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# Modeling Failure Mechanisms in Sands under Extreme Loads Using COMSOL

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## Abstract:

Previous studies carried out by Nishaat (2009) and Nyamutale (2009) showed that Terzaghi's bearing capacity model did not adequately predict the bearing capacity failure in soils [1]. Nishaat carried out her investigation on Philippi Dune sands using a physical model that was built in a geotechnical laboratory. The failure surfaces she observed in the sands did not resemble those predicted by Terzaghi and Meyerhof's models. However the use of a physical model limited how many things she could observe like the failure mechanism, the exact extent of plastic deformation and the stress distribution. In this study a simulation of the laboratory experiments done by Nishaat was developed. The simulation had the same failure load, similar patterns of plastic deformation and illustrated a failure mechanism that was different from that predicted by Terzaghi's model.

**Keywords:** Bearing capacity failure, Philippi Dune sand, Terzaghi

## 1. Introduction

All engineered construction resting on the earth must be carried by a foundation which transmits to, and into, the underlying soil or rock the loads from the structure above [2]. A properly designed foundation should generate stresses in the ground that are within safe levels and the resulting settlement of the structure should be within acceptable limits. Karl Terzaghi (1883 – 1963), known as the father of soil mechanics, developed a theory that estimates the failure load of the soil [1]. He developed his theory using analytical models and by making assumptions about the behavior of the soil.

Previous research by Nishaat (2009), using a physical model (Figure 1), showed that Terzaghi and Meyerhof's theoretical models didn't resemble the failure surfaces that were observed during the laboratory tests (Figure 2). These models overestimated the extent of the shear planes and the lateral dimensions of the failure zone. However the use of a physical model made it difficult for Nishaat to observe the stress distributions, the failure mechanism and the exact extent of plastic deformation.

In this study a simulation of the laboratory tests done by Nishaat was developed to give further insight into the failure mechanisms of sands.

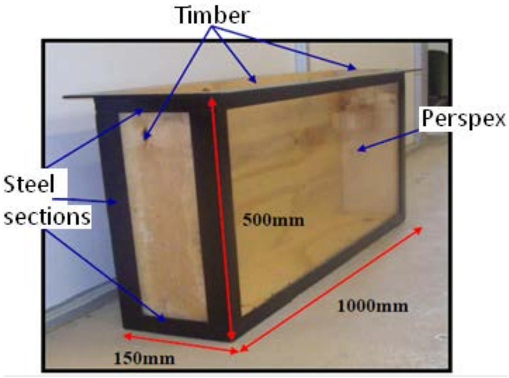

Figure 1: Loading box [1]

Figure 2: Shear failure in Philippi Dune sand [1]

## 2. The Physical model:

It comprised of; a loading box made from 20mm timber with a transparent 16mm Perspex front and angle steel sections as a frame for support. A Zwick Universal Tensile and Compression Machine, model 1474, was used to apply the load. The loading was stopped as soon as failure occurred in the soil. Steel base plates of dimensions 50x148x16mm and 100x148x16mm were used as footings. Sand paper with a roughness to match the roughness of the sand grains was glued to their bases to create a rough foundation.

## 3. Conceptual Model

It was assumed that the loading box used in the validation test was rigid enough to ensure that the displacements normal to the front and the rear faces were negligible hence plane strain conditions apply.

The wooden boards were assumed to act as rigid and rough surfaces to the sand grains hence the sides and the base of the model were considered to be fixed ends. The load was applied as a uniformly distributed force on the steel plate (figure 3) and for simplicity the friction between the steel plate and the sand was assumed to be negligible and soil weightless.

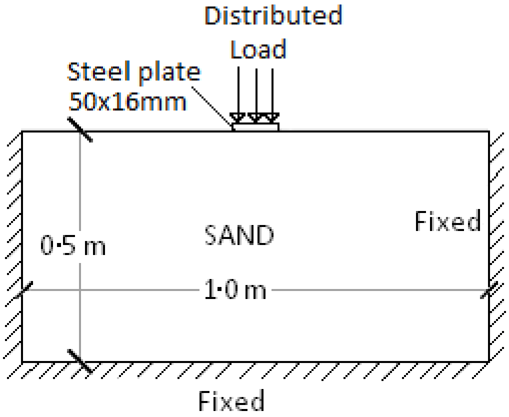

Figure 3: Load and boundary conditions used in COMSOL [3]

The properties of Philippi Dune sand tested with the 100mm long steel plate were used in the model to define the sand. These include: a peak angle of internal friction ($\phi_{peak}$) of 34°, Cohesion (c) of 6.7 KPa, Density (ρ) of 1.713 $Mg/m^3$, Poisson's ratio of 0.3 and Modulus of elasticity of 7.82e6 Pa. Poisson's ratio was estimated basing on the commonly used values of Poisson's ratio for sand i.e. 0.3 and 0.4 [2] and the modulus of elasticity was assumed to be equal to the shear modulus estimated from the shear box test results.

## 4. Constitutive model:

The soil was assumed to behave as an elastic perfectly plastic material [4] that failed according to the Drucker-Prager yield criterion. This yield criterion was chosen because it is a function of the mean stress on which yielding in frictional materials like soil is dependent. The yield criterion was given by [5]:

$$\mathrm{F} = 3\alpha\sigma_{\mathrm{m}} + \sigma_{\mathrm{eqv}} - \mathrm{K} \qquad (1)$$

Where the mean stress $\sigma_m$ was defined by:

$$\sigma_m = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) \qquad (2)$$

The equivalent deviatoric stress $\sigma_{eqv}$ was given by:

$$\sigma_{eqv} = \sqrt{\tfrac{1}{2}[P] + Q} \qquad (3)$$

Where;

$$P = {S_x}^2 + {S_y}^2 + {S_z}^2$$

$$Q = {S_{xy}}^2 + {S_{yz}}^2 + {S_{zx}}^2$$

$$S_x = \sigma_x - \sigma_m \qquad S_{xy} = \tau_{xy}$$

$$S_y = \sigma_y - \sigma_m \qquad S_{yz} = \tau_{yz}$$

$$S_z = \sigma_z - \sigma_m \qquad S_{zx} = \tau_{zx}$$

And the parameters $\alpha$ and $K$ are given by:

$$\alpha = \frac{\tan\phi}{\sqrt{(9+12\tan^2\emptyset)}} \qquad (4)$$

$$\mathrm{K} = \frac{3c}{\sqrt{(9+12\tan^2\emptyset)}} \qquad (5)$$

## 5. Modeling in COMSOL

The modeling was done in 2D using the Structural Mechanics Module-Static analysis elasto-plastic material. First the geometry was drawn and then the material properties, constraints and loads were specified. Under the elasto plastic material settings the perfectly plastic hardening model was chosen. The mesh was initialized, refined and a parametric solver was used to calculate the solution for every small increment in load till a maximum value.

## 6. Results

The COMSOL model simulated the bearing capacity failure of Philippi Dune sand by showing the development of plastic deformation below the footing. The region of soil that experienced plastic deformation below the footing in the simulation (Figure 4) had an elliptical shape similar to that observed in the validation test (Figure 5) and the maximum traction force below the edge of the footing at the point of failure was similar to the bearing capacity measured in the laboratory test carried out by Nishaat (Figure 6).

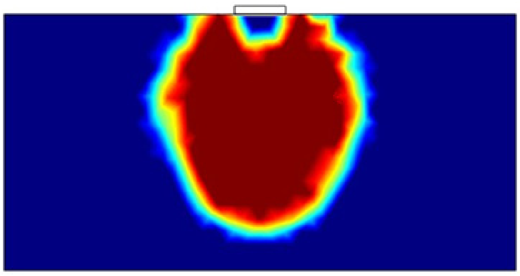

Figure 4: Plastically deformed soil below the footing

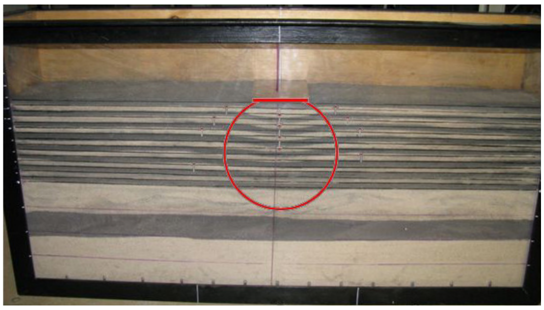

Figure 5: Plastically deformed soil in Philippi Dune sand [3]

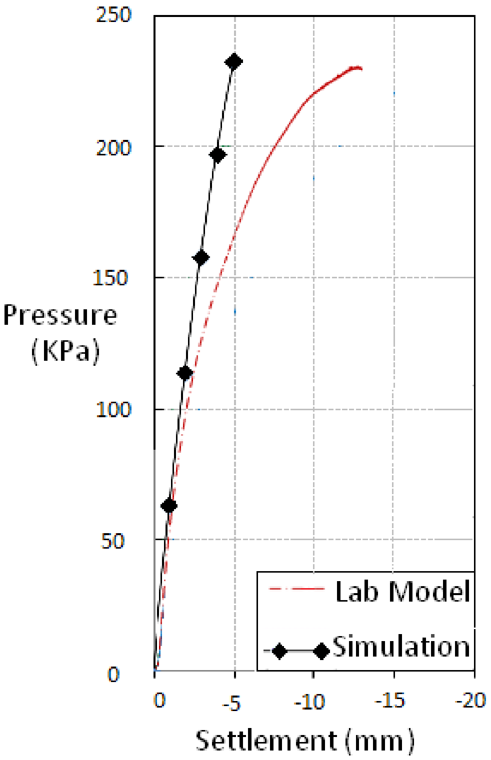

Figure 6: Applied Pressure against the Plate Settlement [3]

## 6.1 Failure Mechanism

Plastic deformation started at the corners of the footing (figure 7) and progressed downwards below along paths that were at approximately $45 + {}^{\varphi}/_{2}$ to the base of the footing. This formed a triangular region (Figure 8) that remained in elastic state similar to zone 1 in Terzaghi's model.

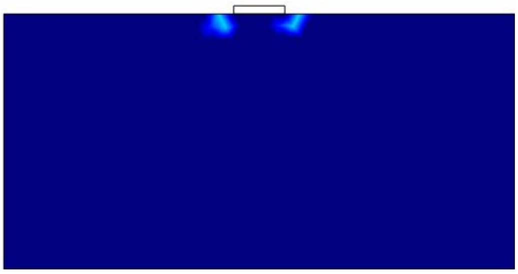

Figure 7: Plastic deformation at the edges of the footing [3]

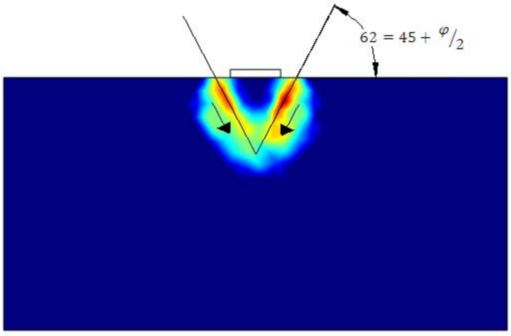

Figure 8: Triangular region in elastic state [3]

The edges of this triangle then expanded outwards into the soil forming a bulb that was an approximate ellipse (Figure 9)

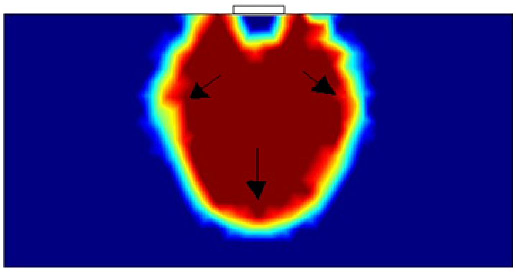

Figure 9: Development of the elliptical bulb [3]

But according to Terzaghi's bearing capacity theory the plastic flow occurs along log spirals [6] (Figure 10) however this was not observed in the simulation. This variation could possibly be the result of using the Drager-Prager yield criterion

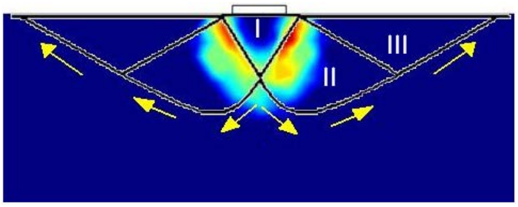

Figure 10: Plastic flow according to Terzhagi's bearing capacity theory [3]

## 7. Conclusions

The developed model simulated the bearing capacity failure in Phillipi dune sand by showing the region of plastically deformed soil below the footing and the maximum applied force at the point of failure. The failure mechanism of the simulation showed that Terzaghi's theory doesn't adequately predict the failure mechanism in Phillipi dune sand. This model can be used to predict the bearing capacity of different soils and their failure mechanisms under applied vertical loading.

## 8. References

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