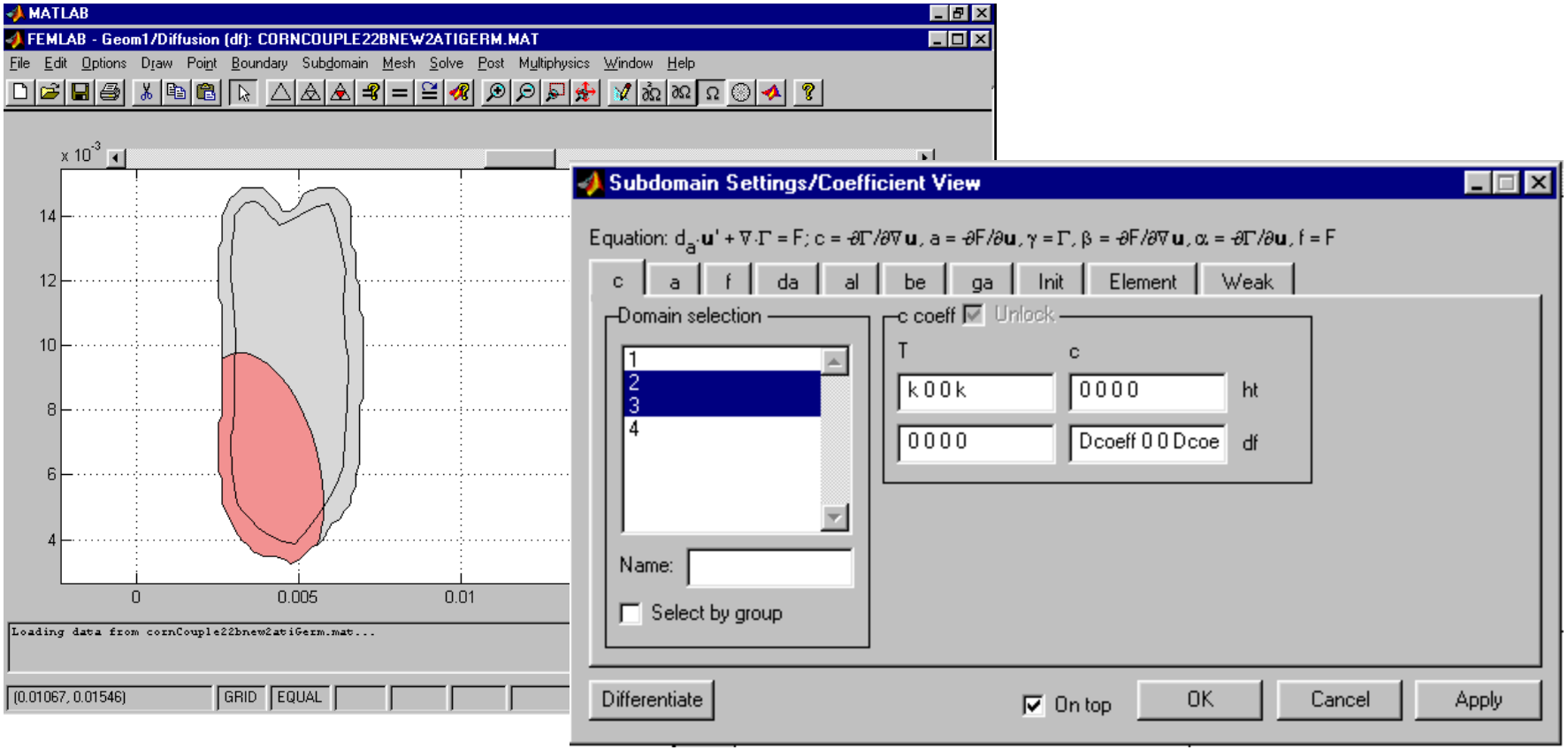
COMSOL MULTIPHYSICS simulation



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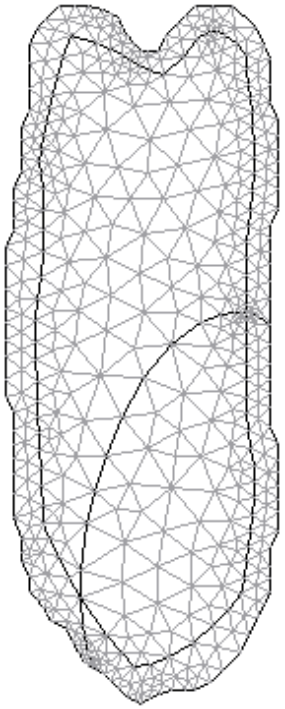
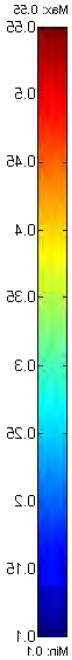
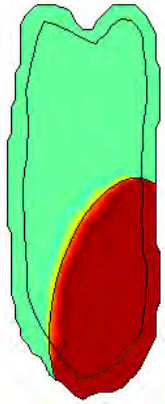
Heat Transfer: $\rho \cdot c \frac{\partial T}{\partial t} = k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + L \cdot \rho \frac{\partial X}{\partial t}$

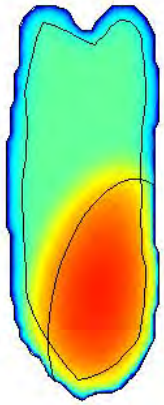
Mass Transfer: $\frac{\partial X}{\partial t} = D(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2})$



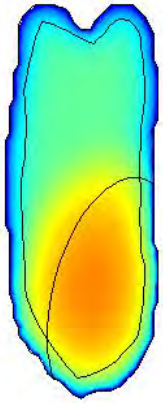




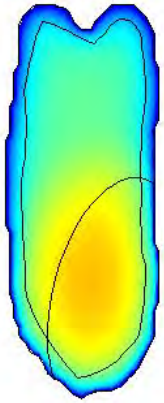
Before



10 min



20 min



30 min of drying

FE modelling with
FEMLAB 2.2

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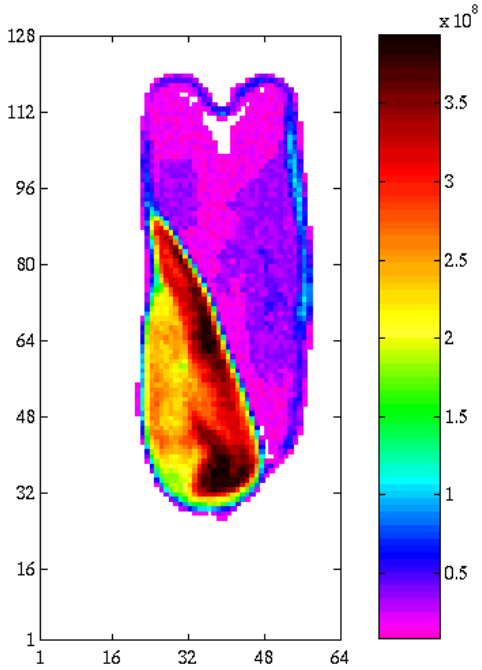
Magnetic resonance imaging (MRI) studies were carried out to evaluate the accuracy of the FE models.

NMR magnet at BRUKER GMBH



MRI image of a corn kernel before drying

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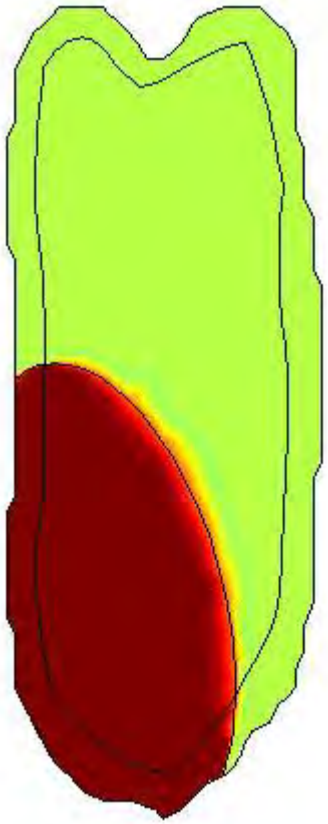
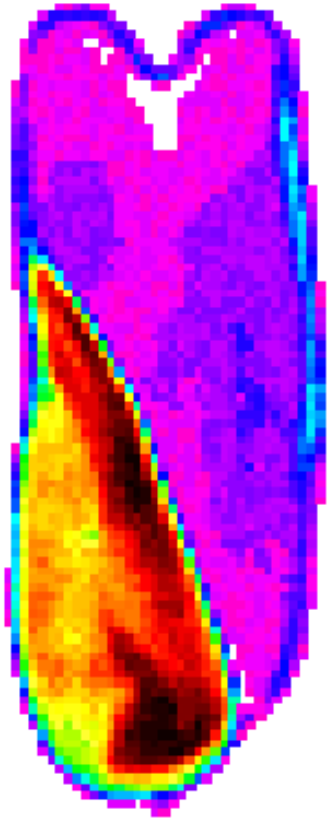
MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**





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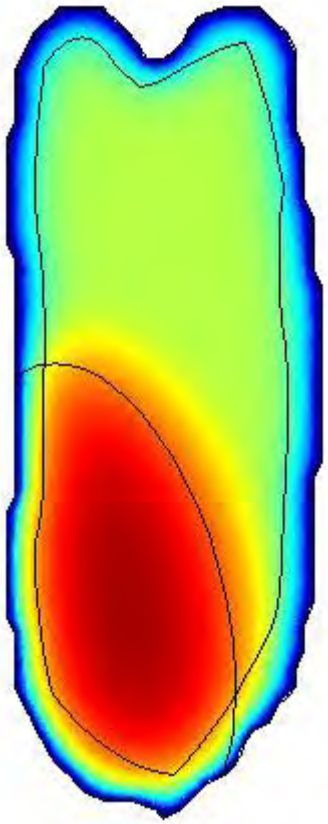

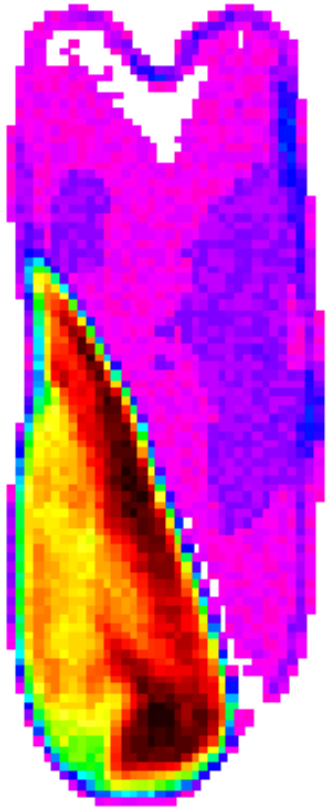

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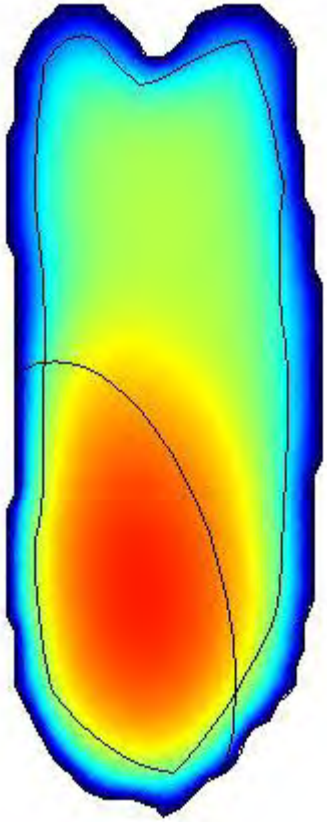

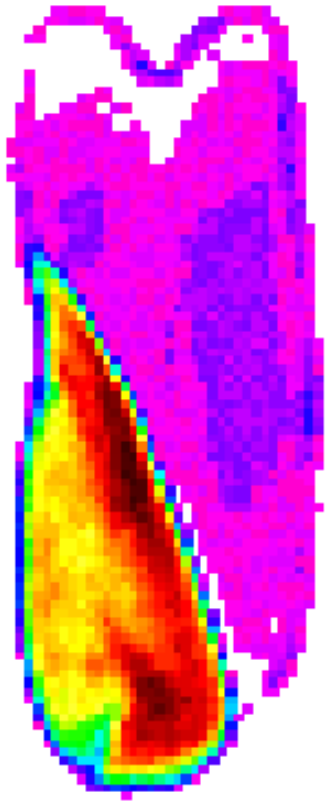

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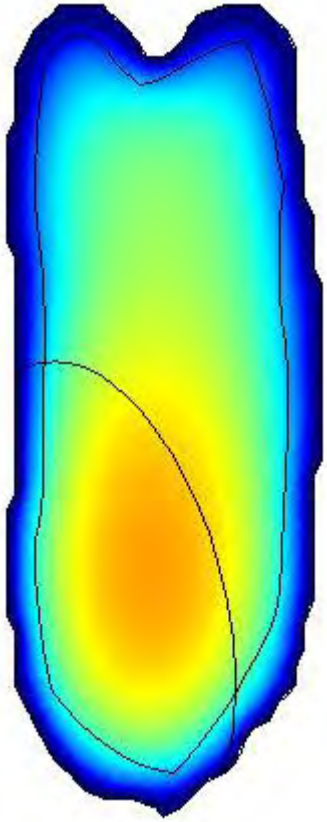

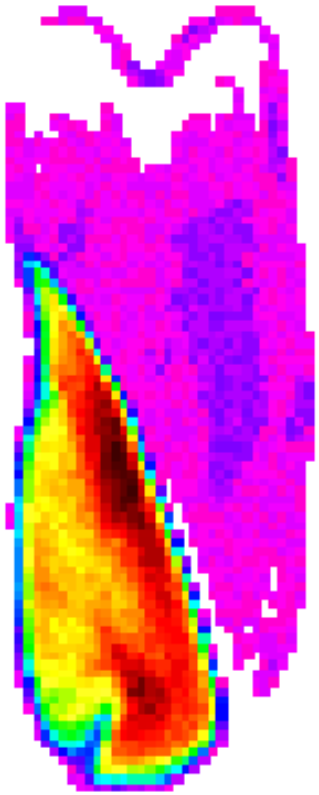

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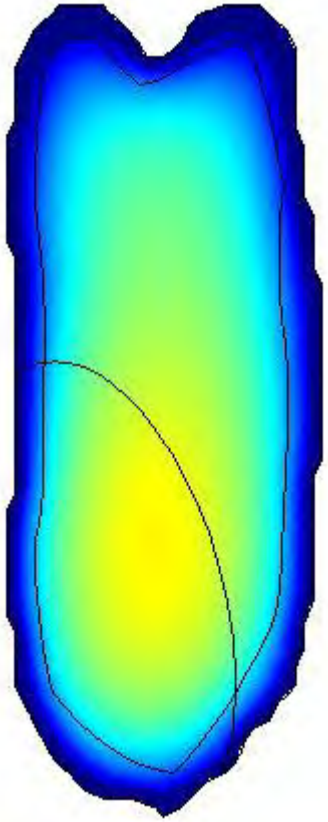

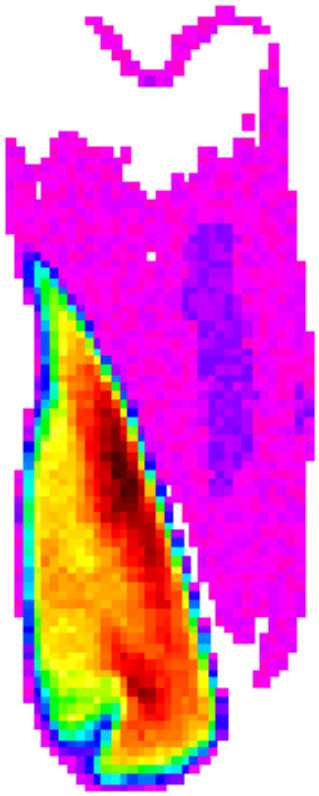

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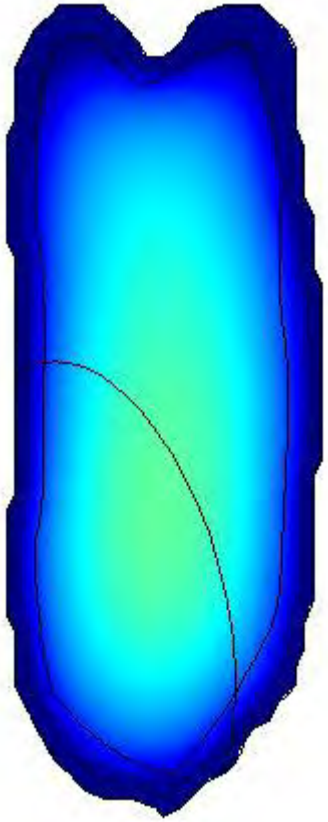

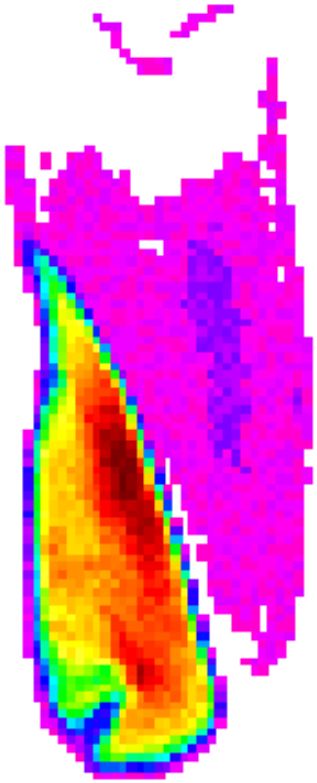

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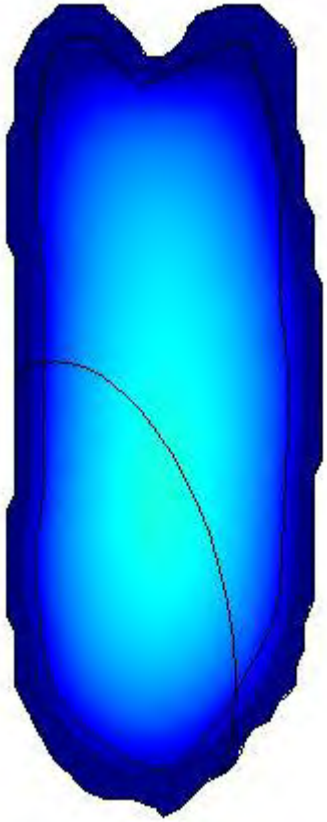

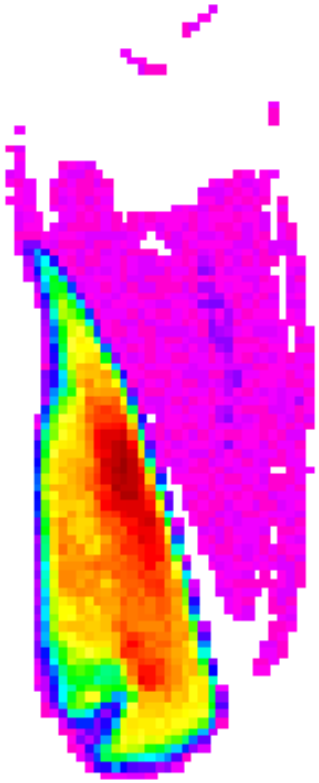

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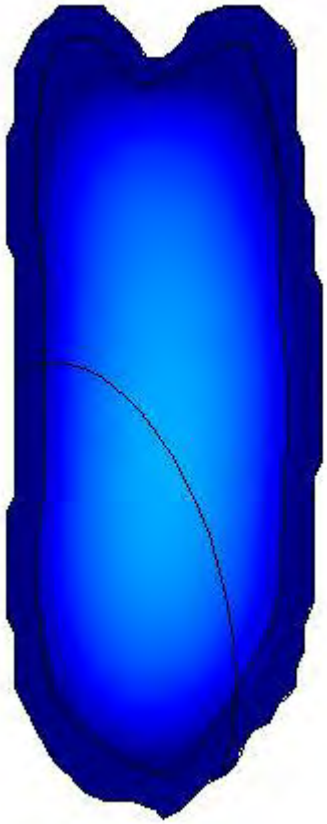

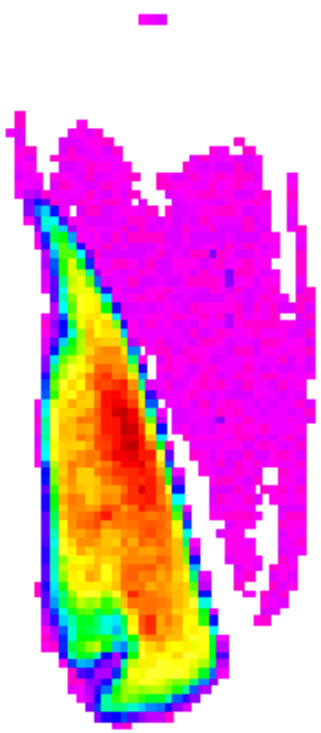

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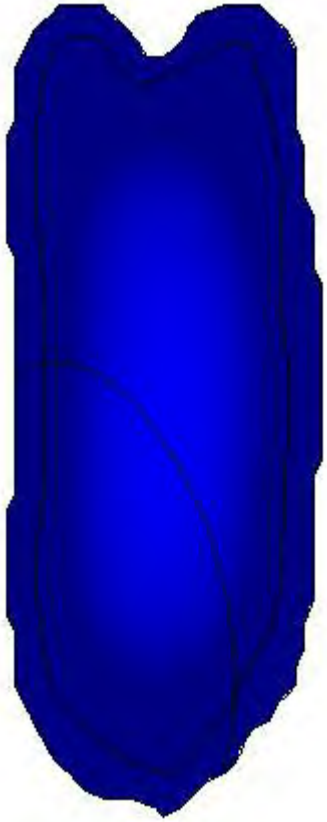

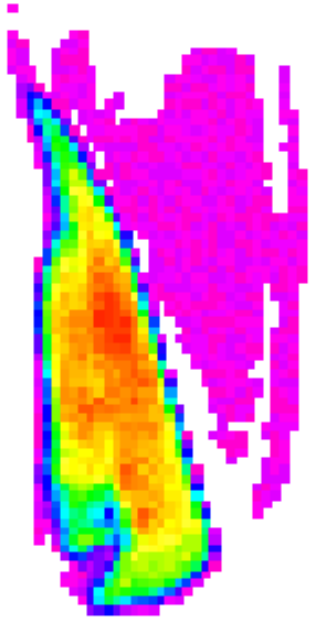

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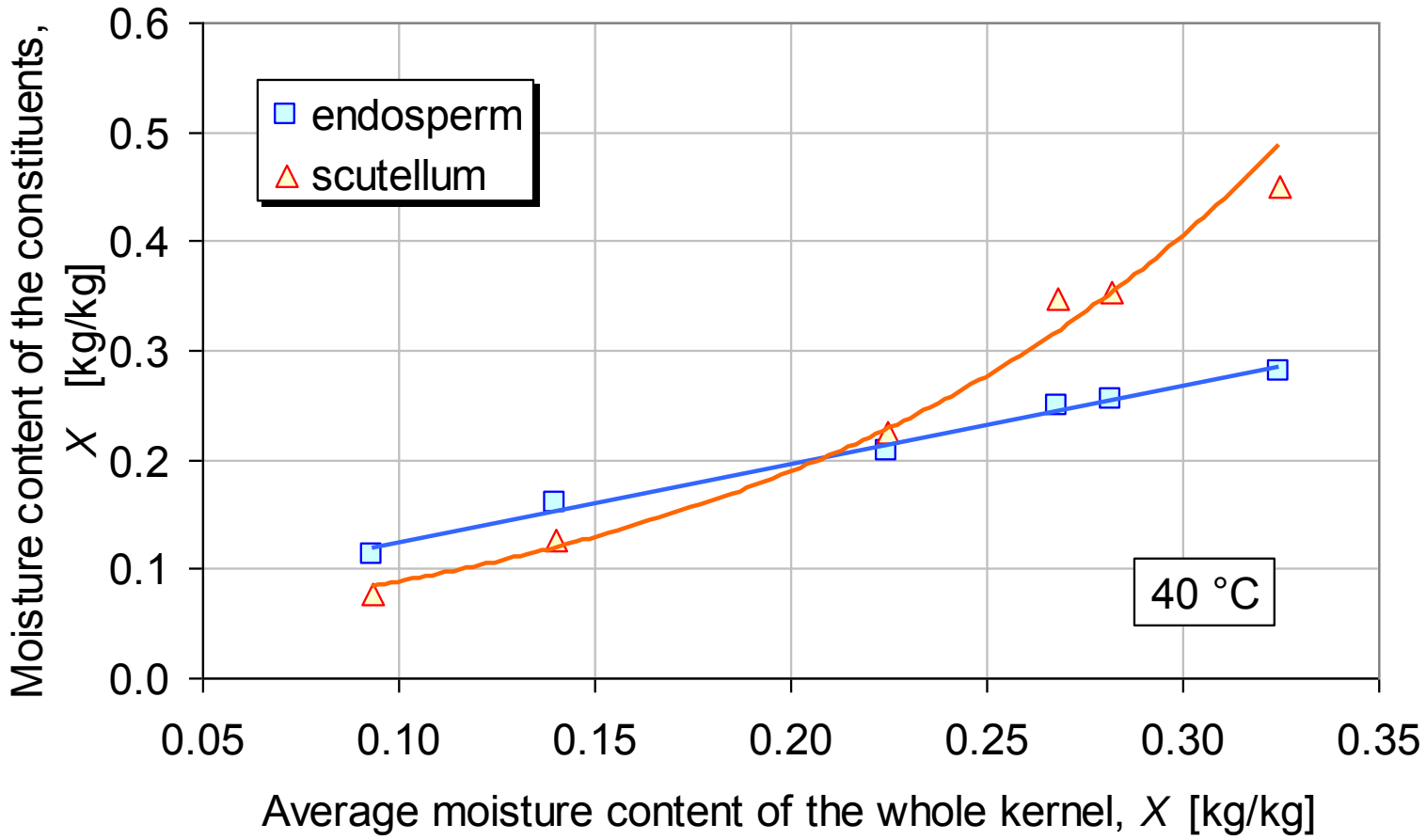
Moisture content of the particles vs. whole maize kernels (Florencia)



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Considering capillary (water) potential

The modified **Luikov's model** of **mass transport**,

$$\frac{\partial X}{\partial \tau} = \mathrm{div} D \cdot \mathrm{grad} X$$

where: X is moisture content;
τ is time;
D is mass diffusion coefficient.

can be **rewritten**:

$$\frac{\partial X}{\partial \Psi}\frac{\partial \Psi}{\partial \tau} = \frac{\partial}{\partial x}\left[D \frac{\partial X}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial x} \right] + \frac{\partial}{\partial y}\left[D \frac{\partial X}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial y} \right] + \frac{\partial}{\partial z}\left[D \frac{\partial X}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial z} \right]$$

and thus the equation **gives**:

$$\frac{\partial X}{\partial \Psi}\frac{\partial \Psi}{\partial \tau} = \mathrm{div} K \cdot \mathrm{grad}\, \psi$$

where: Ψ is capillary (water) potential;
K is hydraulic conductivity ($K = D \cdot \partial X / \partial \Upsilon$).

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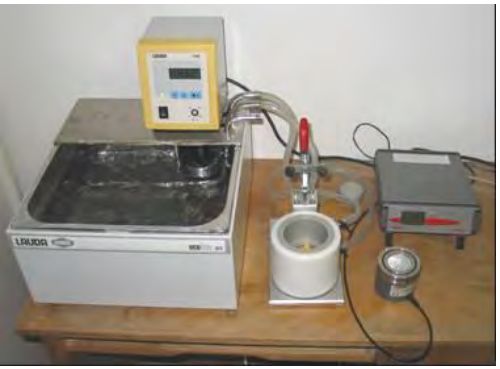
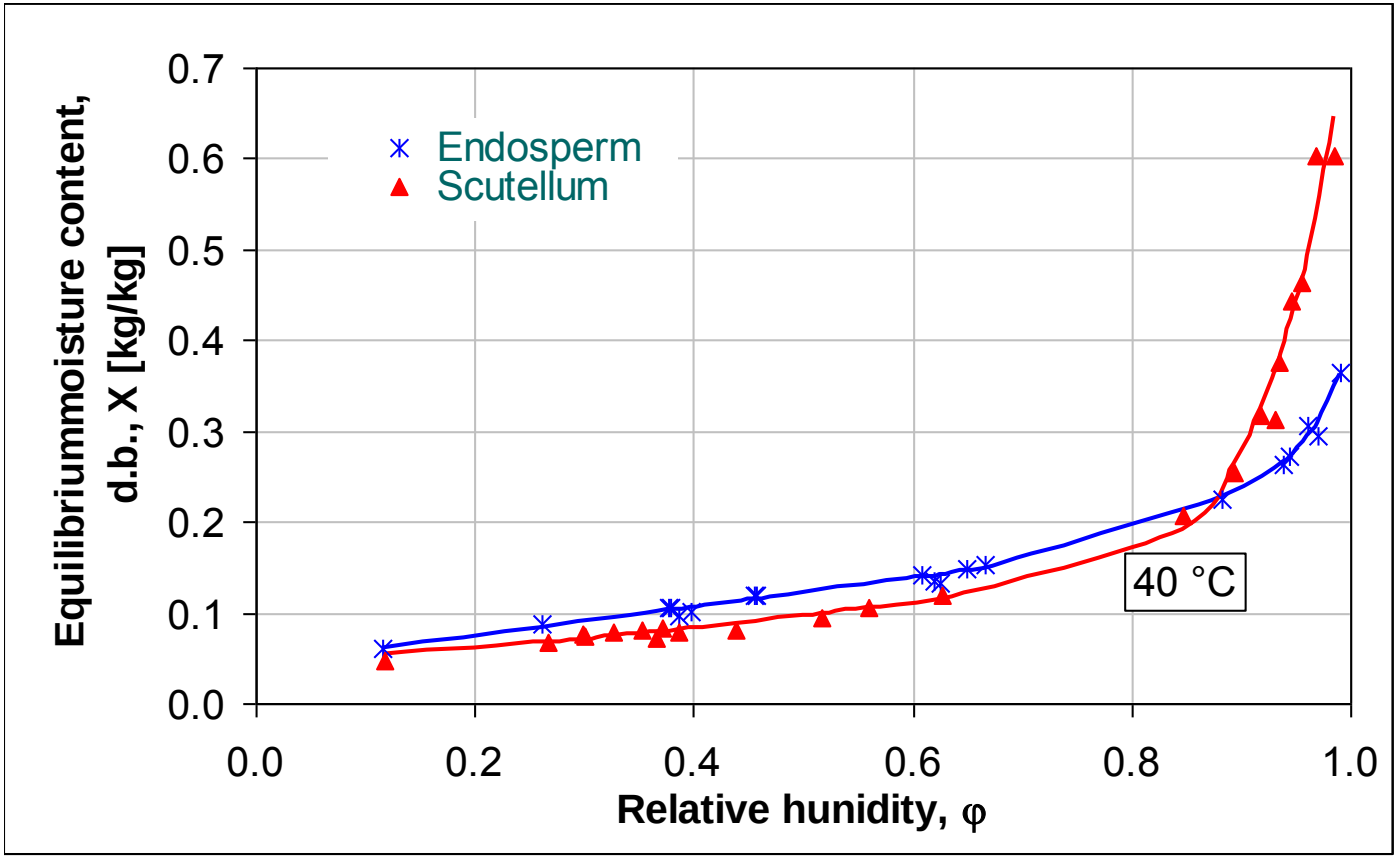
Desorption curves of maize kernel's particles



ROTRONIC water activity meter



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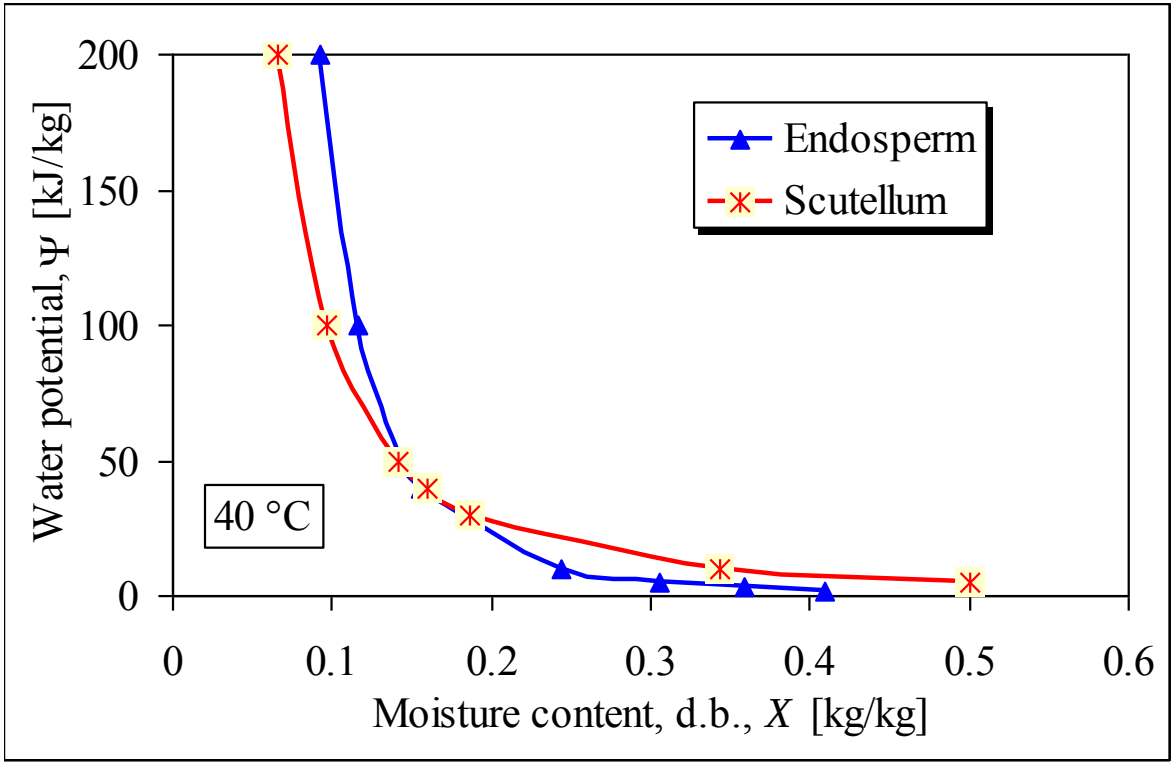
Water potential curves of maize particles as a function of moisture content

$$[\Psi]_X = \frac{R_U \cdot T}{m_V} \ln \varphi$$

where:

- φ is the relative humidity of the air (decimal);
- R_U is the universal gas constant;
- m_v is the molecular weight of vapour;
- T is the absolute temperature;
- X is moisture content, d.b.



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Boundary conditions

Neumann type:

$$\underline{n} \cdot (c \nabla u) + qu = qu_{\infty}$$

where:

u is stands for the water potential (Ψ);

$\underline{n}$ is the normal vector.

$$d_a = \frac{\partial X}{\partial \psi}$$

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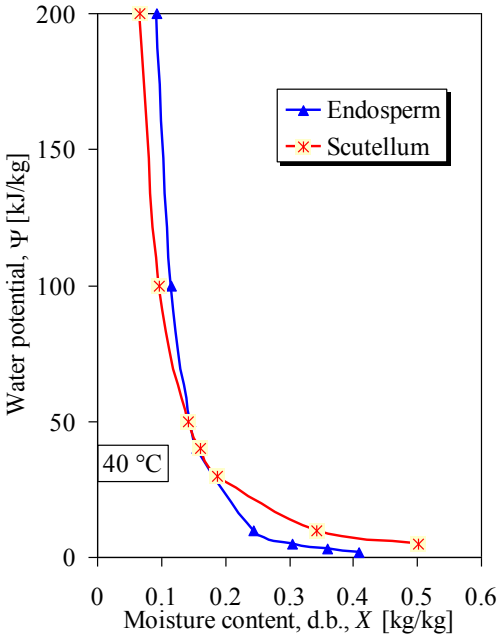
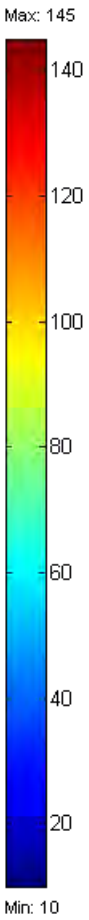
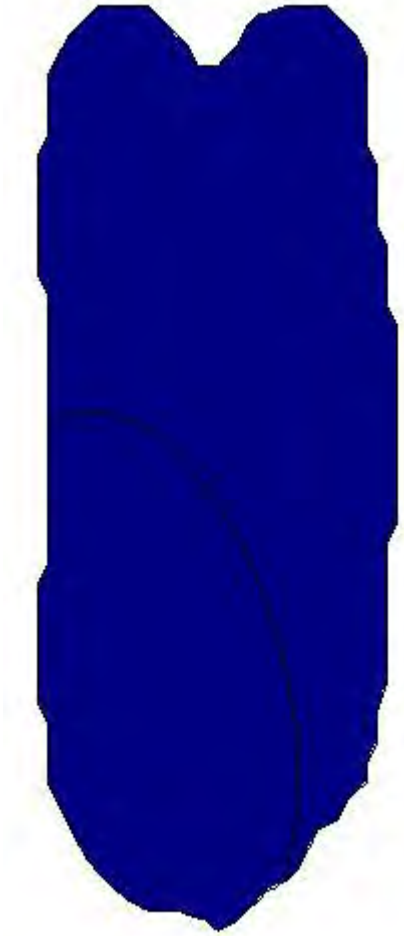
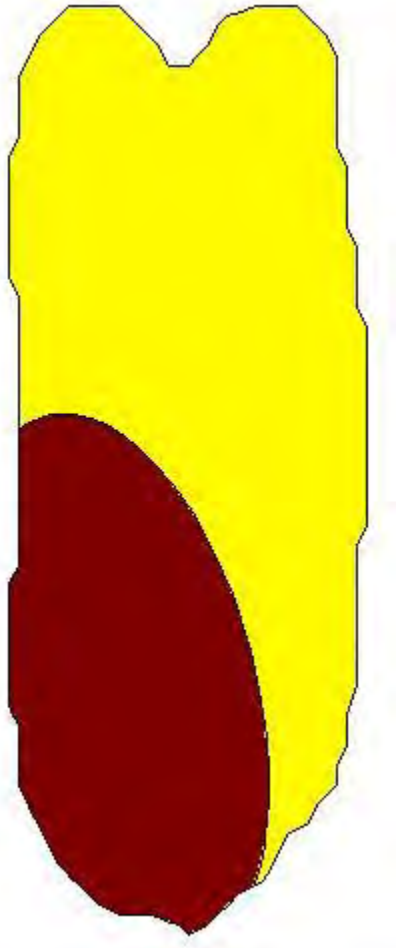
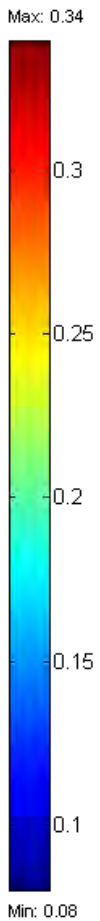






Water potential, [kJ/kg]





Moisture cotent, [kg/kg]

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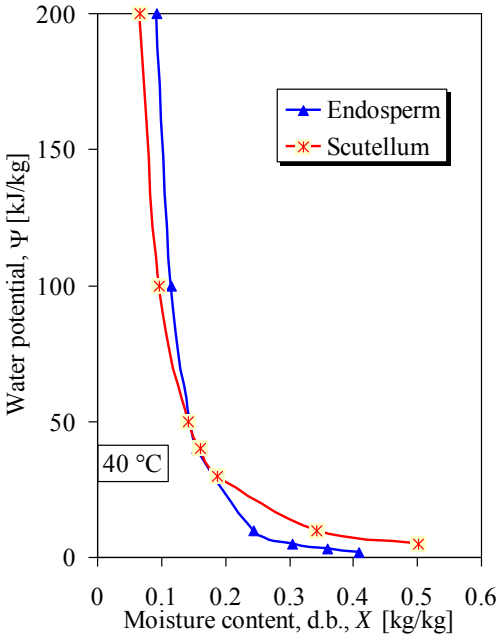

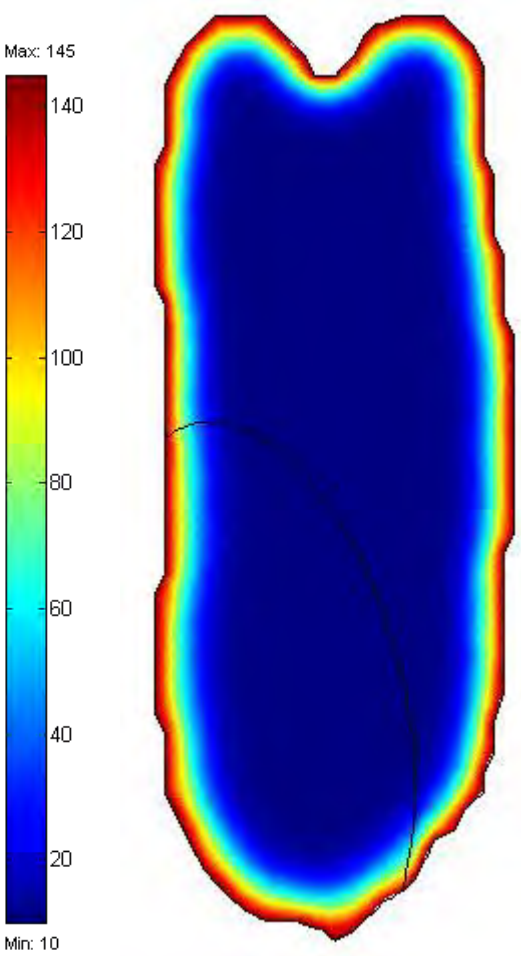

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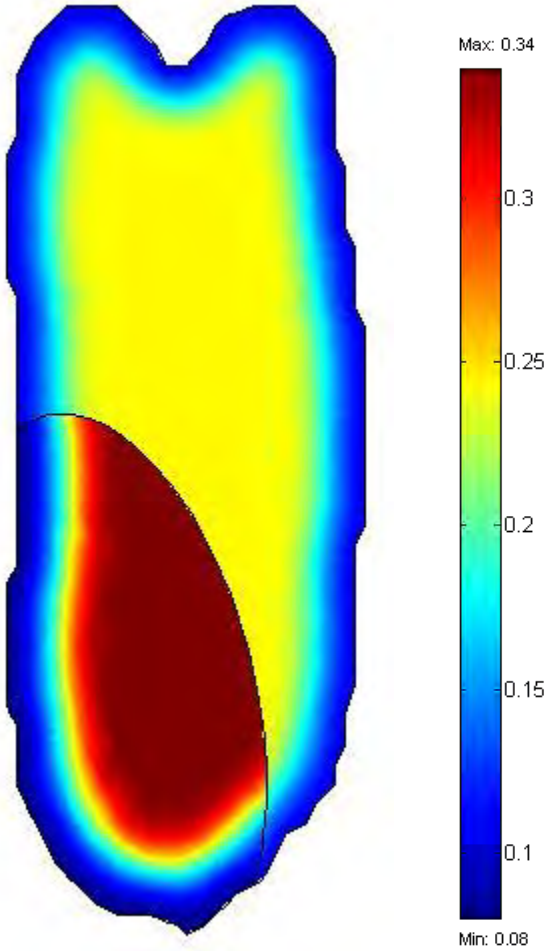

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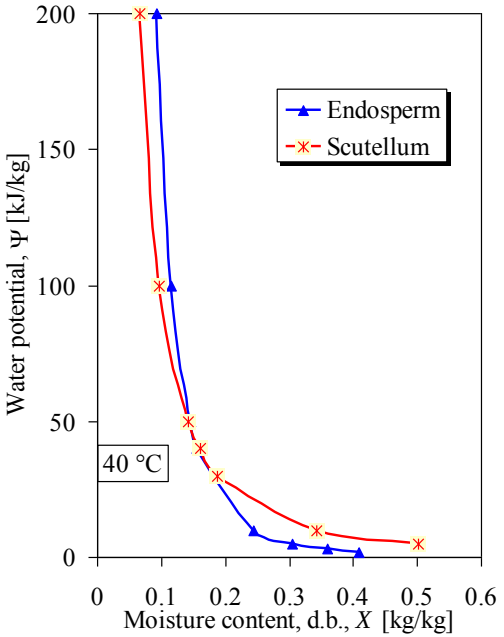

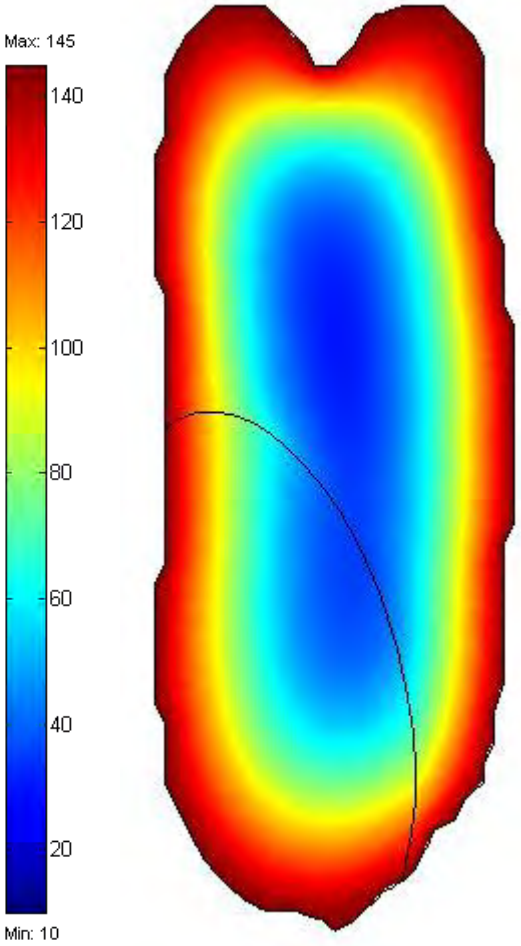

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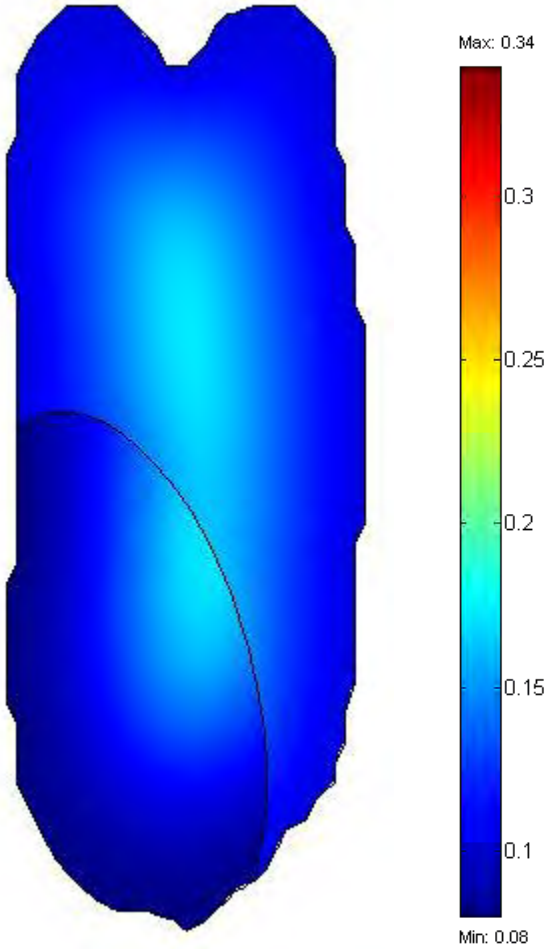

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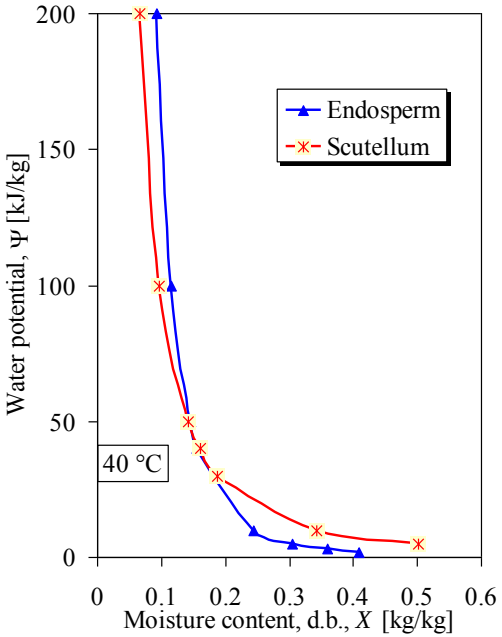

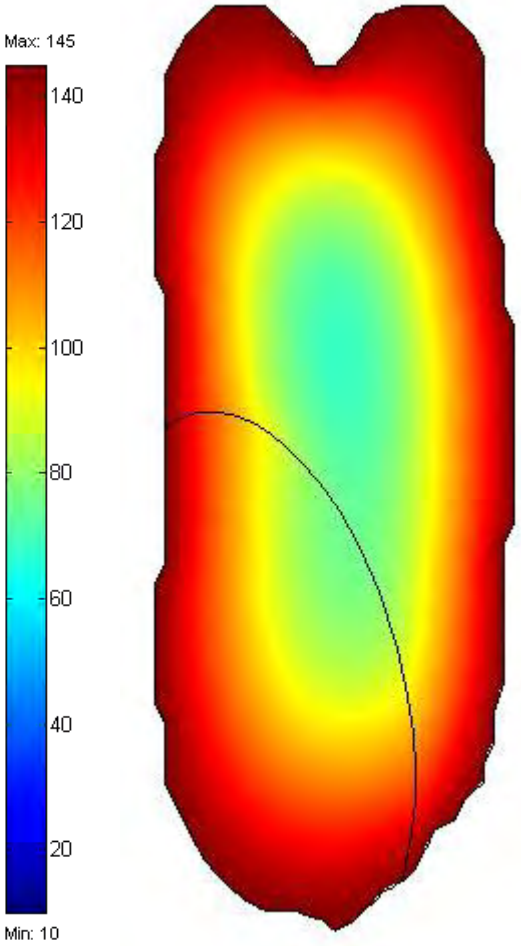

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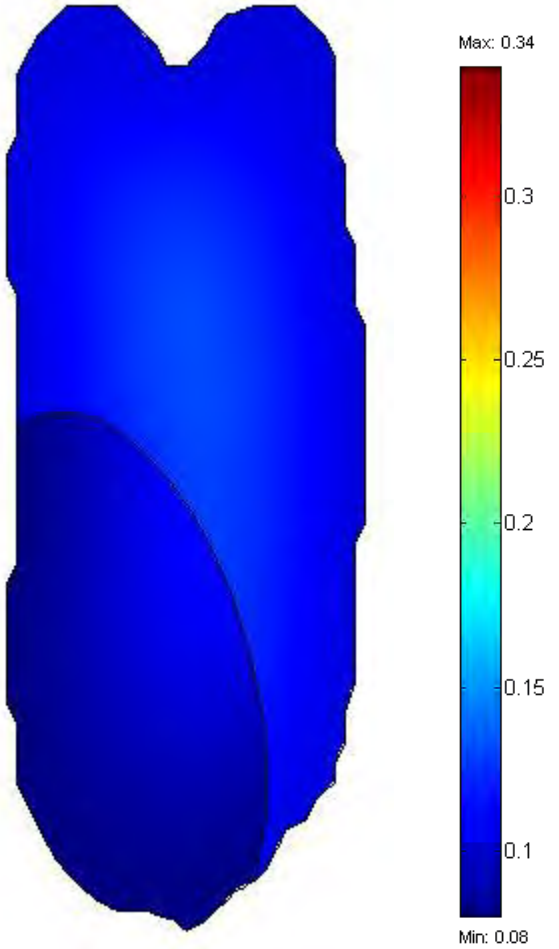

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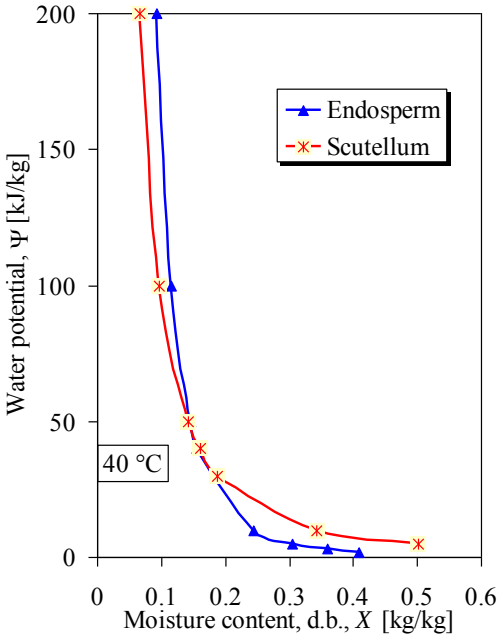

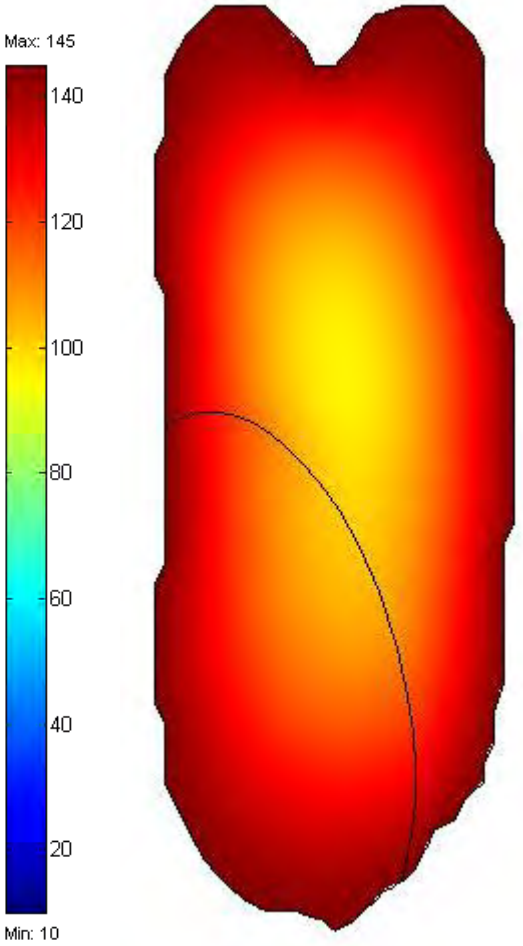

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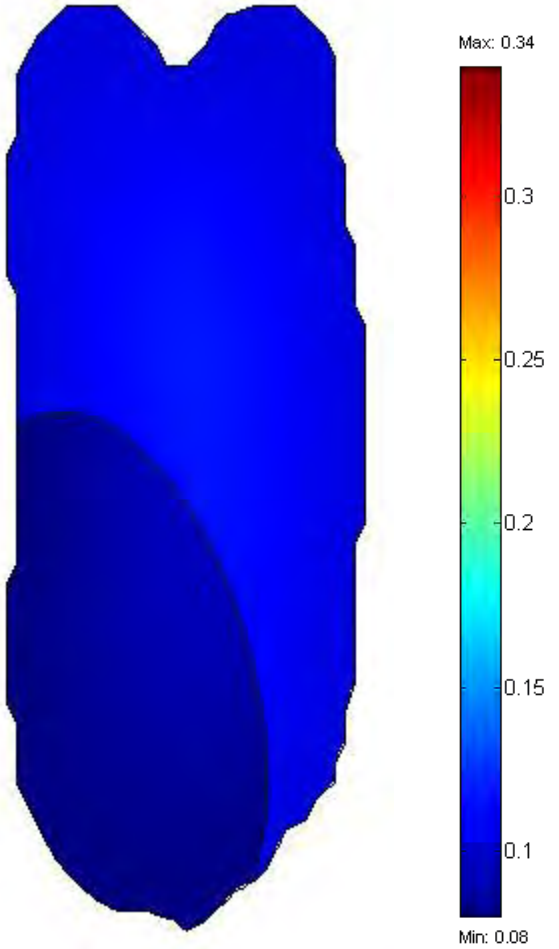

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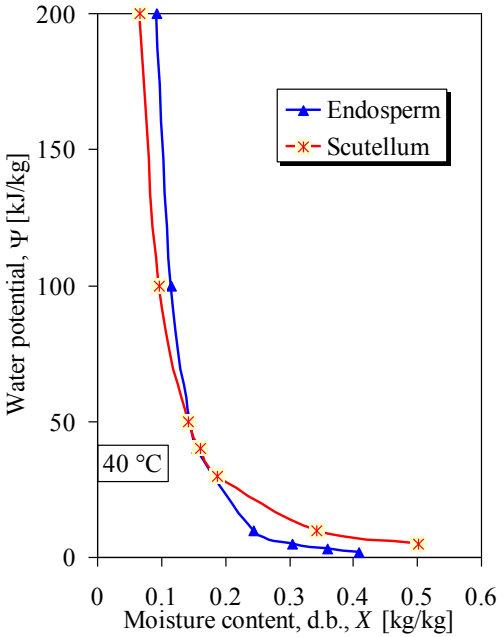

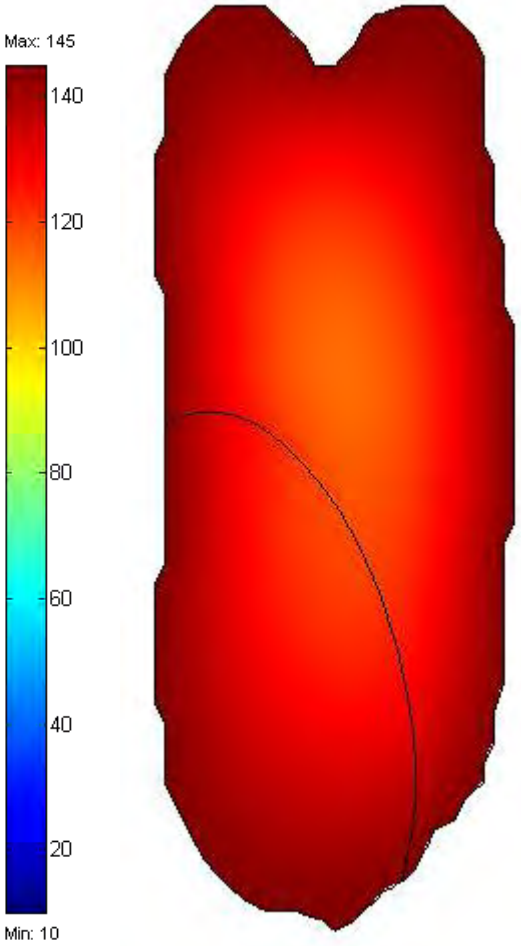

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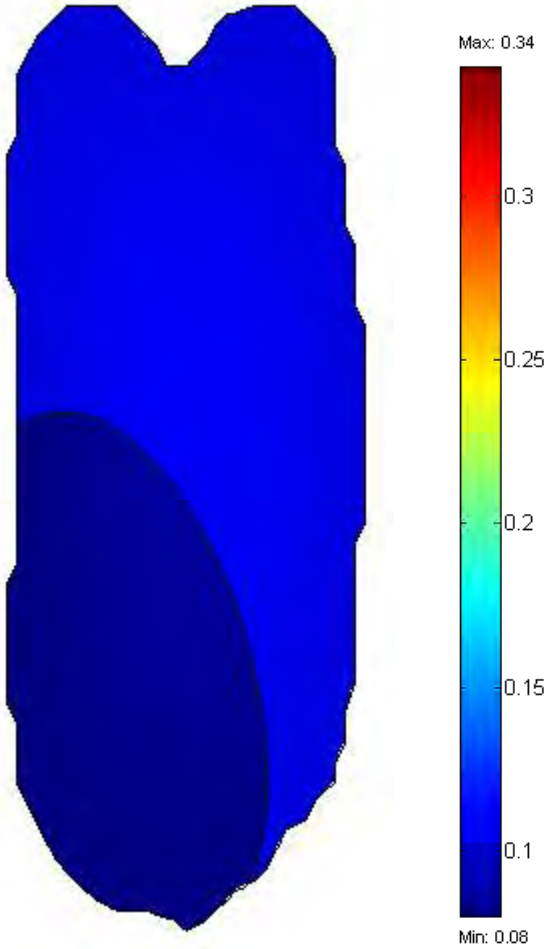

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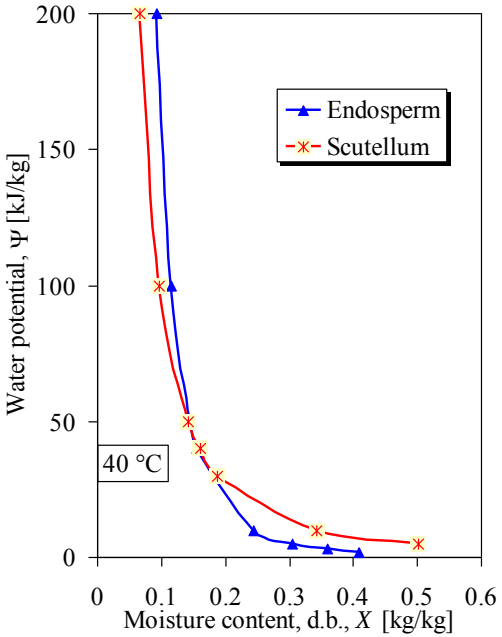

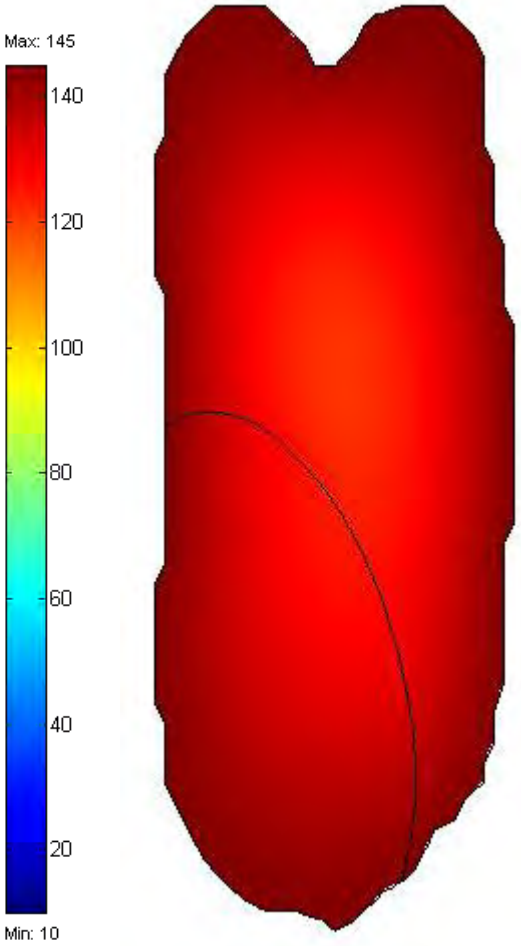

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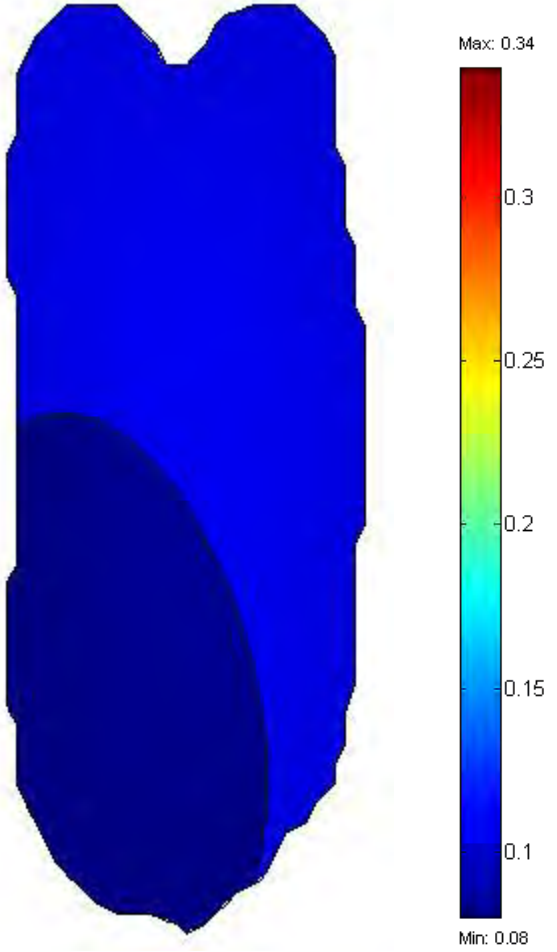

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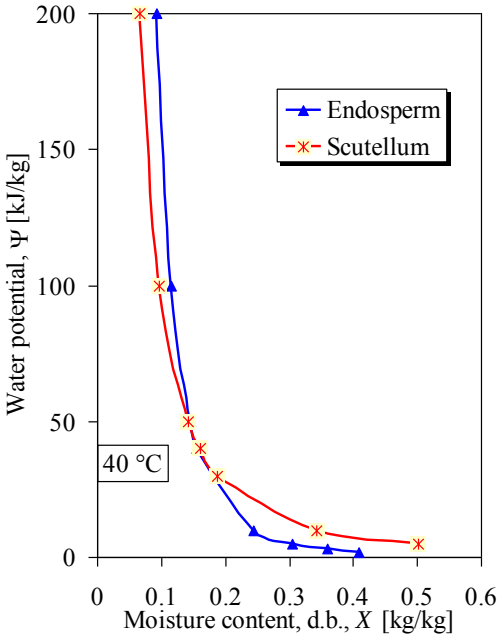

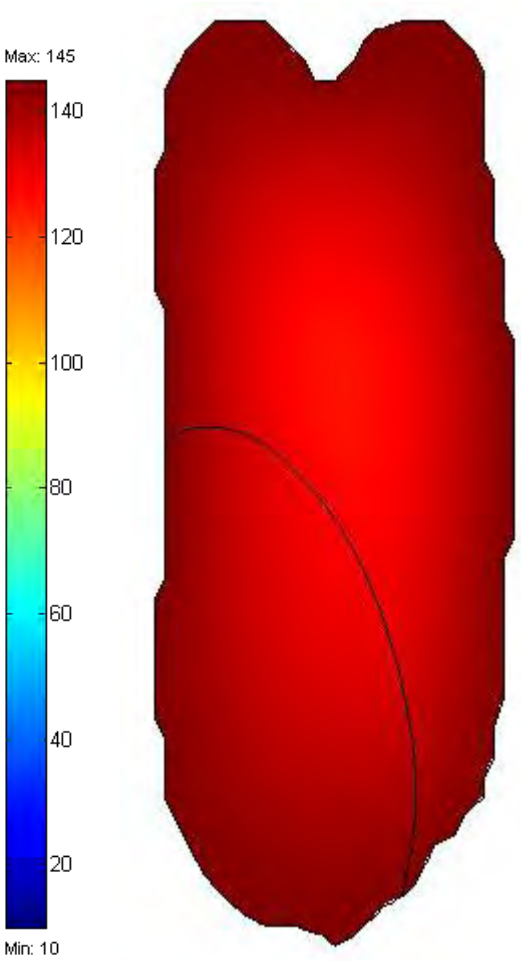

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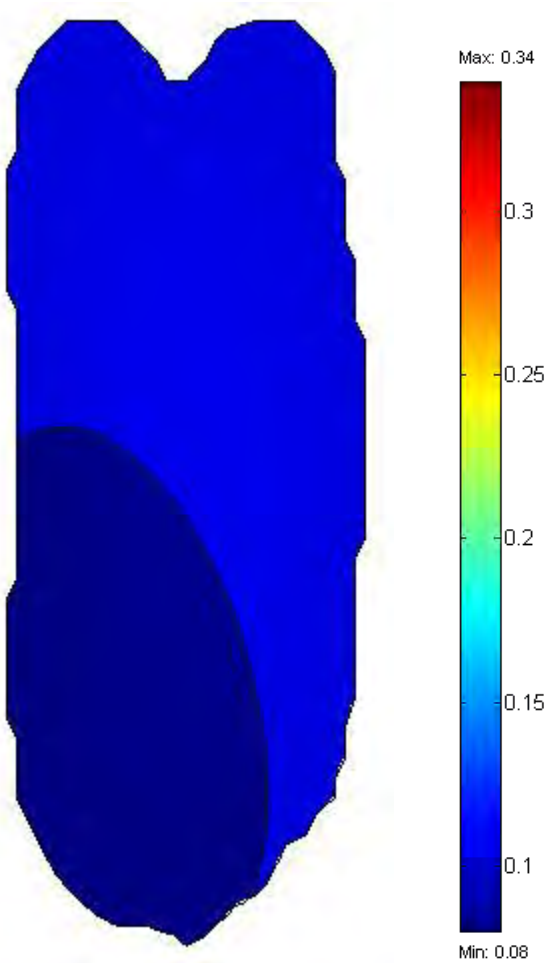

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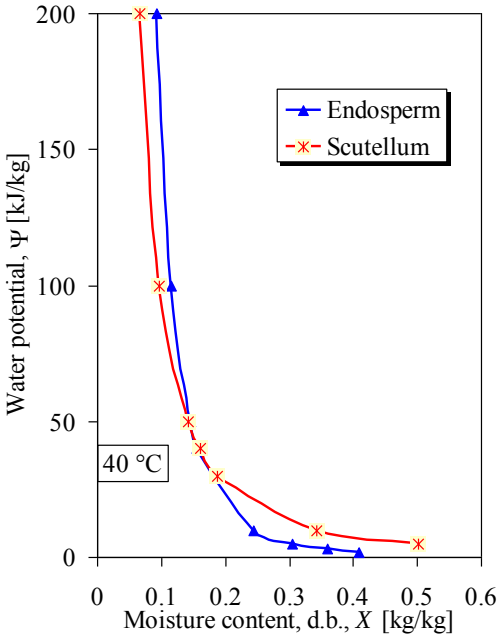

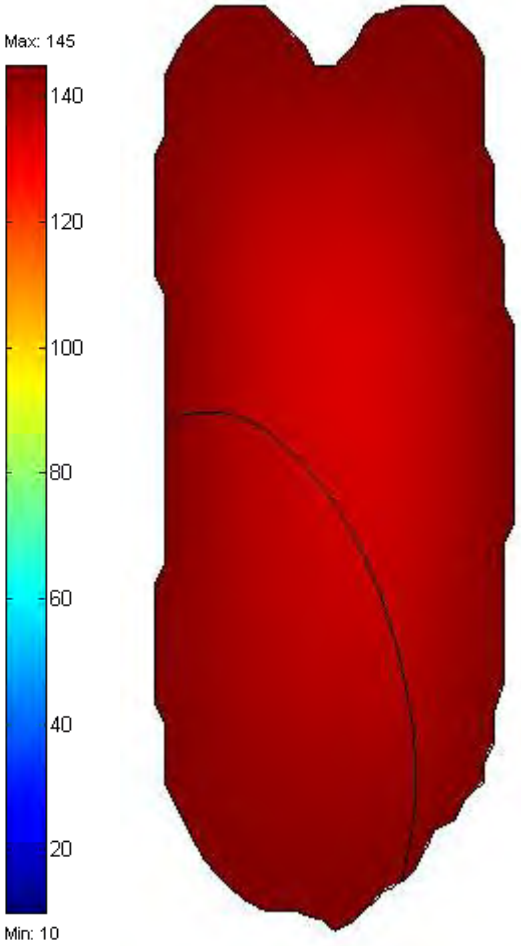

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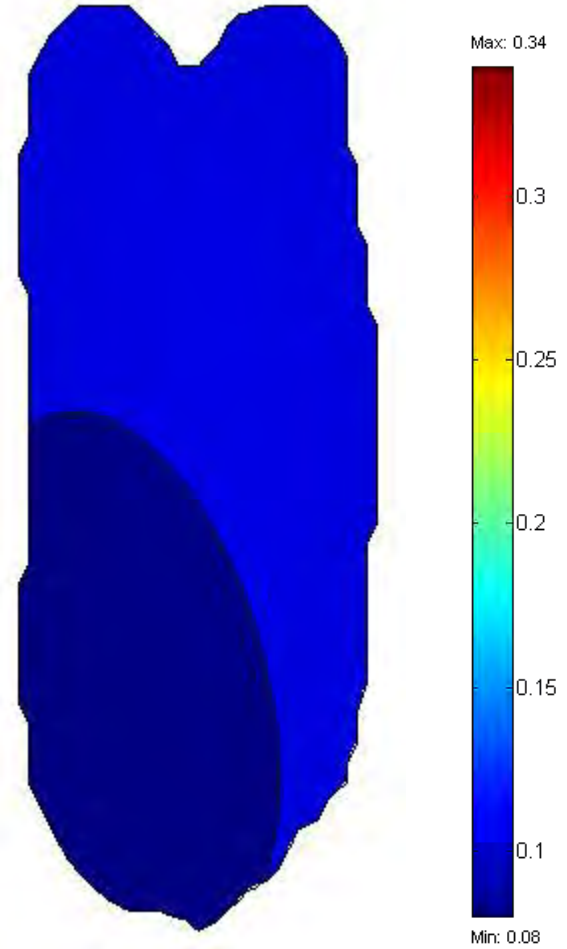

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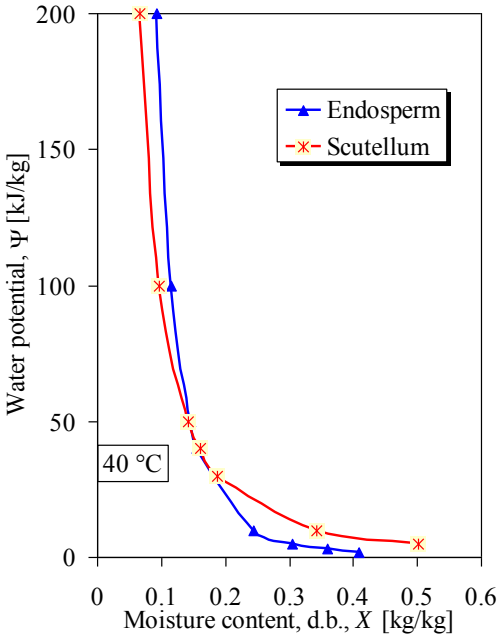

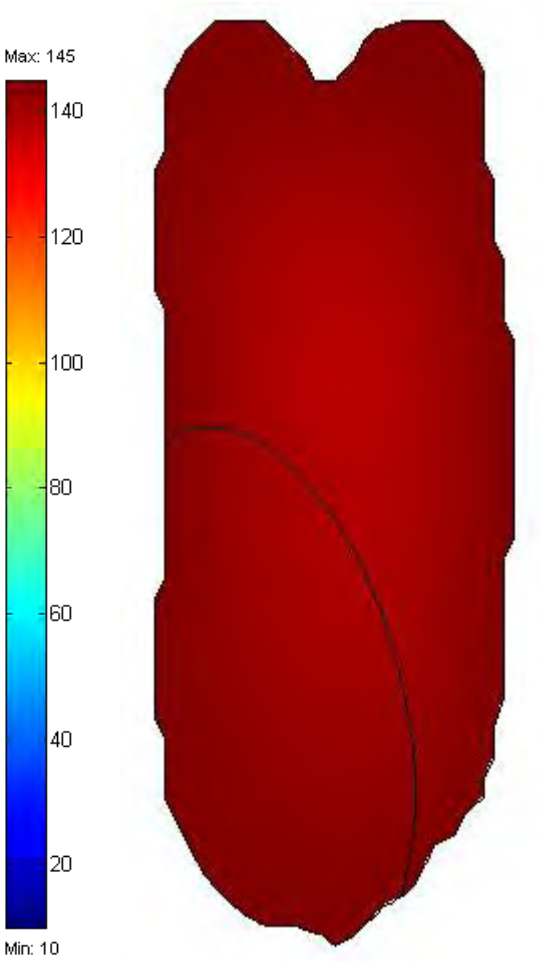

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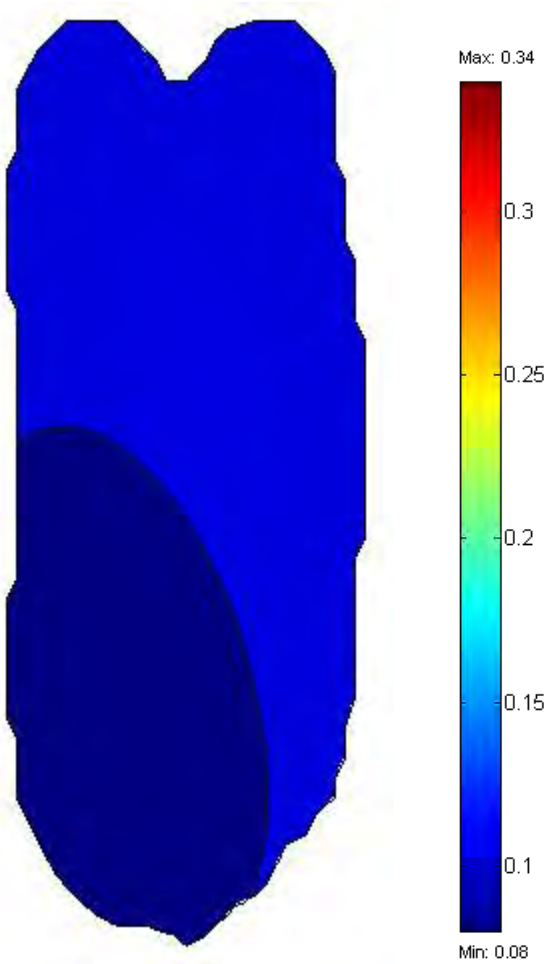

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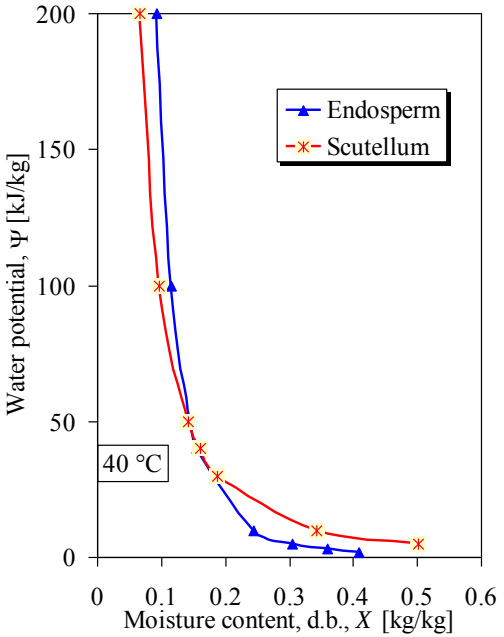

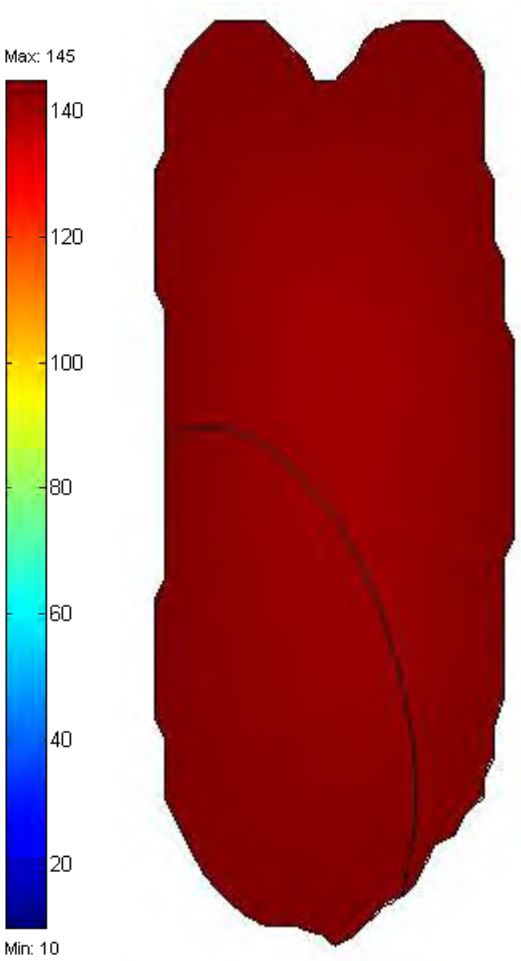

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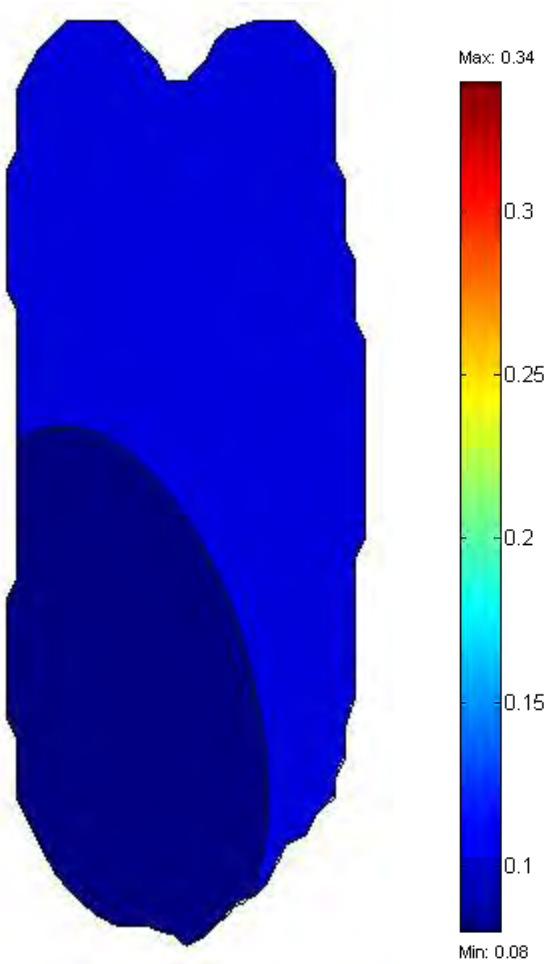

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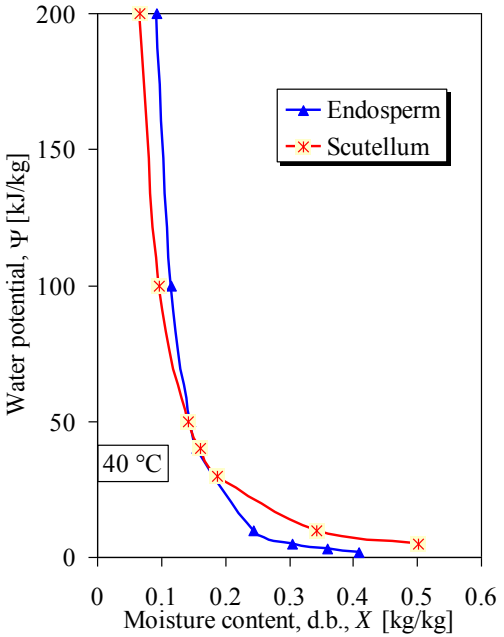

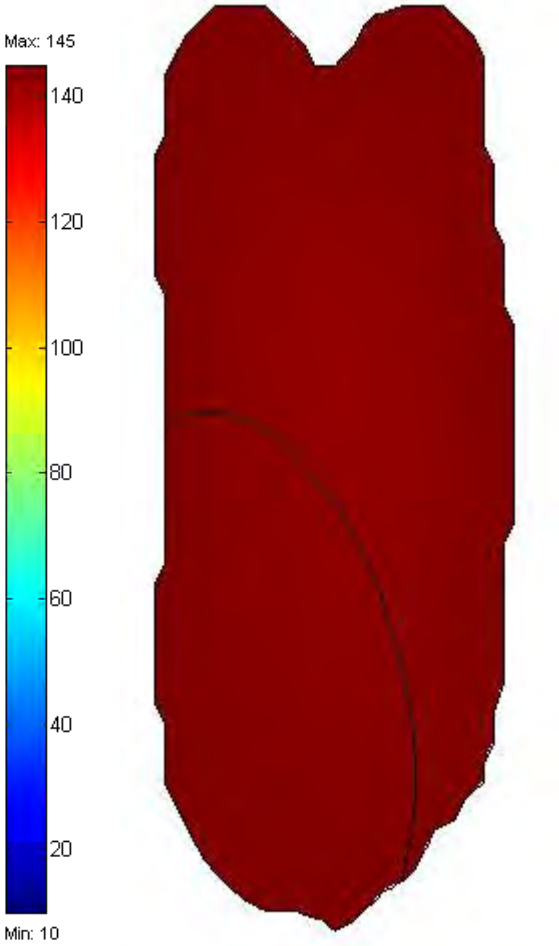

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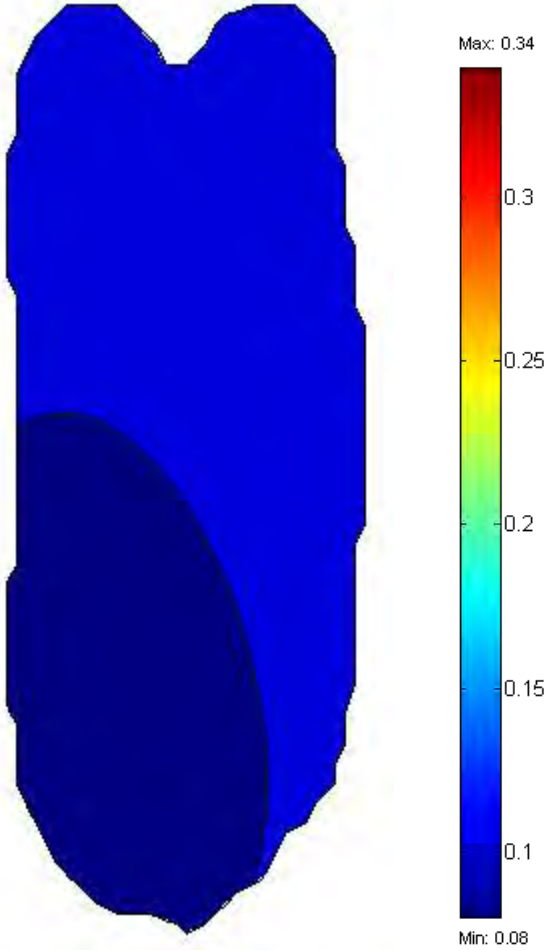

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CONCLUSIONS

- In the governing equation using water potential as the driving force has exact physical meaning, instead of the moisture gradient drying force.
- One multiphysical equation system gives more accurate solutions instead of iteration solutions.
- Further studies needs to be done to determine water potential above hygroscopic moisture content.

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THANK YOU FOR YOUR ATTENTION!

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