Computational Acoustic Attenuation Performance of Helicoidal Resonators Comparable to Experiment

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Introduction

• Helicoidal resonator is a newly developed acoustic resonator near well-known Helmholtz resonator

Figure 1. Basic components of resonators as a part of simple acoustic system - cylindrical duct.

• As it can be seen in Figure 1 the main difference between both solutions is that the Helmholtz resonator is located outside the duct, and the helicoidal resonator stands inside the duct.
Introduction

• The main purpose of this work is to show computationally obtained the acoustic attenuation performance of helicoidal resonators which can be comparable to experimental results.

**Figure 4.** Cylindrical duct with helicoidal resonator inside.

• The most important question is:
How to simulate in computational environment the acoustic systems to obtain results, which can be comparable to experiment?

**Figure 5.** Parameters of helicoidal profile.

\[
\frac{s}{d} = 1.976 \quad \frac{d_m}{d} = 0.24 \quad \frac{g}{d} = 0.04
\]

\[n = 0.671, n = 0.695\]
Numerical environment

Finite Element Method (FEM) was used to solve three-dimensional (3D) numerical models in the frequency domain by the use of time-harmonic Pressure Acoustics application mode in a COMSOL Multiphysics - Acoustics Module computational application. The final solving parameter is the acoustic pressure $p$ [Pa], which can be computed by the use of slightly modified Helmholtz equation:

$$
\nabla \cdot \left( -\frac{\nabla p}{\rho_0} \right) - \frac{\omega^2 p}{c_s^2 \rho_0} = 0
$$

(1)

where $\rho_0$ is the density of air ($\rho_0=1,23$ kg/m$^3$), $c_s$ is the speed of sound in air ($c_s=343$ m/s), and $\omega$ gives the angular frequency.

For investigated models in this work the boundary conditions are of three types:

For **acoustically hard walls at the solid boundaries**, which are the walls of the cavity volume, neck, helicoidal profile, mandrel and circular duct, the model uses sound hard (wall) boundary conditions:

$$
\left( \frac{\nabla p}{\rho_0} \right) \cdot \mathbf{n} = 0
$$

(2)
Numerical environment

The boundary condition at the inlet surface (sound source) of circular duct is a combination of incoming and outgoing plane waves:

$$ \mathbf{n} \cdot \frac{1}{\rho_0} \nabla p + ik \frac{p}{\rho_0} + i \frac{\Delta_T p}{2k} = \left( \frac{i}{2k} \Delta_T p_0 + (1 - (\mathbf{k} \cdot \mathbf{n}))ik \frac{p_0}{\rho_0} \right) e^{-ik(\mathbf{k} \cdot \mathbf{r})} \quad (3) $$

where $\Delta_T$ denotes the boundary tangential Laplace operator, $k=\omega/c_s$ is the wave number, $\mathbf{n}$ is the normal direction vector for investigated circular duct, and wave vector is defined as $\mathbf{k}=\mathbf{n}_k$, where $\mathbf{n}_k$ is the wave-direction vector. In equation (3), $p_0$ represents the applied outer pressure, and $i$ denotes the imaginary unit. The inlet boundary condition is valid as long as the frequency is kept below the cutoff frequency for the second propagating mode in the cylindrical duct.
Numerical environment

Second type of the inlet boundary condition (sound source) of cylindrical duct is radiation condition of a spherical wave with an incoming plane wave included, as follows:

$$
\mathbf{n} \cdot \left( \frac{1}{\rho_0} \left( \nabla p - \mathbf{q} \right) \right) + \left( i k + \frac{1}{r} \right) \frac{p}{\rho_0} - \frac{r \Delta_T p}{2 \rho_0 (ikr + 1)} = 0
$$

$$
= \left( - \frac{r \Delta_T p}{2 \rho_0 (ikr + 1)} + \left( ik + \frac{1}{r} - i \mathbf{k} \cdot \mathbf{n} \right) \frac{p_0}{\rho_0} \right) e^{-i \mathbf{k} \cdot \mathbf{r}}
$$

which emanates from inlet circular boundary and allows a radiated wave to leave the modeling domain without reflections.

At the outlet boundary is set as the radiation boundary condition which allows an outgoing wave to leave the modeling domain with no or minimal reflections:

$$
\mathbf{n} \cdot \frac{1}{\rho_0} \nabla p + i \frac{k}{\rho_0} p + \frac{i}{2k} \Delta_T p = 0
$$
Performance parameters

As an acoustic attenuation performance parameter is used transmission loss (TL), given by:

\[ TL = 10 \log_{10} \left( \frac{w_i}{w_o} \right), \text{[dB]} \]  

(6)

where:

\[ w_o = \int_{\Omega} \frac{|p_c|^2}{2\rho_0 c_s} dS \]  

is the outgoing power at the outlet \( w_o \),

\[ w_i = \int_{\Omega} \frac{p_0^2}{2\rho_0 c_s} dS \]  

is the incoming power at the inlet \( w_i \).

Insertion Loss (IL), used both in computations and experiment, given by:

\[ IL = SPL_1 - SPL_2, \text{ [dB]} \]  

(7)

\( SPL_1 \) – the sound pressure level at the outlet of cylindrical duct without filter, [dB]  
\( SPL_2 \) – the sound pressure level at the outlet of cylindrical duct with filter, [dB].
Sound attenuation model – used in NM1

Well-known Delany and Bazley model: complex impedance $Z_c$ and wave number $k_c$

$$Z_c = \rho_0 c_s \left[ 1 + C_5 \left( \frac{\rho_0 f}{R_f} \right)^{-c_6} - iC_7 \left( \frac{\rho_0 f}{R_f} \right)^{-c_8} \right]$$

$$k_c = \frac{\omega}{c_s} \left[ 1 + C_1 \left( \frac{\rho_0 f}{R_f} \right)^{-c_2} - iC_3 \left( \frac{\rho_0 f}{R_f} \right)^{-c_4} \right]$$

(7)

$c_1=0,0978$, $c_2=0,7$, $c_3=0,189$, $c_4=0,595$, $c_5=0,0571$, $c_6=0,754$, $c_7=0,087$, $c_8=0,732$.

Bies and Hansen empirical correlation for flow resistivity $R_f$:

$$R_f = \frac{3,18 \cdot 10^{-9} \cdot \rho_{ap}^{1.53}}{d_{av}^2} \left[ \frac{Pa \cdot s}{m^2} \right] \text{ or } \left[ \frac{rayls}{m} \right]$$

(8)
Experimental set-up

Source signal was the white noise. Both helicoidal resonators were made by the use of a three dimensional rapid prototyping technique SLS. Thus, the shape of resonators is exactly the same as in the simulation. To measure sound pressure levels at the outlet of cylindrical ducts were used Brüel & Kjær’s platform PULSE for noise and vibration analysis equipped with Brüel & Kjær’s 1/2 inch acoustical microphone type 4190-C-001.

Figure 6. Example view on the experimental set up.

Figure 7. Schematic view of the experimental set up in the laboratory experiment.
Results

In case of experimental results that are represented only by insertion loss (IL), the research made by the use of numerical calculations in COMSOL Multiphysics are focused on achieving the best convergence.

Figure 8. Numerical model 1 (NM1) - 1,16m long pipe with helicoidal resonator surrounded at the inlet side in the distance of 16cm by an absorptive material.

Figure 9. Comparison of results for NM1 for helicoidal resonators with number of turns: a) n=0,671, b) n=0,695.
Results

In case of experimental results that are represented only by insertion loss (IL), the research made by the use of numerical calculations in COMSOL Multiphysics are focused on achieving the best convergence.

Figure 10. Numerical model 2 (NM2) - 1,16m long pipe with helicoidal resonator and 1m cubic box.

Figure 11. Comparison of results for NM2 for helicoidal resonators with number of turns: a) n=0,671, b) n=0,695.
Results

In case of experimental results that are represented only by insertion loss (IL), the research made by the use of numerical calculations in COMSOL Multiphysics are focused on achieving the best convergence.

Figure 12. Numerical model 3 (NM3) - 1m long pipe with helicoidal resonator and 1m cubic box.

Figure 13. Comparison of results for NM3 for helicoidal resonators with number of turns: a) n=0.671, b) n=0.695.
Conclusions

As it can be observed from presented results, the best convergence between experimental and numerical ILs is achieved by modeling just 1m long pipe with helicoidal resonator inside and inlet boundary condition set as spherical wave. This numerical acoustic system geometrically differs a lot from experimental set up.

It was also observed that in all cases of numerical models (from NM1 to NM3) the characteristic for presented helicoidal resonators sound attenuation frequency range between about 1150Hz to about 1340Hz can be simply identified. But the boundary conditions play a key role when identifying lower frequency range of acoustic characteristics.

This study shows that modeling acoustic systems with helicoidal resonators in COMSOL Multiphysics don't need to include all of experimentally used environment to achieve similar results, but the boundary conditions should be properly used. Also it is a great knowledge to know which exact numerical model should be used to achieve results comparable to experiment.
Thank You for Your kind attention...

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